Propagation-induced polarization changes in partially coherent optical beams

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Propagation of a partially coherent optical beam inside a linear, nondispersive, dielectric medium is studied, taking into account the vector nature of the electromagnetic field. Propagation-induced polarization changes are studied by using the Gaussian–Schell model for the cross-spectral-density tensor. The degree of polarization changes with propagation and also becomes nonuniform across the beam cross section. The extent of these changes depends on the coherence radius associated with the cross-correlation function. For optical beams with symmetric spectra, the bandwidth of the source spectra is found to play a relatively minor role.

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1. INTRODUCTION

It is well known[1–3] that the coherence properties of an optical field can change during the field's propagation in free space. An example is provided by the van Cittert–Zernike theorem (see Ref. 3; Sec. 10.4.2), which predicts quantitatively how a field generated by a spatially incoherent source becomes partially coherent on propagation in free space. Closely related to such changes is the fact that that the spectrum of a partially coherent field can change as the field propagates, even in free space.[4,5] One might expect that the polarization of a partially polarized beam will also change with propagation since the degree of polarization of an optical field is a measure of the correlation between two orthogonally polarized components of the beam. Indeed, this was shown to be the case as early as 1973.[6] Except for this early work, propagation-induced polarization changes in optical beams were considered mostly after 1993.[7–14] James reexamined the polarization of light radiated by blackbody sources using vector diffraction theory[8] and found that light remains unpolarized in all directions of the far zone, contrary to the prediction based on a quasi-scalar theory.[6] Several authors have used modified versions of the Gaussian–Schell model to study propagation-induced polarization changes of optical fields.[7,10]

In this paper we study the propagation of a partially coherent optical beam in a linear, homogeneous, isotropic, dielectric medium with a constant refractive index n. In Section 2 we develop the general formalism, and in Section 3 we apply it to the propagation of Gaussian beams by using a vectorial extension of the Schell source model.[15] In Section 4 we show that the polarization of an optical beam becomes spatially nonuniform even if the degree of polarization is initially constant across the entire Gaussian beam. The degree of polarization is also found to depend on the source spectrum. In the far zone, the degree of polarization is shown not only to become nearly uniform but also to be independent of the spectral bandwidth of the source.

2. GENERAL FORMALISM

Polarization properties of a partially coherent optical beam may be characterized by the coherence matrices with matrix elements[1]

$$\Gamma_{ij}(\mathbf{r}_1, \mathbf{r}_2, \tau) = \langle E_i^*(\mathbf{r}_1, t) E_j^*(\mathbf{r}_2, t + \tau)\rangle, \tag{1}$$

where $i, j = x, y, z$ label the Cartesian components of the electric field. For a beam the component of the electric field along the axis of the beam (say, the z direction) is often small enough to be neglected. Polarization properties of a beam are then governed by a 2×2 coherence matrix [$i, j = x, y$ in Eq. (1)].

The polarization aspects are deduced from the coherence matrix by setting $\mathbf{r}_1 = \mathbf{r}_2 = \mathbf{r}$ and $\tau = 0$ in Eq. (1). The resulting matrix is often called the polarization matrix $\mathbf{J}$; its elements are given by

$$J_{ij}(\mathbf{r}) = \Gamma_{ij}(\mathbf{r}, \mathbf{r}, 0), \qquad (i, j = x, y). \tag{2}$$

The degree of polarization at a point $\mathbf{r}$ is then given by the expression (Ref. 1, p. 353)

$$P(\mathbf{r}) = \left(1 - \frac{4 \det \mathbf{J}}{(\operatorname{tr} \mathbf{J})^2}\right)^{1/2}, \tag{3}$$

where $\det \mathbf{J}$ and $\operatorname{tr} \mathbf{J}$ are, respectively, the determinant and the trace of the polarization matrix $\mathbf{J}$. Propagation-induced changes in the degree of polarization can be studied by determining the polarization matrix $\mathbf{J}$ after the beam has propagated a certain distance from the input plane $z = 0$.

