# Effects of Polarization-Mode Dispersion in Dual-Pump Fiber-Optic Parametric Amplifiers

F. Yaman, Q. Lin, and Govind P. Agrawal

***Abstract*—Effects of polarization-mode dispersion (PMD) on dual-pump parametric amplifiers are investigated numerically using a set of four vector equations. It is found that PMD induces large fluctuations in the signal power that can affect the system performance by enhancing the outage probability. The average gain itself is reduced by more than 10 dB even for a relatively small value of 0.05 ps/$\sqrt{\text{km}}$ for the PMD parameter. For larger values of the PMD parameter, the gain spectrum begins to distort and loses its flatness. We also show that the polarization dependence of gain cannot be eliminated by using orthogonally polarized pumps.**

***Index Terms*—Fiber parametric amplifier, polarization-dependent gain, polarization-mode dispersion (PMD).**

FIBER-optic parametric amplifiers (FOPAs) not only are useful for wavelength conversion but they can also offer a large, flat gain over a wide bandwidth when designed suitably [1]–[6]. The underlying mechanism is the nonlinear phenomenon of four-wave mixing (FWM) in which two photons from a single pump or two pumps interact with a signal photon to create a fourth photon at the idler frequency in such a way that the total energy and momentum is conserved [1]. The strict phase-matching condition required for FWM by the conservation of momentum and the anisotropic nature of the third-order nonlinearity make the FOPA susceptible to fiber imperfections. One such fiber imperfection, which is the focus of this work, is randomly fluctuating residual birefringence ever present in fibers longer than a few meters. This imperfection leads to polarization-mode dispersion (PMD), a phenomenon that has been studied extensively in recent years [7] as it limits the performance of high-capacity lightwave systems. In the context of FWM, it introduces residual phase mismatch and changes the state of polarization (SOP) of all optical fields randomly. It has been shown experimentally that PMD not only reduces the peak gain but also makes the FOPA gain to depend on the input signal SOP [6].

In a recent study, an analytic vector theory was developed to investigate the PMD effects in FOPAs pumped at a single wavelength [8]. However, in practice, dual-pump FOPAs are preferred since they provide a uniform gain over a much larger bandwidth [4]–[6]. A dual-pump scheme has also been suggested as a way to eliminate the dependence of gain on the initial SOP of the signal [9], [10]. It is difficult to extend the analytic vector theory developed for single-pump FOPAs to the case of dual-wavelength pumping because of the complexity of the problem. For this reason, we employ numerical simulations in this letter to study the impact of PMD on the performance of such amplifiers. Two separate issues are investigated. First, how much the overall gain and its flatness are degraded because of PMD; second, whether it is possible to make FOPAs in such a way that gain is independent of the signal SOP.

The vector equations governing the evolution of the pump, signal, and idler fields can be derived using the general form of third-order nonlinear polarization for silica glass [1] provided a number of assumptions and simplifications are made. In the following analysis, we neglect the effects of stimulated Raman scattering and assume that all optical fields participating in the FWM process can be treated as quasi-monochromatic. We also neglect the FWM effects arising from a signal interacting with each pump separately (degenerate FWM). This is a reasonable assumption when the two pump wavelengths are located on opposite sides of the zero-dispersion wavelength $\lambda_0$ and are relatively far apart. Degenerate FWM then contributes only in a narrow spectral region around the pumps, and it does not affect the flat portion of the gain spectrum between the two pumps which is of primary interest in most experimental situations. This allows us to consider a single idler field generated through nondegenerate FWM. The pump powers are assumed to be high enough that they are not depleted by the FWM-induced power transfer. At the same time, the signal and idler are assumed to be weak enough that self- and cross-phase modulation induced by them are negligible. Finally, fiber lengths used for making FOPAs are typically under 2 km, allowing us to neglect fiber losses for all four waves.

With these simplifications and adopting the Jones vector notation of [7], the vector equations governing the FWM process can be written in the following form:

$$\begin{aligned}\frac{\partial|A_j\rangle}{\partial z} =& [(b(z,\omega_j)\sigma_1+\beta_j]|A_j\rangle \\ &+\frac{i\gamma}{3}\Big(|A_j^*\rangle\langle A_j^*|+2|A_k\rangle\langle A_k|+2\,|A_k^*\rangle\langle A_k^*|\Big)|A_j\rangle \quad (1)\end{aligned}$$

