Polarization mode dispersion–induced fluctuations during Raman amplifications in optical fibers

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We present a vector theory of Raman amplification and use it to discuss the effect of polarization-mode dispersion (PMD) on the Raman gain process inside the optical fiber used for stimulated Raman scattering. We show that the PMD induces large fluctuations in the amplified signal and reduces the average value of the amplifier gain. In the case of forward pumping, fluctuations are expected to be more than 15% under typical operating conditions and can exceed 50% for fiber with a relatively low value of the PMD parameter. The PMD effects are much less severe in the case of backward pumping. Signal fluctuations reduce to less than 1% in this case when the PMD parameter exceeds 0.05 ps/$\sqrt{\text{km}}$.

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Raman amplification, based on stimulated Raman scattering occurring inside optical fibers, attracted considerable attention recently[1–3] because of its potential for providing a relatively flat gain over a wide bandwidth. Its theoretical treatment is often based on a scalar approach[2] even though the Raman gain is known to be polarization dependent.[4–6] A scalar approach can be justified if the polarization states of the pump and the signal fields do not change along the fiber. This is, however, not the case in most fibers in which birefringence fluctuations lead to randomization of the state of polarization (SOP). This effect is known as polarization-mode dispersion (PMD) and has been studied extensively in recent years.[7] Although the effects of PMD on Raman amplification have been observed experimentally,[8] a vector theory of the stimulated Raman scattering process has not yet been fully developed. In this Letter we present a vector theory of Raman amplifiers by including the PMD-induced random evolution of the pump and signal polarization states. We use this theory to show that the amplifier gain fluctuates over a wide range because of PMD, and the average gain is significantly lower than that expected in the absence of PMD.

As is common in discussing the PMD effects,[7] it is helpful to describe the Raman process in the Stokes space. Introducing the Stokes vectors $\mathbf{P}$ and $\mathbf{S}$ for the pump and signal fields in the usual way,[2] we obtain the following set of two vector equations:

$$\eta \frac{\mathrm{d}\mathbf{P}}{\mathrm{d}z} = -\alpha_p \mathbf{P} - \frac{\omega_p}{2\omega_s} g_R (P_0 \mathbf{S} + S_0 \mathbf{P}) + (\omega_p \boldsymbol{\beta} + \gamma_p \mathbf{W}_p^{\mathrm{NL}}) \times \mathbf{P}, \tag{1}$$

$$\frac{\mathrm{d}\mathbf{S}}{\mathrm{d}z} = -\alpha_s \mathbf{S} + \frac{1}{2} g_R (S_0 \mathbf{P} + P_0 \mathbf{S}) + (\omega_s \boldsymbol{\beta} + \gamma_s \mathbf{W}_s^{\mathrm{NL}}) \times \mathbf{S}, \tag{2}$$

where α_j and $\gamma_j = n_2 \omega_j / c A_{\mathrm{eff}}$ $(j = p, s)$ account for fiber losses and nonlinearities at the pump and signal wavelengths. A_{eff} is the effective core area of the fiber. The Raman gain coefficient depends on the Raman shift defined as $\Omega_R = \omega_p - \omega_s$. The birefringence vector β governs the PMD-induced rotation of the Stokes vectors on the Poincaré sphere.[9] The vectors $\mathbf{W}_p^{\mathrm{NL}}$ and $\mathbf{W}_s^{\mathrm{NL}}$ account for nonlinear polarization rotation (NPR) induced by self-phase modulation and cross-phase modulation and are given by $\mathbf{W}_p^{\mathrm{NL}} = 2(\mathbf{P}_3 - 2\mathbf{S} + 2\mathbf{S}_3)/3$ and $\mathbf{W}_s^{\mathrm{NL}} = 2(\mathbf{S}_3 - 2\mathrm{P} + 2\mathrm{P}_3)/3$. In Eq. (2), $\eta = \pm 1$ for the forward- and backward-pumping configurations, respectively.

The magnitudes of $\mathbf{P}$ and $\mathbf{S}$, $P_0 \equiv \mathbf{P}$ and $S_0 \equiv |\mathbf{S}|$, represent the total pump and signal powers, respectively. From Eqs. (1) and (2), they are found to satisfy

$$\eta \frac{\mathrm{d}P_0}{\mathrm{d}z} = -\left(\frac{\omega_p}{2\omega_s} g_R S_0 + \alpha_p\right) P_0 - \frac{\omega_p}{2\omega_s} g_R \mathbf{P} \cdot \mathbf{S}, \tag{3}$$

$$\frac{\mathrm{d}S_0}{\mathrm{d}z} = \left(\frac{1}{2} g_R P_0 - \alpha_s\right) S_0 + \frac{1}{2} g_R \mathbf{P} \cdot \mathbf{S}. \tag{4}$$