Propagation of coherence functions has been studied extensively (for a comprehensive account see Refs. 1–3). It is often convenient to introduce the cross-spectral-density matrix whose components are the temporal Fourier transform of Γ_{ij}:

$$W_{ij}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \int_{-\infty}^{\infty} \Gamma_{ij}(\mathbf{r}_1, \mathbf{r}_2, \tau)\exp(i\omega\tau)\mathrm{d}\tau. \qquad (4)$$

Its propagation in a nondispersive dielectric medium with constant refractive index n is governed by the formula

$$W_{ij}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \int\int K^*(\mathbf{r}_1, \boldsymbol{\rho}_1, \omega)K(\mathbf{r}_2, \boldsymbol{\rho}_2, \omega) \times W_{ij}^s(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega)\mathrm{d}\boldsymbol{\rho}_1\mathrm{d}\boldsymbol{\rho}_2, \qquad (5)$$

where W_{ij}^s denotes the cross-spectral density at the source plane $z = 0$. The propagation kernel in Eq. (5) is given by

$$K(\mathbf{r}, \boldsymbol{\rho}, \omega) = \frac{k^2}{4\pi^2}\int\int_{-\infty}^{\infty} \exp\{ik[p(x - \xi) + q(y - \eta) + mz]\}\mathrm{d}p\,\mathrm{d}q, \qquad (6)$$

where $\boldsymbol{\rho} = (\xi, \eta)$ is a vector in the input plane $z = 0$ and m is defined by the formulas

$$m = \begin{cases} (1 - p^2 - q^2)^{1/2} & \text{if } p^2 + q^2 \leqslant 1 \\ i(p^2 + q^2 - 1)^{1/2} & \text{otherwise} \end{cases}. \qquad (7)$$

The parameter $k = n\omega/c$ is the wave number inside the dielectric medium assumed to have a constant refractive index n. The medium is assumed to occupy the half-space $z > 0$. For free-space propagation, $n = 1$. In Eq. (5) the integration extends over the entire plane $z = 0$.

Equation (5) can be used to study polarization changes in optical beams on propagation. However, its direct use may not always be useful because of the multiple integrations involved. It may be preferable to solve the propagation problem approximately by making some simplifying assumptions. We make two such assumptions. First, since the integration region $p^2 + q^2 > 1$ corresponds to the contribution of evanescent waves, we can ignore them at distances $z \gg \lambda$, where $\lambda = 2\pi c/(n\omega)$ is the wavelength in the medium and c is the speed of light in vacuum. Second, we make the paraxial approximation, which is usually adequate for beams. The integrals over p and q in Eq. (6) can then be evaluated analytically, and one finds that (Ref. 1, Sec. 3.2)

$$k(\mathbf{r}, \boldsymbol{\rho}, \omega) \approx \frac{k\exp(ikz)}{2\pi iz}\exp\left\{\frac{ik}{2z}[(x - \xi)^2 + (y - \eta)^2]\right\}. \qquad (8)$$

3. VECTOR GAUSSIAN–SCHELL MODEL

We assume that the coherence properties of the optical beam are known at the plane $z = 0$, which we will refer to as the plane of a secondary planar source, and we model the source as a vector generalization of the scalar Gaussian–Schell-model source (Ref. 1, p. 242). Instead of the usual scalar form of the cross-spectral density, we now have a matrix whose elements are

$$W_{xx}^s(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = S(\omega)[I_x(\boldsymbol{\rho}_1)I_x(\boldsymbol{\rho}_2)]^{1/2}\gamma_a(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2), \qquad (9)$$

$$W_{yy}^s(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = S(\omega)[I_y(\boldsymbol{\rho}_1)I_y(\boldsymbol{\rho}_2)]^{1/2}\gamma_a(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2), \qquad (10)$$

$$W_{xy}^s(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = S(\omega)[I_x(\boldsymbol{\rho}_1)I_y(\boldsymbol{\rho}_2)]^{1/2}\gamma_c(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2). \qquad (11)$$

The off-diagonal elements satisfy the relation $W_{yx} = W_{xy}^*$. Here $S(\omega)$ is the spectrum of the beam, assumed to be normalized so that $\int_{-\infty}^{\infty} S(\omega)\mathrm{d}\omega = 1$, and I_x and I_y are the spatial intensity profiles of the two polarization components. The autocorrelation and cross-correlations functions of the Cartesian components of the optical field, γ_a and γ_c, depend only on the difference $\boldsymbol{\rho} = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$, as appropriate for a Schell-model source. For simplicity, the two diagonal components are assumed to have the same degree of spatial coherence. Since the off-diagonal components represent cross correlation between the polarization components, γ_c differs, in general, from γ_a. In particular, $\gamma_c(0)$ is not necessarily unity and has zero value when the two components are uncorrelated, as is the case when the beam is unpolarized.[7] One can easily verify, by setting $\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2$ in Eqs. (9)–(11) and using Eq. (3), that when the two components have the same intensity profiles, the degree of polarization of the optical field at the source plane $z = 0$ is given by $P_0 = |\gamma_c(0)|$.

