Impact of Dispersion Fluctuations on Dual-Pump Fiber-Optic Parametric Amplifiers

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***Abstract*—Effects of zero-dispersion wavelength (ZDWL) fluctuations on dual-pump fiber-optic parametric amplifiers are investigated analytically and numerically. It is found that the signal gain varies considerably from fiber to fiber even though each fiber may have the same ZDWL on average. Moreover, the gain spectrum becomes highly nonuniform for a given fiber because of such dispersion fluctuations. Numerical simulations show that this problem can be solved to a large extent by reducing wavelength separation between the two pumps, but only at the expense of a reduced amplifier bandwidth.**

***Index Terms*—Dispersion fluctuations, four-wave mixing (FWM), parametric amplification.**

FIBER-OPTIC parametric amplifiers (FOPAs), based on four-wave mixing (FWM), can provide uniform gain over a relatively wide bandwidth when they are pumped at two wavelengths located on each side of the zero-dispersion wavelength (ZDWL) of the fiber [1]–[5]. However, the core diameter of any fiber exhibits random variations related to manufacturing, resulting in a randomly varying ZDWL along the fiber. Since FWM is sensitive to phase mismatch among the interacting waves, it is important to know how ZDWL variations affect the FOPA performance. This issue was addressed in [6] for single-pump FOPAs. In this letter, we study the effects of ZDWL fluctuations on dual-pump FOPAs and show that the amount as well as the uniformity of gain reduces considerably because of ZDWL fluctuations. We also show that the problem can be solved to a large extent by reducing the wavelength separation between the two pumps at the expense of a reduced gain bandwidth.

Although a complete description of dual-pump FOPA is quite complicated [4], the uniform portion of the gain spectrum actually stems from the nondegenerate FWM process for which $\omega_1+\omega_2=\omega_3+\omega_4$, where ω_j (j = 1–4) are the frequencies of the two pumps, signal, and idler waves, respectively. We focus on this process and assume that the pump powers P_1 and P_2 are not depleted. The evolution of the signal and idler waves is then governed by the following two equations [7]:

$$\frac{dB_3}{dz}=\frac{i}{2}\kappa B_3+2i\gamma\sqrt{P_1P_2}B_4^* \tag{1}$$

$$\frac{dB_4}{dz}=\frac{i}{2}\kappa B_4+2i\gamma\sqrt{P_1P_2}B_3^* \tag{2}$$

where γ is the nonlinear parameter, $\kappa=\Delta\beta+\gamma(P_1+P_2)$ describes the total phase mismatch, and $\Delta\beta=\beta(\omega_3)+\beta(\omega_4)-\beta(\omega_1)-\beta(\omega_2)$ is the wave-vector mismatch related to dispersion parameters as

$$\Delta\beta\approx\beta_{2c}\left[(\omega_3-\omega_c)^2-\omega_d^2\right]+\beta_{4c}\left[(\omega_3-\omega_c)^4-\omega_d^4\right]/12 \tag{3}$$

where $\omega_c=(\omega_1+\omega_2)/2$ is the center frequency of the two pumps and $\omega_d=(\omega_1-\omega_2)/2$ is half their frequency difference. The parameters β_{2c} and β_{4c} govern the second- and fourth-order dispersion at the center frequency ω_c. They are related to the third- and fourth-order dispersion at the fiber's ZDWL λ_0 as $\beta_{2c}\approx\beta_3(\omega_c-\omega_0)+\beta_4(\omega_c-\omega_0)^2/2$ and $\beta_{4c}\approx\beta_4$, where $\omega_0=2\pi c/\lambda_0$.

When the ZDWL is constant along the fiber, a broad gain spectrum is obtained by optimizing FOPAs such that κ remains close to zero over a relatively broad spectral range. Random variations in the ZDWL cause β_{2c} to vary randomly along the fiber, and it becomes difficult to maintain $\kappa=0$. As a result, the FOPA gain spectrum becomes considerably nonuniform even if the fiber is otherwise perfect. As FWM is sensitive to local phase mismatch, optimization of other design parameters (such as average ZDWL, pump powers and wavelengths, strength of nonlinearity, etc.) does not guarantee a uniform and wide gain spectrum when ZDWL varies randomly along the fiber even by a small amount $\sim$1 nm.

