1 June 2002

Optics Communications 206 (2002) 313–317

OPTICS COMMUNICATIONS

www.elsevier.com/locate/optcom

Pulse broadening induced by dispersion fluctuations in optical fibers

Q. Lin *, Govind P. Agrawal

Institute of Optics, University of Rochester, Rochester, NY 14627, USA

Received 23 January 2002; received in revised form 8 March 2002; accepted 11 March 2002

Abstract

A general theory is presented to describe the effects of dispersion fluctuations on optical pulses propagating inside single-mode fibers modeled as a linear dispersive medium. It is shown that the pulse broadening induced by dispersion fluctuations can be quite large in dispersion-managed lightwave systems, especially at high bit rates, and can exceed that induced by third-order dispersion and polarization mode dispersion.

Keywords: Dispersion fluctuations; Pulse broadening; Dispersion-management

1. Introduction

It is well known that pulse broadening induced by group-velocity dispersion (GVD) can limit the performance of fiber-optic communication systems [1]. The use of a dispersion-management technique in which the GVD is alternated along the fiber link in such a way that the average GVD remains close to zero solves this problem to a large extent [2]. In modern dispersion-managed lightwave systems, the single-channel bit rate can exceed 40 Gb/s. At such high bit rates, higher-order dispersive effects become quite important. Indeed, considerable attention is being paid to the effects of third-order dispersion (TOD) and polarization-mode dispersion (PMD) in this context [1–4]. However, the effects of dispersion fluctuations that are inherent in any real optical fiber have attracted much less attention [5]. Dispersion fluctuations can result from two different sources. First, unintentional variations in the core diameter, or dopant distributions, of the fiber generally change fiber dispersion along the fiber length. Several measurements of fiber dispersion have shown that random dispersion variations along the fiber length can be quite large [6–10]. Such variations are static, i.e., they do not change with time. Second, environmental changes such as temperature fluctuations can introduce time-dependent changes in the fiber dispersion [11–14]. Such changes are relatively small but can add to considerable fluctuations for long-haul fiber links, especially at high bit rates, for which the requirements on accumulative dispersion become quite stringent. In this paper, we address the issue of pulse broadening induced by dispersion fluctua-

* Corresponding author. Tel.: +1-716-2754721; fax: +1-716-2444936.

E-mail address: linq@optics.rochester.edu (Q. Lin).

0030-4018/02/$ - see front matter
PII: S0030-4018(02)01422-0

tions and show that their impact on the system performance can become severe and might exceed those of PMD and TOD at a bit rate of 40 Gb/s or more.

2. Theory

To focus on the effect of dispersion fluctuations, we ignore the PMD effects initially. In a single-mode fiber, the optical field can be written in the form [1]

$$E(x,y,z,t) = F(x,y)A(z,t) = F(x,y)\left(\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} S(z,\omega)\,e^{-i\omega t}\,d\omega\right), \tag{1}$$

where $F(x,y)$ represents the spatial distribution of the fiber mode. The spectral amplitude $S(z,\omega)$ changes with z as [2]

$$S(z,\omega) = S(0,\omega)\exp\left[i\int_0^z \beta(\omega,z)\,dz\right] \equiv \left[S_0(\omega)\,e^{i\theta(\omega)}\right]\exp[i\varphi(z,\omega)], \tag{2}$$

where $S_0(\omega)$ and $\theta(\omega)$ are the amplitude and phase of the initial pulse spectrum, $\varphi(z,\omega) = \int_0^z \beta(\omega,z)\,dz$ is the propagation-induced phase shift, and β is the propagation constant of the fiber mode. If the core radius of the fiber or the distribution of dopants inside the fiber varies along the fiber length in a random fashion, β varies with z because of the waveguide contribution to the effective mode index of the fiber [1]. If the environmental variables such as temperature fluctuate at different locations of fiber, $\beta(z)$ fluctuates with time as well, because the material dispersion of fiber depends on temperature [11–14]. Our objective is to find pulse broadening induced by such fluctuations in addition to that resulting from the average value of β.