We assume that the input beam incident at the plane $z = 0$ has a Gaussian intensity profile for both polarization components but that the peak intensity may be different:

$$I_x(\boldsymbol{\rho}) = I_1\exp\left(-\frac{|\boldsymbol{\rho}|^2}{2\sigma_I^2}\right), \qquad I_y(\boldsymbol{\rho}) = I_2\exp\left(-\frac{|\boldsymbol{\rho}|^2}{2\sigma_I^2}\right). \qquad (12)$$

Here σ_I is the root-mean-square beam width. The autocorrelation and cross-correlations coefficients, γ_a and γ_c, respectively, are also assumed to have Gaussian forms:

$$\gamma_a(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) = \exp\left(-\frac{|\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2|^2}{2\sigma_a^2}\right), \qquad (13)$$

$$\gamma_c(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) = \gamma_0\exp\left(-\frac{|\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2|^2}{2\sigma_c^2}\right). \qquad (14)$$

Here σ_a and σ_c are the coherence radii, which can take any physically allowed values,[14] and $\gamma_0 \equiv \gamma_c(0)$ represents the normalized cross correlation between the two polarization components at the same spatial point. The absolute value of γ_0 can vary in the range $0 \leqslant |\gamma_0| \leqslant 1$. The values 0 and 1 correspond to the extreme cases of totally uncorrelated and completely correlated field components, respectively. For such a Gaussian input beam, the degree of polarization is constant across the entire beam (uniform polarization) at the plane $z = 0$; using Eq. (3), one can show that it has the value

$$P_0 = \left(1 - (1 - |\gamma_0|^2)\frac{4I_1I_2}{(I_1 + I_2)^2}\right)^{1/2}. \qquad (15)$$

As expected, $P_0 = 1$ when $|\gamma_0| = 1$. However, P_0 has a finite value $|I_1 - I_2|/(I_1 + I_2)$ even when $\gamma_0 = 0$.

4. PROPAGATION-INDUCED POLARIZATION CHANGES

In this section we determine the components of the polarization matrix after the Gaussian beam of Section 3 has propagated a distance z from the source plane $z = 0$ inside the homogeneous dielectric medium. The spatial integrals appearing in Eq. (5) can be evaluated analytically.[16] Using Eqs. (2) and (4), one finds that

$$J_{xx}(\mathbf{r}) = \int_{-\infty}^{\infty} \frac{I_1}{1 + d_a^2} \exp\left(-\frac{x^2 + y^2}{2\sigma_I^2(1 + d_a^2)}\right) S(\omega)\mathrm{d}\omega, \quad (16)$$

$$J_{xy}(\mathbf{r}) = \int_{-\infty}^{\infty} \frac{\gamma_0(I_1 I_2)^{1/2}}{1 + d_c^2} \exp\left(-\frac{x^2 + y^2}{2\sigma_I^2(1 + d_c^2)}\right) S(\omega)\mathrm{d}\omega, \quad (17)$$

where d_a and d_c depend on frequency and are defined by the expressions

$$d_\mu(\omega) = \frac{cz}{2n\omega\sigma_I^2}\left(1 + \frac{4\sigma_I^2}{\sigma_\mu^2}\right)^{1/2} \qquad (\mu = a, c). \quad (18)$$

J_{yy} is obtained by replacing I_1 by I_2 in the expression (16) for J_{xx}.

The frequency integral in Eq. (16) and the dependence of d_a and d_c on the frequency ω make it clear that, in general, polarization properties of the optical beam will depend on the spectrum of the beam. In some cases (e.g., for a quasi-monochromatic beam), the spectrum may be so narrow that d_a and d_c remain nearly constant over the source spectrum. We first consider this case, which can be treated analytically.