$$\begin{aligned}\frac{\partial|A_l\rangle}{\partial z} =& [(b(z,\omega_l)\sigma_1+\beta_l]|A_l\rangle \\ &+\frac{2i\gamma}{3}\left[\sum_{h=1}^{2}|A_h\rangle\langle A_h|+|A_h^*\rangle\langle A_h^*|\right]|A_l\rangle \\ &+\frac{2i\gamma}{3}\Big(|A_1\rangle\langle A_2^*|+|A_2\rangle\langle A_1^*|+\langle A_1^*|A_2\rangle\Big)|A_m^*\rangle \quad (2)\end{aligned}$$

where $j,k=1,2\,(j\neq k)$ denotes two pumps and $l,m=3,4\,(l\neq m)$ denotes the signal and idler. $|A_j\rangle$ is the Jones vector for the field at frequency $\omega_j$ propagating with the average propagation constant $\beta_j$. The quantity

Manuscript received June 13, 2003; revised July 31, 2003. This work is supported by the National Science Foundation under Grants ECS-0320816 and DMS-0073923.

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Digital Object Identifier 10.1109/LPT.2003.821055

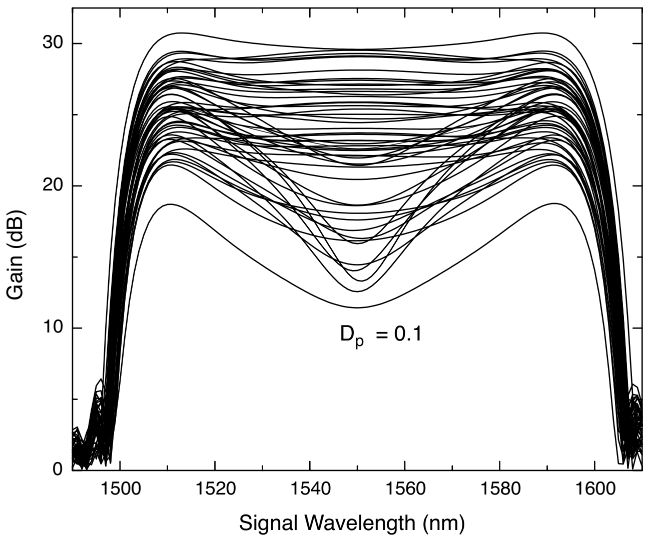


Fig. 1. Changes in gain spectra with birefringence fluctuations for a dual-pump FOPA for $D_p = 0.1$ ps/$\sqrt{\text{km}}$. Other parameters are given in the text. Both pumps and signal are copolarized initially.

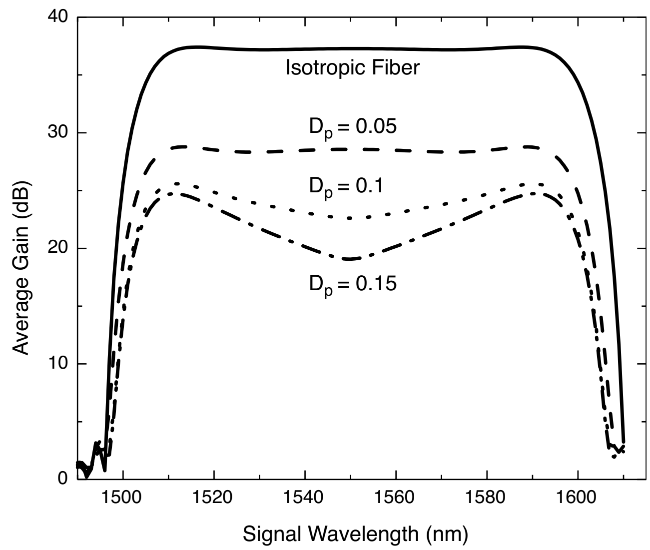


Fig. 2. Average gain spectra for three values of PMD parameters for the same parameter values used in Fig. 1. For comparison, the solid curve shows the case without birefringence.

$b(z,\omega_j) = \omega_j \delta n(z,\omega_j)/c$, where $\delta n(z,\omega_j)$ represents birefringence, can be Taylor expanded around the pump frequency $\omega_1$ as $b(z,\omega_j) \approx b_0(z) + b_1(z)(\omega_j - \omega_1)$. $\sigma_1$ is the diagonal Pauli matrix with components 1 and $-1$ [7]. In (1) and (2), a common phase factor $\exp(2i\gamma P_t z/3)$ was removed for simplicity, where $P_t$ is the total pump power.