To obtain a general expression for the pulse width valid for an arbitrary pulse shape, we calculate the root-mean-square (RMS) width σ of the pulse defined as $\sigma^2 \equiv \widehat{t^2} - (\widehat{t})^2$, where the hat denotes an average over the pulse intensity profile. It is useful to express the two temporal moments in terms of the pulse spectrum $S(z,\omega)$ as

$$\widehat{t} \equiv \int_{-\infty}^{+\infty} t|A(z,t)|^2\,dt = -i\int_{-\infty}^{+\infty} S^*(z,\omega)S_\omega(z,\omega)\,d\omega, \tag{3}$$

$$\widehat{t^2} \equiv \int_{-\infty}^{+\infty} t^2|A(z,t)|^2\,dt = \int_{-\infty}^{+\infty} S_\omega^*(z,\omega)S_\omega(z,\omega)\,d\omega, \tag{4}$$

where the subscript ω denotes a frequency derivative, i.e., $S_\omega = \partial S/\partial\omega$. We have assumed that $A(z,t)$ and $S(z,\omega)$ are normalized such that

$$\int_{-\infty}^{+\infty} |A(z,t)|^2\,dt = \int_{-\infty}^{+\infty} |S(z,\omega)|^2\,d\omega = 1.$$

We substitute $S(z,\omega)$ from Eq. (2) in Eqs. (3) and (4) and express σ^2 in the form

$$\sigma^2 = \sigma_0^2 + \overline{\tau^2} - (\overline{\tau})^2 + 2\left(\overline{\tau\theta_\omega} - \overline{\tau}\,\overline{\theta_\omega}\right), \tag{5}$$

where σ_0 is the initial RMS width of the pulse at $z = 0$, $\theta_\omega = d\theta/d\omega$, and τ represents the group delay defined as

$$\tau \equiv \frac{\partial\varphi}{\partial\omega} = \int_0^z \beta_\omega(\omega,z)\,dz. \tag{6}$$

In Eq. (5), an overbar denotes average over the initial pulse spectrum, i.e., $\overline{f} = \int_{-\infty}^{+\infty} f(\omega)|S(0,\omega)|^2\,d\omega$. It is obvious that the pulse broadening is determined by the accumulative dispersion of the whole fiber link. Because of dispersion fluctuations, σ^2 itself is a random quantity.

Since $\beta(\omega,z)$ fluctuates randomly only along z, we assume that it can be written in the form

$$\beta(\omega,z) = \beta_d(\omega)[1 + \varepsilon(z)], \tag{7}$$

where $\varepsilon(z)$ is a random process whose first two moments are given by

$$\langle\varepsilon(z)\rangle = 0, \quad \langle\varepsilon(z_1)\varepsilon(z_2)\rangle = R(z_2 - z_1). \tag{8}$$

The angle brackets denote an ensemble average over dispersion fluctuations and $\beta_d(\omega) \equiv \langle\beta(\omega,z)\rangle$ is the average value of $\beta(\omega,z)$. In the case of a dispersion-managed fiber link, $\beta_d(\omega)$ itself becomes a function of z because it varies from fiber to fiber in a stepwise fashion.

To calculate $\langle\sigma^2\rangle$, we substitute Eq. (7) in Eq. (6) and separate the group delay $\tau = \tau_d + \tau_r$ into its deterministic and random parts defined as

$$\tau_d \equiv \int_0^z \beta_{d1}(\omega)\,dz, \quad \tau_r \equiv \int_0^z \beta_{d1}(\omega)\varepsilon(z)\,dz, \tag{9}$$

where $\beta_{d1} = d\beta_d/d\omega = v_g^{-1}$ and v_g is the group velocity. The average value of σ^2 can then be written as

$$\langle\sigma^2\rangle = \sigma_0^2 + \sigma_d^2 + \langle\sigma_r^2\rangle, \tag{10}$$

where

$$\sigma_d^2 = \overline{\tau_d^2} - (\overline{\tau_d})^2 + 2\left(\overline{\theta_\omega\tau_d} - \overline{\theta_\omega}\,\overline{\tau_d}\right) \tag{11}$$

is the broadening induced by the deterministic part of $\beta(\omega, z)$. The additional broadening resulting from dispersion fluctuations is given by

$$\begin{aligned}\langle\sigma_r^2\rangle &= \langle\overline{\tau_r^2} - (\overline{\tau_r})^2\rangle \\ &= [\overline{\beta_{d1}^2} - (\overline{\beta_{d1}})^2]\int_0^z dz_1 \int_0^z dz_2\, R(z_2 - z_1).\end{aligned} \tag{12}$$

Eqs. (10)–(12) constitute our main result. They are valid for optical pulses of any shape and take into account the effects of initial frequency chirp. Moreover, they include fiber dispersion to all orders.