Mathematically, random ZDWL variations along the fiber render κ random and transform (1) and (2) into two stochastic differential equations with multiplicative noise whose solution generally requires a numerical approach. Each realization of the random process corresponds to one fiber with a specific form of ZDWL variations. FOPAs made using different fiber pieces from the same spool would exhibit different gain spectra because each corresponds to a different realization of the stochastic process. Such fiber-to-fiber variations in the gain spectra can be predicted by solving (1) and (2) numerically. We consider a FOPA made using 500 m of high-nonlinearity fiber (γ = 10 W^{-1}/km) for which the average ZDWL is $\bar{\lambda}_0$ = 1550 nm with dispersion parameters β_3 = 100 ps^3/m and β_4 = 0.1 ps^4/m. The FOPA is assumed to be pumped at 1502.6 and 1600.6 nm with a power of 0.5 W at each wavelength. We assume that ZDWL fluctuations follow a Gaussian distribution with a standard deviation σ_λ = 1 nm and a correlation length l_c = 5 m. Fig. 1 shows the gain spectra for 100 realizations of the random process. It is evident that amplified signal

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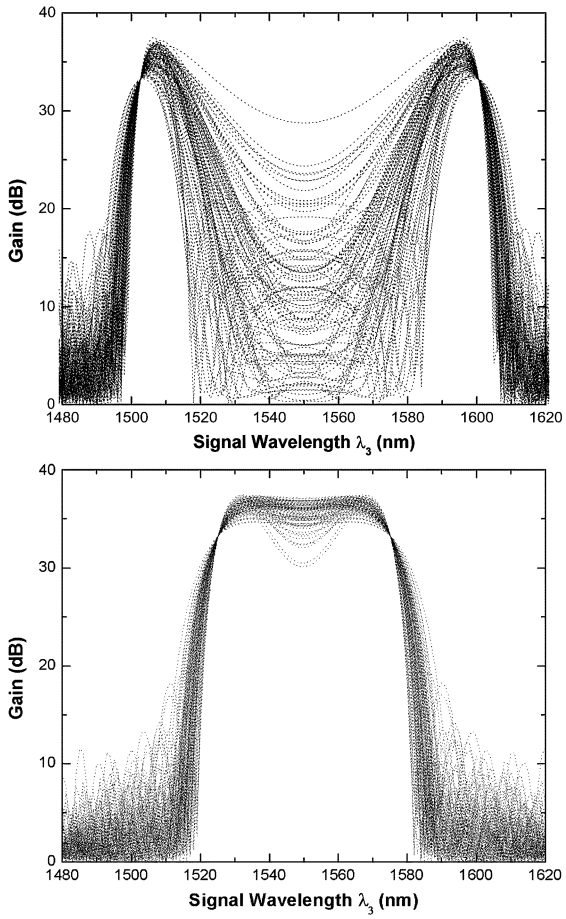

Fig. 1. Fiber-to-fiber variations in the FOPA gain spectra caused by random variations in ZDWL along the fiber for the pump wavelength separation of 98 (top) and 50 nm (bottom). FOPA parameters are given in the text.

can fluctuate over a wide range for different members of the ensemble even when $\sigma_\lambda = 1$ nm. Such fluctuations indicate that it is not easy to predict the gain profile based only on the average value of ZDWL of the fiber. Moreover, the gain spectrum is highly nonuniform for any single fiber. Such large variations would be unacceptable in practice.