A. Quasi-Monochromatic Gaussian Beams

If we take in Eq. (16) $S(\omega) = \delta(\omega - \omega_0)$, where δ is the Dirac delta function and ω_0 is the frequency at which the spectral line is centered, we obtain relatively simple analytic expressions for the components of the polarization matrix. Using them, we can calculate the degree of polarization $P(\mathbf{r})$ at any point in the transverse plane $z = z_0 > 0$, z_0 being a constant. The most important qualitative change that is found is that the beam polarization is not uniform across the beam cross section, even though it was so at the source plane $z = 0$. Figure 1 shows, for a selected set of parameters, how P varies with σ_c/σ_I for three values of the normalized transverse distance $R = (x^2 + y^2)^{1/2}/\sigma_I$ after the field has propagated over a distance z equal to one diffraction length $L_d = 2(n\omega_0/c)\sigma_I^2$ (also known as the Rayleigh range). Among several features seen in Fig. 1, one is of particular interest. For a given value of σ_c, the degree of polarization is different for different values of R and generally becomes larger off axis compared with its on-axis value. For a given value of R, the degree of polarization increases with increasing σ_c. The minimum value of P occurs when $\sigma_c = 0$. The off-diagonal components of the polarization matrix, J_{xy} and J_{yx}, then vanish, as seen from Eq. (17).

The degree of polarization also changes with the propagation distance. This feature is shown in Fig. 2 where P is plotted as a function of z/L_d for $R = 0$ (dotted line) and $R = 1$ (solid curve). We chose $\sigma_c/\sigma_I = 0.5$ for these curves. Other parameters are the same as those pertaining to Fig. 1. The main conclusion that is evident from these results is that light becomes depolarized with propagation and becomes nonuniform across the cross section of the beam. However, polarization is found to become uniform for propagation distances much larger than the diffraction length.

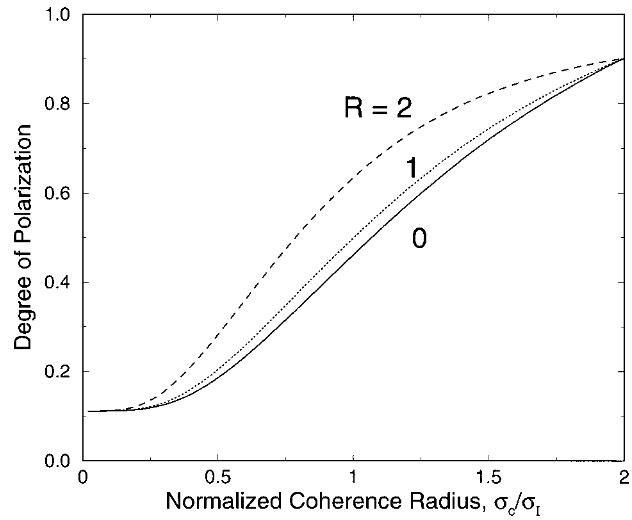

Fig. 1. Degree of polarization P plotted as a function of the ratio σ_c/σ_I for three values of $R = (x^2 + y^2)^{1/2}/\sigma_I$ at $z = L_d$, for a partially coherent Gaussian beam. $R = 0$ (solid curve) corresponds to the beam center. Values of other parameters are $|\gamma_0| = 0.9$, $I_2/I_1 = 0.8$, and $\sigma_a/\sigma_I = 2$.

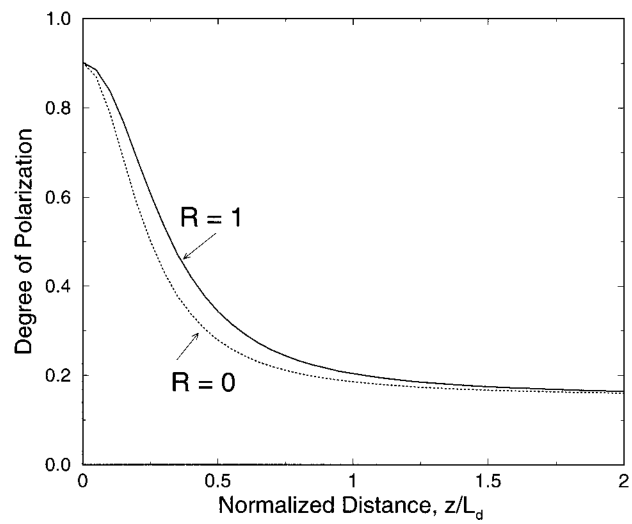

Fig. 2. Degree of polarization P plotted as a function of propagation distance for two values of $R = 0$ and $R = 1$ with the choice $\sigma_c/\sigma_I = 0.5$. $R = 0$ (dotted curve) corresponds to beam center. Values of other parameters are same as in Fig. 1.