3. Application to chirp Gaussian pulses

To illustrate the significance of Eq. (10), we apply it to the case of chirped Gaussian pulses and consider fiber dispersion up to third order using $\beta_d(\omega) \approx \beta_0 + \beta_1\omega + \beta_2\omega^2/2 + \beta_3\omega^3/6$. The input field in Eq. (1) is of the form $A(0,t) = A_0 \exp[-(1+iC)t^2/4\sigma_0^2]$. As the input pulse spectrum is also Gaussian, all frequency integrals can be done analytically. In fact, in the absence of dispersion fluctuations, $\sigma^2 = \sigma_0^2 + \sigma_d^2$, and we recover the well-known result [1]

$$\frac{\sigma^2}{\sigma_0^2} = \left(\frac{\beta_2 z C}{2\sigma_0^2} + 1\right)^2 + \left(\frac{\beta_2 z}{2\sigma_0^2}\right)^2 + \frac{1}{2}\left[\frac{\beta_3 z(1+C^2)}{4\sigma_0^3}\right]^2. \tag{13}$$

To calculate the additional broadening $\langle\sigma_r^2\rangle$ produced by dispersion fluctuations, we need a specific form of the correlation function $R(z_2 - z_1)$ appearing in Eq. (12). Using $R(z_2 - z_1) = \eta^2 e^{-|z_2 - z_1|/l_c}$, where η indicates the strength of dispersion fluctuations and l_c is the length over which fluctuations remain correlated, we obtain

$$\begin{aligned}\langle\sigma_r^2\rangle = &\left[\frac{\beta_2^2(1+C^2)}{2\sigma_0^2} + \frac{\beta_3^2(1+C^2)^2}{16\sigma_0^4}\right] \\ &\times \eta^2 l_c[z - l_c(1 - \exp(-z/l_c))].\end{aligned} \tag{14}$$

Eq. (14) applies to fibers with constant average dispersion. In the case of dispersion-managed lightwave system, Eq. (14) can be applied separately for each fiber section used to form the dispersion map. If the section length $L_i \gg l_c$, the last factor in Eq. (14) is approximately equal to L_i. Noting that the ratio $\sqrt{1+C^2}/\sigma_0$ is related to the spectral width of the pulse that does not change in a linear medium, the total broadening can be obtained by summing over all fiber segments. The final result is given by

$$\begin{aligned}\langle\sigma_r^2\rangle = &\frac{(1+C^2)}{2\sigma_0^2}\sum_{i=1}^{N}\beta_{2i}^2\eta_i^2 l_{ci} L_i \\ &+ \frac{(1+C^2)^2}{16\sigma_0^4}\sum_{i=1}^{N}\beta_{3i}^2\eta_i^2 l_{ci} L_i.\end{aligned} \tag{15}$$

If we assume that the relative level η of dispersion fluctuations and the correlation length l_c are the same for all fibers used to form the fiber link, this equation reduces to

$$\begin{aligned}\langle\sigma_r^2\rangle = &\left[\frac{(1+C^2)\left(\beta_{21}^2 L_1 + \beta_{22}^2 L_2\right)}{2\sigma_0^2(L_1+L_2)}\right. \\ &\left. + \frac{(1+C^2)^2\left(\beta_{31}^2 L_1 + \beta_{32}^2 L_2\right)}{16\sigma_0^4(L_1+L_2)}\right]\eta^2 l_c L\end{aligned} \tag{16}$$

for a two-section map, where L is the total link length.

To illustrate the importance of dispersion fluctuations, we consider a specific dispersion map made using 60 km of standard single-mode fiber ($\beta_2 = -22$ ps^2/km) and ≈ 13.2 km of dispersion-compensating fiber ($\beta_2 = 100$ ps^2/km) such that average GVD β_2^{av} of the link is zero or has a small value. Both fibers have a constant TOD, $\beta_3 = 0.1$ ps^3/km. Fig. 1 shows the broadening

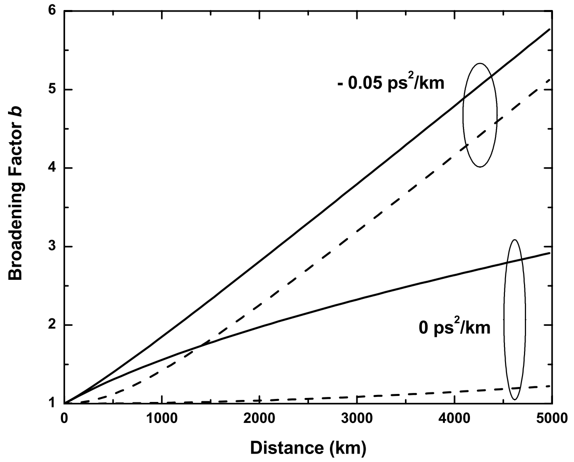

Fig. 1. Broadening factor for unchirped Gaussian pulses as a function of propagation distance with (solid lines) and without (dashed lines) dispersion fluctuations for a dispersion map having average dispersion of 0 and -0.05 ps^2/km.