The important question is how FOPAs can be used in practice in spite of ZDWL variations. We have found that the impact of random ZDWL variations can be mitigated significantly by reducing the wavelength separation between the two pumps and optimizing the average frequency ω_c. Fig. 1 shows the improvement realized by changing the pump wavelengths to 1525.12 and 1575.12 nm so that the two pumps are only 50 nm apart. With this choice, the signal power fluctuates over a much reduced range. More importantly, the gain spectrum is uniform to within a few decibels for each fiber over a 40-nm region between two pumps. A closer look at (3) reveals why it is necessary to reduce the gain bandwidth to retain the gain uniformity. In this equation, ZDWL appears only through β_{2c}. When the signal wavelength is close to a pump wavelength, $\Delta\beta$ nearly vanishes regardless of the ZDWL. However, as signal wavelength moves away, fluctuations in $\Delta\beta$ increase and become maximum for $\omega_3 = \omega_c$. Reducing the pump separation guarantees that fluctuations in $\Delta\beta$ and the parametric gain can be kept below a reasonable limit. The same reasoning applies to single-pump FOPAs but $\Delta\beta$ is obtained from (3) after setting $\omega_1 = \omega_2 = \omega_c$. However, single-pump FOPA gain cannot be made as uniform as that for dual-pump FOPAs.

A simple way to quantify the FOPA performance is to calculate the *average* gain by averaging the signal gain, defined as $G(\omega_3) = P_3(L)/P_3(0)$, where $P_3 = |B_3|^2$ and L is the FOPA length, over random ZDWL variations. Although such an "averaged" gain spectrum does not correspond to any real FOPA, it provides a good indication of the impact of ZDWL variations on FOPA performance. The average gain $G_{\rm av}$ can be calculated analytically when the correlation length l_c is much smaller than the FOPA length [6]. In the following, we derive an expression for $G_{\rm av}$ and use it to optimize the FOPA design. We first expand $\Delta\beta$ in a Taylor series as $\Delta\beta = b_a + b\delta\lambda_0$, where $b_a = \langle\Delta\beta\rangle$ and $b = \langle d\Delta\beta/d\lambda_0\rangle$. The random variable $\delta\lambda_0$ represents fluctuations in the ZDWL. It can be modeled as a Gaussian stochastic process whose first and second moments are given by

$$\langle\delta\lambda_0\rangle = 0, \qquad \langle\delta\lambda_0(z)\delta\lambda_0(z')\rangle = 2D_\lambda\delta(z - z') \tag{4}$$

where $D_\lambda = \sigma_\lambda^2 l_c/2$ is the diffusion coefficient and σ_λ is the standard deviation of ZDWL fluctuations.

From (1) and (2), the signal power is found to evolve as $dP_3/dz = 4\gamma\sqrt{P_1P_2}\,\mathrm{Im}(B_3B_4)$. The same equations can also be used to find an equation for the product B_3B_4. Averaging these two equations over ZDWL fluctuations [8], we obtain the following set of three linear equations

$$\frac{d\langle P_3\rangle}{dz} = g\Gamma_i, \qquad \frac{d\Gamma_r}{dz} = -D_\lambda b^2\Gamma_r - \kappa_a\Gamma_i \tag{5}$$

$$\frac{d\Gamma_i}{dz} = g\langle P_3\rangle - \frac{g}{2}P_3(0) - D_\lambda b^2\Gamma_i + \kappa_a\Gamma_r \tag{6}$$

where $g = 4\gamma\sqrt{P_1P_2}$ is the gain coefficient and $\kappa_a = b_a + \gamma(P_1 + P_2)$ represents the average phase-mismatch. We also introduced the correlation between the signal and idler fields as $\Gamma = \langle B_3B_4\rangle$ and used $\Gamma = \Gamma_r + i\Gamma_i$. The average gain $G_{\rm av} = \langle P_3(L)\rangle/P_3(0)$ can be obtained by solving the linear equations (5) and (6) and is given by

$$G_{\rm av} = \frac{1}{2}\left[\sum_{i=1}^{3}\frac{(g^2 + a_ja_k)e^{a_iL}}{(a_i - a_j)(a_i - a_k)} + 1\right] \tag{7}$$

where $i \neq j \neq k$ and a_i are the roots of the cubic polynomial $a^3 + 2(D_\lambda b^2)a^2 + (D_\lambda^2 b^4 + \kappa_a^2 - g^2)a - D_\lambda(bg)^2$.