Several special cases are of interest and provide further physical insight. Consider the case in which two polarization components have identical intensities, i.e., when $I_1 = I_2 = I_0$. In this particular case the degree of polarization $P(x, y)$ is found to be given by the expression

$$P(x, y) = |\gamma_0|\left(\frac{1 + d_a^2}{1 + d_c^2}\right) \exp\left[\frac{(x^2 + y^2)(d_c^2 - d_a^2)}{2\sigma_I^2(1 + d_a^2)(1 + d_c^2)}\right], \quad (19)$$

where d_a and d_c are defined as in Eq. (18) but with ω replaced by ω_0. The degree of polarization at the source plane is obtained by setting $z = 0$ and has the value $P_0 = |\gamma_0|$. As noted earlier, polarization is uniform across the beam initially since P_0 is independent of x and y.

It is evident from Eq. (19) that beam polarization becomes nonuniform across the beam after propagation. The degree of polarization depends not only on the propagation distance but also on the two coherence radii σ_a and σ_c. An interesting situation occurs when $|\gamma_0| = 1$. The input Gaussian beam is then completely and uniformly polarized ($P_0 = 1$). However, as seen from Eq. (19), the Gaussian beam becomes partially polarized on propagation, and the polarization is spatially nonuniform across the beam. Figure 3 shows the extent of depolarization occurring at $z/L_d = 1$ when $\sigma_a/\sigma_I = 2$ for several values of the ratio σ_c/σ_I. As in Fig. 1, the degree of polarization is seen to depend on σ_c and can become zero if $\sigma_c = 0$. On the other hand, if $\sigma_c = \sigma_a$, i.e., if the cross-correlation and the autocorrelation functions have the same spatial domain, the beam remains completely and uniformly polarized on propagation. For other values of σ_c, light is only partially polarized, with a nonuniform degree of polarization across the beam. This result may appear somewhat surprising for the following reason. It is well known that a scalar field that is completely coherent (spatially) in the source plane remains so after propagation (Ref. 3, Sec. 10.4). Here we find that a completely polarized, but partially coherent, optical field maintains its degree of polarization only if $\sigma_c = \sigma_a$ so that $d_c = d_a$. Note, however, that if the optical field is fully spatially coherent ($\sigma_c \to \infty$, $\sigma_a \to \infty$), its degree of polarization does not change on propagation.

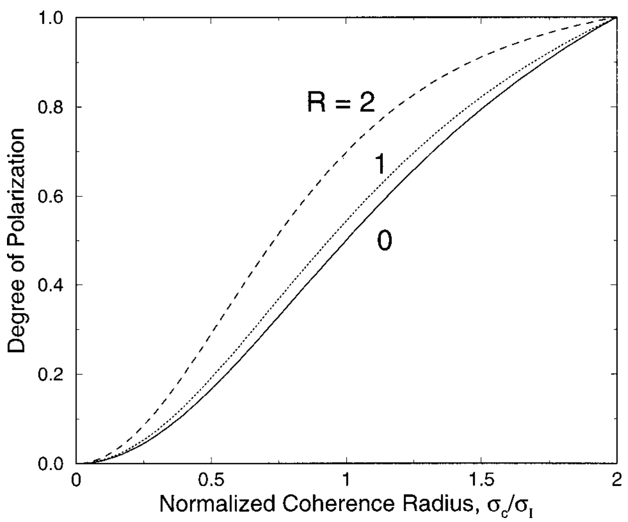

Fig. 3. Same as in Fig. 1 except that $|\gamma_0| = 1$, $I_2/I_1 = 1$. The Gaussian input beam is completely polarized initially under these conditions. Not only does it become partially polarized when $\sigma_c < \sigma_0$, but the degree of polarization also becomes nonuniform across the beam.