Equations (1)–(4) describe the Raman amplification process under quite general conditions. We make two simplifications in the following analysis. First, we neglect pump depletion and the signal-induced NPR. This is justified because $P_0 \gg S_0$ in practice. Second, we average the NPR terms induced by $\mathbf{P}_3$ in Eqs. (1) and (2). One can understand the reason for this by noting that the beat length of NPR (~10 km) is much longer than both the birefringence beat length (~1 m) and the PMD correlation length (~10 m). The analysis can be further simplified by use of a rotating frame in which pump polarization remains fixed. Physically, we justify this by noting that even though polarization vectors $\mathbf{P}$ and $\mathbf{S}$ rotate randomly on the Poincaré sphere because of random birefringence changes, the Raman gain depends only on their relative orientation. Mathematically, this transformation is equivalent to dropping the $\omega\boldsymbol{\beta}$ term in Eq. (1) and replacing it with $(\omega_s - \eta\omega_p)\mathbf{b}$ in Eq. (2), where the vector $\mathbf{b}$ is related to $\boldsymbol{\beta}$ by a rotation and accounts for random birefringence responsible for PMD. As optical fibers used for Raman amplifiers are much longer than the PMD correlation length, we can model $\mathbf{b}$ as a three-dimensional Gaussian random process with the first- and second-order moments given by

$$\langle \mathbf{b}(z)\rangle = 0\,,$$
$$\langle \mathbf{b}(z_1)\mathbf{b}(z_2)\rangle = \frac{1}{3}D_p^2 \overleftrightarrow{I}\,\delta(z_2 - z_1)\,, \tag{5}$$

where the angle brackets denote an ensemble average over birefringence fluctuations, $\overleftrightarrow{I}$ is the second-order unit tensor, and D_p is the PMD parameter. Note that the relative orientation of $\mathbf{S}$ and $\mathbf{P}$ depends not only on D_p but also on their frequency separation, Ω_R.

We focus first on the forward-pumping case and set $\eta = 1$ in Eq. (1). Since the pump's SOP is fixed in the rotating frame, as a final simplification we make the transformation $\mathrm{P} = \hat{P}P_{\mathrm{in}}\exp(-\alpha_p z)$ and $\mathbf{S} = \mathbf{S}\exp\{\int_0^z[(g_R/2)P_{\mathrm{in}}\exp(-\alpha_p z) - \alpha_s]\,\mathrm{d}z\}$ to obtain the following set of simple equations:

$$\frac{\mathrm{d}S_0}{\mathrm{d}z} = \frac{1}{2}g_R P_{\mathrm{in}}\exp(-\alpha_p z)\hat{P}\cdot\mathbf{S}\,, \tag{6}$$

$$\frac{\mathrm{d}\mathbf{S}}{\mathrm{d}z} = \frac{1}{2}g_R P_{\mathrm{in}}\exp(-\alpha_p z)S_0\hat{P} - \Omega_R\mathbf{b}\times\mathbf{S} - \frac{8\gamma_s}{9}\mathbf{P}\times\mathbf{S}\,, \tag{7}$$

where P_{in} is the input pump power and $\hat{P}$ is the unit vector in the direction in which the pump is polarized. The last term in Eq. (7) accounts for the cross-phase modulation–induced NPR in the rotating frame; this term rotates the signal's SOP around that of the pump deterministically and does not affect the Raman gain. The second term in Eq. (7) accounts for the PMD effects governed by random variations in the birefringence vector $\mathbf{b}$. The random nature of $\mathbf{b}$ makes $\mathbf{S}$ random and introduces fluctuations into the amplified signal power. Since the PMD correlation length is much smaller than typical fiber lengths, it is appropriate to treat Eqs. (6) and (7) in the Stratonovich sense.[10]

The average gain and signal-power fluctuations can be obtained from Eqs. (6) and (7) by use of

$$G_{\mathrm{av}} = \frac{\langle S_0(L)\rangle}{S_0(0)}\,, \qquad \sigma_s^2 = \frac{\langle S_0^2(L)\rangle - \langle S_0(L)\rangle^2}{\langle S_0(L)\rangle^2}\,. \tag{8}$$