Fig. 2 shows the "average" gain spectra obtained analytically (solid curves) and numerically (dashed curves) for the same FOPA used for Fig. 1 for $l_c = 5$ and 50 m. In the case of a constant ZDWL (dotted curve), the gain spectrum is flat over an 80-nm bandwidth. However, variations in the ZDWL of even ± 1 nm deteriorate the flat region of the gain spectrum severely. For $l_c = 5$ m, the average gain is reduced from 38 to 18 dB in the center and varies by as much as 20 dB over the central region. For $l_c = 50$ m, a flat region reappears, but the gain is reduced from 38 to 7 dB. When $l_c = 50$ m, ZDWL fluctuations cannot be assumed to be delta-correlated. This is the reason behind the discrepancy between the analytical and numerical curves for $l_c = 50$ m.

The innermost solid (analytical) and dashed (numerical) curves in Fig. 2 show the average gain for the same FOPA

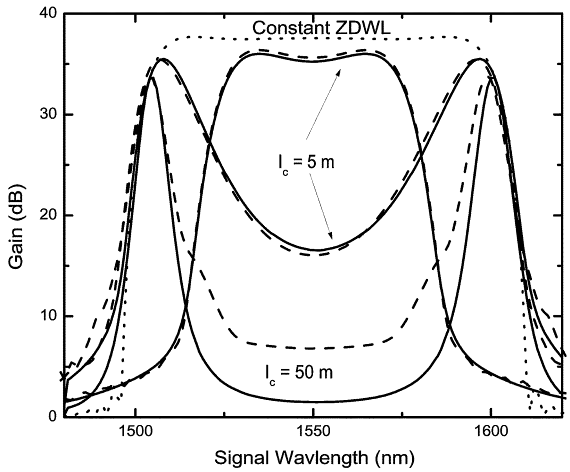

Fig. 2. Average FOPA gain spectra in the case of 98-nm pump separation for $l_c = 5$ and 50 m. Solid curves show the analytical prediction. Dashed curves represent an average of 100 gain spectra. The dotted curve shows the expected gain in the absence of dispersion fluctuations. The innermost curves show the average gain when pump separation is reduced to 50 nm using $\sigma_\lambda = 1$ nm and $l_c = 5$ m.

when pump separation is reduced from 98 to 50 nm, The pump wavelengths were optimized to make the gain curve as flat as possible. For this choice of pump spacing, the average gain remains nearly uniform although its bandwidth is reduced to around 40 nm and the amount of gain is reduced by about 2 dB compared with the case of a constant ZDWL. Noting from Fig. 1 that the gain fluctuations are much smaller for 50-nm pump spacing, we conclude that the flatness of the average gain and the variance of gain fluctuations are related, and flatness of $G_{\rm av}$ is a desirable design goal. We have verified this relationship through extensive numerical simulations. Our main conclusion is that the effects of ZDWL variations can be mitigated significantly by reducing the pump separation in the 40–50-nm range. This conclusion is consistent with the recent experiments in which pump separation was chosen to be in this range [4], [5]. The reduced pump spacing lowers the usable FOPA bandwidth but makes its performance relatively immune to ZDWL variations.

As a further guide to FOPA design, we introduce a measure of the flatness through a "degree of flatness" defined as $F = G_{\min}/G_{\max}$, where $G_{\min}$ and $G_{\max}$ are the minimum and maximum values of the average gain in the spectral region between the two pumps. Fig. 3 shows how F varies as a function of pump-wavelength separation for different levels of ZDWL fluctuations quantified through the standard deviation σ_λ. For each point in the curves, the center pump frequency ω_c was also optimized. Other parameters of FOPA were the same as those used in Fig. 2. As seen in Fig. 3, for any value of σ_λ, it is possible to retain the flatness of the average gain spectrum as long as the pump separation is reduced below a critical value. This critical value depends on the level of ZDWL fluctuations. In particular, pump separation becomes increasingly smaller as σ_λ becomes larger to maintain the same degree of flatness. For $\sigma_\lambda = 1$ nm, the maximum tolerable pump separation is about 50 nm.