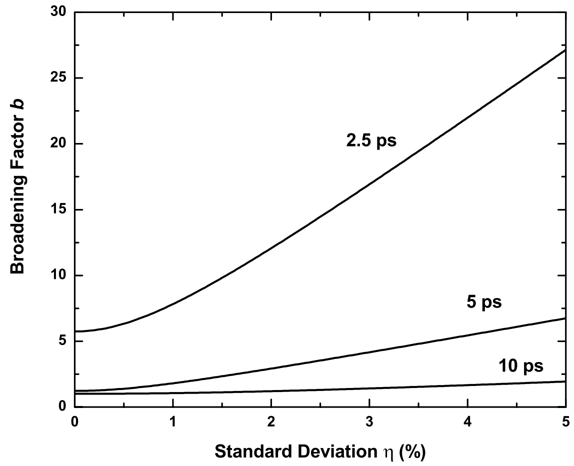

Fig. 2. Broadening factor at 5000 km as a function of the standard deviation of dispersion fluctuations for three values of the RMS width σ_0. Average dispersion of the map is zero. Other parameters are the same as in Fig. 1.

factor $b = \langle\sigma^2\rangle^{1/2}/\sigma_0$ for $\beta_2^{av} = 0$ and -0.05 ps^2/km when $\sigma_0 = 5$ ps and C $= 0$. The $\eta = 0$ case is shown by dashed lines for comparison. We assume $\eta = 2\%$ and $l_c = 2$ km as representative of typical dispersion fluctuations in optical fibers. The TOD effects are relatively small for such wide pulses (full width at half maximum ≈ 12 ps) and the pulse broadens by only 20% at 5000 km when $\beta_2^{av} = 0$. However, even 2% fluctuations produce broadening by a factor of 2.9 at 5000 km. When the average GVD is not zero, the effect is not as dramatic but still remains quite large. For example, when $\beta_2^{av} = -0.05$ ps^2/km, broadening factor at 5000 km increases from 5.1 to 5.8.

To study the dependence of pulse broadening on the level of dispersion fluctuations, we show in Fig. 2 the broadening factor at 5000 km as a function of η for three different pulse widths ($C = 0$) and the same map used in Fig. 1 with $\beta_2^{av} = 0$. When $\eta = 0$ (no dispersion fluctuations) the effects of TOD lead to considerable pulse broadening for 2.5-ps pulses. The broadening factor does not change much when η is below 1% but increases rapidly for $\eta > 2\%$. The rate of increase depends inversely on the pulse width. For 2.5-ps pulses, broadening factor increases from 6 to 27 when $\eta = 5\%$. These results show that the impact of dispersion fluctuations would become severe for systems operating at single-channel bit rates of 40 Gb/s or more.

Finally, one may ask how broadening induced by dispersion fluctuations compares with that induced by PMD. We use Eq. (7) of Ref. [4] to include PMD effects. Fig. 3 compares the broadening induced by dispersion fluctuations

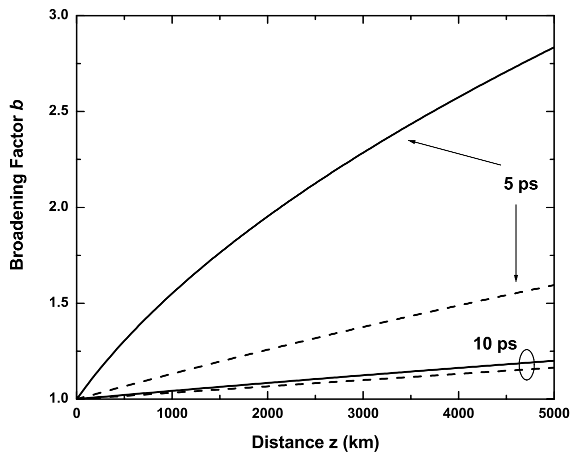

Fig. 3. Broadening factor as a function of distance for two values of σ_0. The dispersion map is the same as in Figs. 1 and 2 but average dispersion equals zero. Other parameters are the same as in Fig. 1. Dashed lines show for comparison broadening induced by first-order PMD in the absence of dispersion fluctuation ($\eta = 0$).