The average signal power at the end of a fiber of length L is obtained by averaging of Eqs. (6) and (7). We multiply Eq. (7) by $\hat{P}$ so that the last term vanishes. Writing $\hat{P}\cdot\mathbf{S}$ as $S_0\cos\theta$ and averaging Eqs. (6) and (7) with a standard technique,[10] we obtain the following two coupled but deterministic equations:

$$\frac{\mathrm{d}\langle S_0\rangle}{\mathrm{d}z} = \frac{1}{2}g_R P_{\mathrm{in}}\exp(-\alpha_p z)\langle S_0\cos\theta\rangle\,, \tag{9}$$

$$\frac{\mathrm{d}\langle S_0\cos\theta\rangle}{\mathrm{d}z} = \frac{1}{2}g_R P_{\mathrm{in}}\exp(-\alpha_p z)\langle S_0\rangle - \frac{1}{3}D_p^2\Omega_R^2\langle S_0\cos\theta\rangle\,, \tag{10}$$

where θ is the angle between $\mathbf{P}$ and $\mathbf{S}$. We have solved these equations numerically. Figure 1 shows how the average Raman gain changes with the PMD parameter at two pump levels when the input signal is copolarized or orthogonally polarized to the pump. The signal is located at the Raman gain peak ($\Omega_R/2\pi = 13.2$ THz) and the Raman gain coefficient $g_R = 0.61\ \mathrm{W}^{-1}/\mathrm{km}$.[1] The fiber is 10 km long, with losses of 0.273 and 0.2 dB/km for the pump and signal, respectively. In the absence of PMD, the pump and signal maintain their SOPs, and the copolarized signal experiences a maximum gain of 7.8 dB for $P_{\mathrm{in}} = 0.5$ W and 17.7 dB for $P_{\mathrm{in}} = 1.0$ W, but the orthogonally polarized signal experiences no Raman gain. As PMD increases, the gain difference between the copolarized and orthogonally polarized cases decreases and disappears eventually for $D_p > 0.1\ \mathrm{ps}/\sqrt{\mathrm{km}}$. For smaller values of D_p, even the average gain is polarization dependent. Note that this average gain difference is not the usual polarization-dependent gain, which is defined as the difference between the maximum and the minimum gain for each realization of PMD. For $D_p > 0.1\ \mathrm{ps}/\sqrt{\mathrm{km}}$, the average gain becomes polarization independent but is reduced drastically compared with the maximum value found for $D_p = 0$.

The PMD-induced signal fluctuations require the second-order moment $\langle S_0^2(L)\rangle$ of the amplified signal. Following the procedure described earlier, Eqs. (6) and (7) lead to the following set of three equations[10]:

$$\frac{\mathrm{d}\langle S_0^2\rangle}{\mathrm{d}z} = g_R P_{\mathrm{in}}\exp(-\alpha_p z)\langle S_0^2\cos\theta\rangle\,, \tag{11}$$

$$\frac{\mathrm{d}\langle S_0^2\cos\theta\rangle}{\mathrm{d}z} = -\frac{1}{3}D_p^2\Omega_R^2\langle S_0^2\cos\theta\rangle + \frac{1}{2}g_R P_{\mathrm{in}} \times \exp(-\alpha_p z)[\langle S_0^2\rangle + \langle S_0^2\cos^2\theta\rangle]\,, \tag{12}$$

$$\frac{\mathrm{d}\langle S_0^2\cos\theta\rangle}{\mathrm{d}z} = -D_p^2\Omega_R^2\langle S_0^2\cos^2\theta\rangle + \frac{1}{3}D_p^2\Omega_R^2\langle S_0^2\rangle + g_R P_{\mathrm{in}}\exp(-\alpha_p z)\langle S_0^2\cos\theta\rangle\,. \tag{13}$$

We solved Eqs. (11)–(13) numerically, using the same parameter values used for Fig. 1. In Fig. 2, we show the level of signal fluctuations by plotting σ_s as a function of D_p for several values of input pump powers under the same operating conditions. When the PMD parameter is small, signal fluctuations become large because θ does not fluctuate fast enough and the signal's SOP does not change rapidly over the Poincaré sphere. When the PMD parameter becomes large, θ fluctuates rapidly and all θ angles become equally likely over a short length of fiber, resulting in smaller signal fluctuations. The net result is that signal fluctuations increase quickly with the PMD parameter, reach a peak level, and then decrease slowly to zero after that. Since fluctuations depend on the Raman gain, the larger the pump power, the larger the fluctuations. The location of the fluctuation peak depends on the pump level as well as on the initial polarization of the pump. The noise level depends on the pump power and can exceed 20% for $P_{\mathrm{in}} = 1$ W and $D_p = 0.05\ \mathrm{ps}/\sqrt{\mathrm{km}}$. If a fiber with low PMD is used, the noise level can exceed 50% under some conditions. These results suggest that forward-pumped