For numerical simulations, we divide the fiber into many sections such that the amount of birefringence and the direction of principle axes are fixed in each section but vary from section to section [11]. Each section was taken to be 10 m long. As the correlation length of birefringence fluctuations is typically $\sim$10 m, our numerical results should mimic the expected behavior. We assume that $b_0$ follows Gaussian statistics with zero mean and a standard deviation of 0.4 $\text{m}^{-1}$ ($\delta n \approx 10^{-7}$). The random variable $b_1$ also follows Gaussian statistics but its variance is related to $D_p$ as $\langle b_1^2 \rangle = D_p^2/l_c$. The correlation length is equal to the length of each section ($l_c = 10$ m). To model a realistic FOPA, parameters are chosen so that they represent a typical experimental situation as close as possible. More specifically, we consider a 500-m-long high-nonlinearity fiber with $\gamma = 10$ $\text{W}^{-1}$/km pumped with two lasers, each providing 0.5 W of power. The fiber is assumed to have its zero-dispersion wavelength at $\lambda_0 = 1.55$ $\mu$m with $\beta_3 = 0.1$ $\text{ps}^3$/km and $\beta_4 = 10^{-4}$ $\text{ps}^4$/km. The initial SOP of all fields is chosen to be linear and parallel. The pump wavelengths are chosen to obtain the most uniform gain profile in the absence of PMD and have values $\lambda_1 = 1600.6$ nm and $\lambda_2 = 1502.6$ nm.

We first use numerical simulations to investigate the degree that the gain spectrum of a FOPA is degraded by PMD. For this purpose, we solve (1) and (2) and calculate the FOPA gain for three different values of $D_p$ using $G = \langle A_3(L)|A_3(L)\rangle / \langle A_3(0)|A_3(0)\rangle$ and vary the signal wavelength in the range 1.5–1.6 $\mu$m. Because of PMD, $G$ varies over a wide range for each realization of the stochastic process. Fig. 1 shows the range of gain fluctuations for $D_p = 0.1$ ps/$\sqrt{\text{km}}$. We find the average gain after averaging over 50 realizations for each set of parameters. The results are shown in Fig. 2. The ideal case of an isotropic fiber is also shown for comparison (solid curve). PMD reduces the average gain considerably and degrades the flatness of the gain spectrum appreciably for $D_p > 0.1$ ps/$\sqrt{\text{km}}$. For a relatively low-PMD fiber with $D_p = 0.05$ ps/$\sqrt{\text{km}}$, the average gain is reduced by 10 dB but the spectrum remains relatively flat.

To understand the PMD-induced degradation seen in Fig. 2 in more physical terms, we note that random birefringence in fibers consists of a detuning independent part ($b_0$) and a part that depends linearly on frequency detuning from the first pump ($b_1$) as long as higher order PMD effects are negligible. These two parts affect the FOPA in different ways. The detuning-independent part changes the SOP of all fields identically. Thus, in the absence of frequency-dependent part, if all fields start parallel, they will remain parallel along the fiber but their SOP will change randomly. Since the beat length and correlation length of birefringence are much smaller compared with the nonlinear length and the length of FOPA, the detuning-independent fast randomization of SOPs averages the nonlinear effects and reduces the nonlinear parameter $\gamma$ by a factor of $8/9$ [11], leading to a reduction of FOPA gain. For the specific example of Fig. 2, it reduces the peak gain from 37 to 33 dB. Any further reduction from 33 dB represents the contribution of frequency-dependent variations in SOPs and can be attributed to PMD, because of which the four fields no longer retain the initial parallel configuration of their SOPs.

For two optical fields separated in frequency by $\Delta\omega$, the depolarization effects can be characterized by a diffusion length $L_d$ defined as $L_d = 3/(D_p \Delta\omega)^2$ [12]. For $D_p = 0.05$ ps/$\sqrt{\text{km}}$, the diffusion length for the two pumps is $\sim$200 m but it reduces to 50 m for $D_p = 0.1$ ps/$\sqrt{\text{km}}$. The reason why gain spectrum remains relatively flat in Fig. 2 for low PMD values is that the four fields keep their original parallel configuration for a considerable portion of the fiber. For larger values of $D_p$, a dip begins to form at the center of the gain spectrum. Since the center of