(solid curves) with that of PMD (dashed curves) assuming $\eta = 2\%$ and a PMD value of 0.2 ps/$\sqrt{\text{km}}$. The comparison is made for 5 and 10-ps pulses using the same dispersion map used earlier in Fig. 1 with $\beta_2^{\text{av}} = 0$. For 10-ps pulses, dispersion-fluctuation-induced broadening ($b = 1.19$) is almost the same as PMD-induced broadening ($b = 1.16$). However, when pulse width decrease to 5 ps, the dispersion fluctuations induced broadening greatly increases to 2.8, almost three times the initial pulse width, although that induced by PMD still keeps as low as 1.6. The impact is even more severe for shorter pulses. It is clear that high-bit-rate lightwave systems will be affected considerably by dispersion fluctuations.

4. Conclusions

In conclusion, we have derived a general expression for pulse broadening that is valid for chirped pulses of arbitrary shape and includes dispersive effects to all orders. We have used it to study the effect of dispersion fluctuations and found that even a few percent fluctuations from the average value of dispersion can enhance pulse broadening by a large factor, especially for short pulses. Although the manufacturing-induced dispersion fluctuations, such as random variations in fiber core and dopant distribution along the fiber length, are static and can be compensated totally in real systems by pre- or post-compensation techniques, the environment-induced dispersion fluctuations (such as temperature-induced changes) vary with time randomly. The important question is whether such time-dependent fluctuations can be compensated in a practical lightwave system. The answer depends on the time-scale at which fluctuations occur. Temperature-induced dispersion fluctuations are likely to vary slowly and it may be possible to compensate for them by using an adaptive dynamic compensation technique [12–15]. In contrast, any mechanism that leads to rapid dispersion fluctuations may be difficult to eliminate fully. Our results show that dispersion fluctuations may be much more limiting factor than PMD for lightwave systems in which each channel operates at a bit rate of 40 Gb/s or more.

Acknowledgements

This research is supported in part by the National Science Foundation under Grants ECS-9903580 and DMS-0073923.

References

[1] G.P. Agrawal, Fiber-optic Communication Systems, second ed., Wiley, New York, 1997, Chapter 2.
[2] G.P. Agrawal, Nonlinear Fiber Optics, third ed., Academic Press, San Diego, 2001, Chapter 3.
[3] P.K.A. Wai, C.R. Menyuk, J. Lightwave Technol. 14 (1996) 148.
[4] H. Sunnerud, M. Karlsson, P.A. Andrekson, IEEE Photon. Technol. Lett. 12 (2000) 50.
[5] F.K. Abdullaev, B.B. Baizakov, Opt. Lett. 25 (2000) 93.
[6] L.F. Mollenauer, P.V. Mamyshev, M.J. Neubelt, Opt. Lett. 21 (1996) 1724.
[7] M. Eiselt, R.M. Jopson, R.H. Stolen, J. Lightwave Technol. 15 (1997) 135.
[8] K. Nakajima, M. Ohashi, M. Tateda, J. Lightwave Technol. 15 (1997) 1095.
[9] I. Brener, P.P. Mitra, D.D. Lee, D.J. Thomson, D.L. Philen, Opt. Lett. 23 (1998) 1520.
[10] J. Gripp, L.F. Mollenauer, Opt. Lett. 23 (1998) 1603.
[11] W.H. Hatton, M. Nishimura, J. Lightwave Technol. LT-4 (1986) 1552.
[12] H. Onaka, K. Otsuka, H. Miyata, T. Chikama, IEEE Photon. Technol. Lett. 6 (1994) 1454.
[13] G. Ishikawa, H. Ooi, Proc. Eur. Conf. Opt. Commun. (ECOC) 98 (1998) 519.
[14] B.J. Eggleton, B. Mikkelsen, G. Raybon, A. Ahuja, J.A. Rogers, P.S. Westbrook, T.N. Nielsen, S. Stulz, K. Dreyer, IEEE Photon. Technol. Lett. 12 (2000) 1022.
[15] T. Sugihara, K. Ishida, K. Shimomura, K. Shimizu, Y. Kobayashi, IEICE Trans. Electron. E84-C (2001) 527.