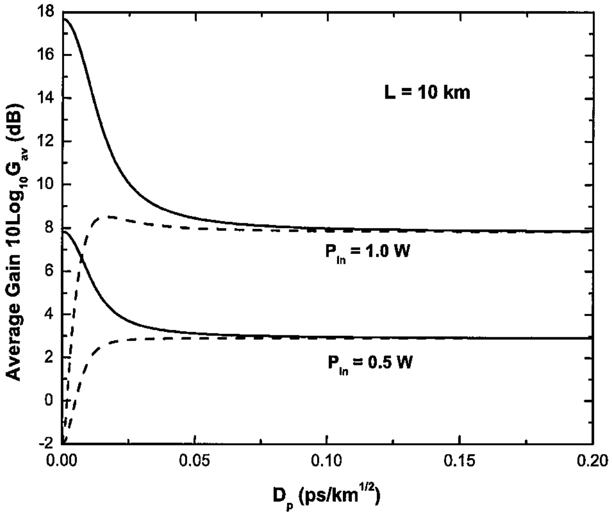

Fig. 1. Average Raman gain as a function of PMD parameters at two pump powers. The solid and dashed curves correspond to the cases in which the pump and signal at the input end are copolarized and orthogonally polarized, respectively. Other parameters are given in the text.

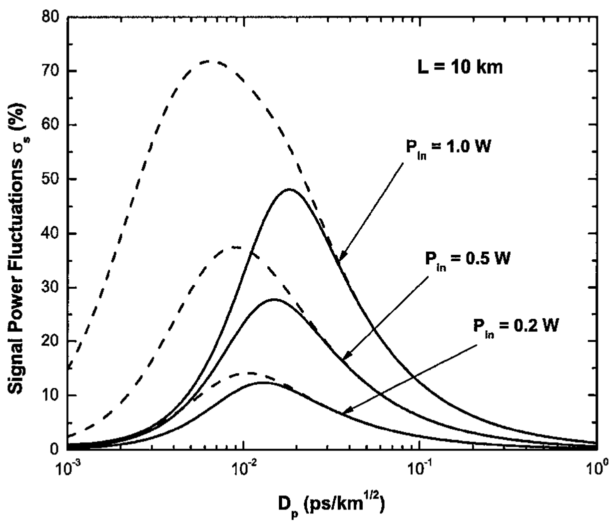

Fig. 2. Fluctuations in the amplified signal power as a function of PMD parameter in the case of forward pumping at three pump powers. The solid curves show the copolarized case and the dashed curves represent the case of orthogonal polarizations initially. All other parameters are the same as in Fig. 1.

Raman amplifiers will perform better if a fiber with $D_p > 0.1$ ps/$\sqrt{\text{km}}$ is used.

In the case of backward pumping ($\eta = -1$), Eqs. (6)–(13) require two modifications. First, the factor $\exp(-\alpha_p z)$ is replaced with $\exp[-\alpha_p(L - z)]$ because of the backward-propagating nature of the pump. Second, Ω_R should be replaced with $-(\omega_p + \omega_s)$. The latter change affects the Raman amplification process considerably because $\omega_p + \omega_s$ is larger than Ω_R by a factor of 30 or so in the 1.55-μm region. The variation of σ_s with D_p in this case follows curves nearly identical to those shown in Fig. 2, except that the peak occurs at a much lower value of D_p (~30 times smaller). Mathematically, one can understand this behavior from Eqs. (10)–(13) by noting that the PMD effects depend on the product $D_p\Omega_R$. If Ω_R increases by a factor of 30, D_p should decrease by the same factor to get the same PMD effect. From a practical standpoint, backward pumping produces only ~1% fluctuation in the signal power for typical values of $D_p \geq 0.05$ ps/$\sqrt{\text{km}}$ found for optical fibers.

In summary, we have developed a general vector theory to include the polarization effects occurring inside optical fibers during the SRS process. We found that PMD reduces the average Raman gain and introduces signal fluctuations because of random changes in the relative angle between the pump and signal SOPs. The signal fluctuations induced by PMD can be quite large, depending on the value of the PMD parameter. Such fluctuations add additional noise to the signal and would degrade the system performance. For typical values of D_p for modern fibers in the range 0.05–0.1 ps/$\sqrt{\text{km}}$, fluctuations are large in the case of forward pumping but can be reduced to less than 1% by use of backward pumping. In long-haul fiber links, fiber PMD is intentionally reduced for minimization of PMD-induced pulse broadening. Our results show that fibers with D_p values of ~0.02 ps/$\sqrt{\text{km}}$ (forward pumping) or ~0.001 ps/$\sqrt{\text{km}}$ (backward pumping) are not suitable for making Raman amplifiers. If SRS is used for distributed amplification, one must use backward pumping to keep signal fluctuations within the acceptable range.

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