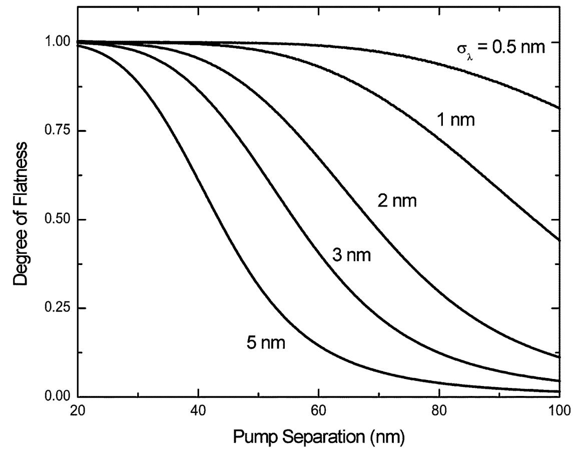

Fig. 3. Degree of flatness plotted as a function of separation between pump wavelengths for several values of $\sigma_\lambda \cdot l_c = 5$m.

In conclusion, random variations of the ZDWL constitute a major limiting factor for modern FOPAs. We have investigated their influence on the performance of dual-pump FOPAs both analytically and numerically. Gain spectrum becomes highly nonuniform for a given FOPA because of such dispersion fluctuations. This problem can be solved to a large extent by reducing wavelength separation between the two pumps but at the expense of a reduced gain bandwidth. Our analytical theory shows that the maximum tolerable pump separation depends on the level of ZDWL variations within the fiber and is about 50 nm for a standard deviation of 1 nm. In most recent experiments, pumps were indeed kept 40 to 50 nm apart [5], [9].

References

[1] K. Inoue, "Polarization independent wavelength conversion using fiber four-wave mixing with two orthogonal pump lights of different frequencies," *J. Lightwave Technol.*, vol. 12, pp. 1916–1920, Nov. 1994.

[2] P. O. Hedekvist, M. Karlsson, and P. A. Andrskson, "Polarization dependence and efficiency in a fiber four-wave mixing phase conjugator with orthogonal pump waves," *IEEE Photon. Technol. Lett.*, vol. 8, pp. 776–778, June 1996.

[3] K. K. Y. Wong, M. E. Marhic, K. Uesaka, and L. G. Kazovsky, "Polarization-independent two-pump fiber optical parametric amplifier," *IEEE Photon. Technol. Lett.*, vol. 14, pp. 911–913, July 2002.

[4] S. Radic and C. J. McKinstrie, "Two-pump fiber parametric amplifiers," *Opt. Fiber Technol.*, vol. 9, pp. 7–23, 2003.

[5] T. Tanemura and K. Kikuchi, "Polarization-independent broad-band wavelength conversion using two-pump fiber optical parametric amplification without idler spectral broadening," *IEEE Photon. Technol. Lett.*, vol. 15, pp. 1573–1575, Nov. 2003.

[6] M. Karlsson, "Four-wave mixing in fibers with randomly varying zero-dispersion wavelength," *J. Opt. Soc. Amer. B*, vol. 15, pp. 2269–2275, 1998.

[7] G. P. Agrawal, *Nonlinear Fiber Optics*, 3rd ed. San Diego, CA: Academic, 2001, ch. 10.

[8] C. W. Gardiner, *Handbook of Stochastic Methods*, 2nd ed. New York: Springer-Verlag, 1985, ch. 4.

[9] S. Radic, C. J. McKinstrie, R. M. Jopson, J. C. Centanni, Q. Lin, and G. P. Agrawal, "Record performance of parametric amplifier constructed with highly nonlinear fiber," *Electron. Lett.*, vol. 39, pp. 838–839, 2003.