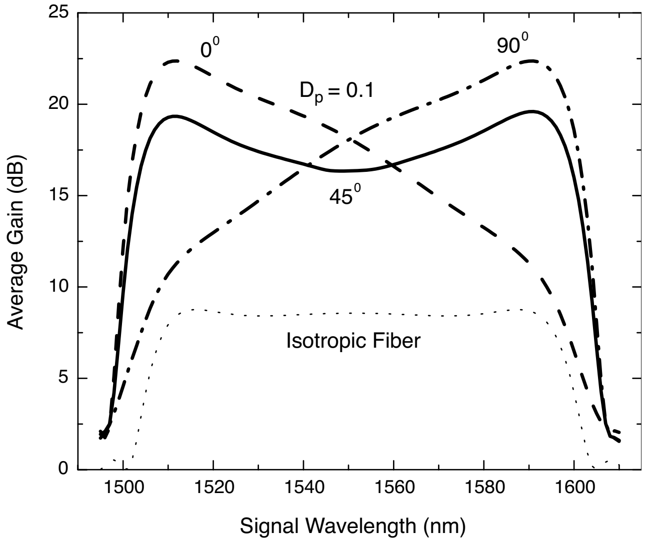


Fig. 3. Average gain versus signal wavelength for three different initial linear SOP of the signal for $D_p = 0.1$ ps/$\sqrt{\text{km}}$; $\theta$ represents the angle in between the linear SOPS of signal and shorter wavelength pump. The other pump is orthogonally polarized. The dotted curve shows, for comparison, the case without birefringence.

the spectrum corresponds to a signal frequency that is the farthest from both pumps, the signal looses its correlation with both pumps faster and thus experiences less gain. The dip becomes deeper as $D_p$ increases. The diffusion length $L_d$ for signal is 780, 200, and 90 m for $D_p$ =0.05, 0.1, and 0.15 ps/$\sqrt{\text{km}}$, respectively. In short, the gain spectrum retain its flatness whenever diffusion length is larger than fiber length. For large values of $D_p$, the diffusion length becomes considerably shorter than fiber length, and the spectrum degrades.

In Figs. 1 and 2, the input SOP of the signal was kept fixed. In many system applications, it is not possible to control the SOP of the incoming signal. Since the FWM efficiency depends on the relative orientations among two pumps and the signal, fluctuations in the input signal SOP become another source of noise through polarization-dependent gain (PDG). In the case of dual-wavelength pumping, the use of linearly but orthogonally polarized pumps with equal powers can eliminate this problem [9], [10]. However, this scheme relies critically on the assumption that once the two pumps are launched into the fiber, they maintain their orthogonality. As discussed earlier, PMD rotates the pump SOPs at different rates such that they no longer stay orthogonal. Thus, eventually gain would depend on the input SOP of the signal, as also observed experimentally [6].

Fig. 3 shows the results of numerical simulations for the same FOPA used for Figs. 1 and 2 but pumps are now orthogonally (and linearly) polarized and $D_p = 0.1$ ps/$\sqrt{\text{km}}$. The initial SOP of the signal is linear but makes an angle of $\theta$ =0°, 45°, and 90° from the pump at the shorter wavelength. For comparison, the gain expected in the absence of PMD effects is also included (dotted curve). For certain signal wavelengths, PDG can be as much as 12 dB, where PDG equals the difference between the maximum and minimum gains as the input signal SOP is varied. The largest PDG occurs for signals close to the pumps in wavelength. The results in Fig. 3 agree well with a recent experiment [6]. The reason for largest PDG to occur close to pump wavelengths can be understood in physical terms as follows. The signal with a wavelength close to one pump remains aligned with that pump but decorrelates with the other pump rapidly because of a large frequency difference. Hence, the signal can see only the averaged effect of the farther pump but experiences the highest or smallest gain depending on if it started parallel or orthogonal to the closer pump. This also explains why for $\theta = 0$ gain peaks close to the pump at the shorter wavelength and decreases as it gets closer in wavelength to the other pump. Noting that FWM efficiency is minimum when the pumps are orthogonal, it is not surprising that in the case of isotropic fiber the gain is minimum. PMD can make the pumps nonorthogonal (and even parallel occasionally) and thus increases the gain as seen in Fig. 3.

To conclude, the effects of PMD on dual-pump parametric amplifiers are investigated numerically using a set of four vector FWM equations. It is found that PMD induces large fluctuations in the signal power that can affect the system performance by enhancing the outage probability. The average gain itself is reduced by more than 10 dB even for a relatively small value of 0.05 ps/$\sqrt{\text{km}}$ for the PMD parameter. For larger values of the PMD parameter, the gain spectrum begins to distort and loses its flatness. We also show that PDG cannot be eliminated by using orthogonally polarized pumps and can exceed 12 dB for $D_p = 0.1$ ps/$\sqrt{\text{km}}$. Our results are in agreement with a recent experiment.

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