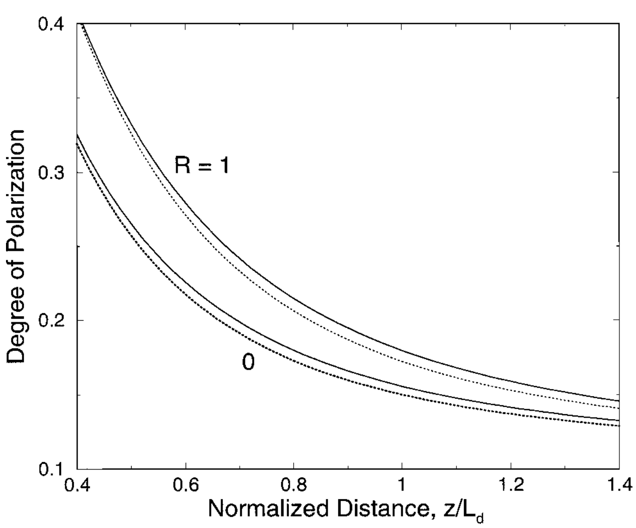

Fig. 4. Effect of the spectral width of the source on beam polarization. Solid curves show the degree of polarization P plotted as a function of z/L_d for two values of $R = 0$ and $R = 1$, with the choice $\sigma_c/\sigma_I = 0.5$, $\sigma_a/\sigma_I = 2$, $|\gamma_0| = 0.9$, $I_2/I_1 = 1$, and $\delta = 0.2$. The case $\delta = 0$ (dotted curve) corresponding to a vanishingly narrow spectrum is included for comparison.

B. Broadband Gaussian Beams

We now consider briefly the effect of the spectrum of the source on the degree of polarization of a Gaussian beam. The integrals appearing in Eqs. (16) and (17) cannot be evaluated analytically, in general. We evaluated them numerically for the case when the source spectrum has a Gaussian form:

$$S(\omega) = \frac{1}{\sqrt{2\pi}\Delta\omega_0} \exp\left[-\frac{(\omega - \omega_0)^2}{2\Delta\omega_0^2}\right]. \tag{20}$$

In this formula ω_0 is the central frequency and $\Delta\omega_0$ is the root-mean-square width of the source spectrum. Let $\delta = \Delta\omega_0/\omega_0$ be normalized bandwidth. Figure 4 shows the effect of source spectrum on the degree of polarization by displaying $P(\mathbf{r})$ as a function of propagation distance z/L_d for the on-axis ($R = 0$) and off-axis ($R = 1$) cases in the range $0 \leqslant \delta \leqslant 0.2$. Dotted and solid curves in each case correspond to a narrow spectrum ($\delta = 0$) and a broad spectrum ($\delta = 0.2$), respectively. The degree of polarization is larger for a broad source spectrum, but the increase is relatively small (a few percent), even for a spectrum whose full width at half-maximum is almost 50% of the central frequency.

To understand why the degree of polarization is relatively unaffected by the spectral bandwidth, we consider the components of the polarization matrix in an analytic form by evaluating the integrals appearing in Eqs. (16) and (17) approximately for points on the z axis located in the far zone. By setting $x = y = 0$ in Eq. (16) and assuming that $d_a \gg 1$, we find that

$$J_{xx}(0, 0, z) \approx \frac{I_1}{d_a^2(\omega_0)} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (1 + \delta f)^2 \exp(-f^2/2) \mathrm{d}f, \tag{21}$$

where we set $\omega = \omega_0(1 + \delta f)$ and $d_a(\omega_0)$ is obtained from Eq. (18) on setting $\omega = \omega_0$. The integration over f can be performed analytically, and one finds that

$$J_{xx}(0, 0, z) \approx (1 + \delta^2) I_1 \frac{L_d^2}{z^2} \left(1 + \frac{4\sigma_I^2}{\sigma_a^2}\right). \tag{22}$$

A similar calculation provides expressions for the remaining three components of the polarization matrix. Using them in Eq. (3), we obtain the following approximate expression for the degree of polarization:

$$P(0, 0, z) \approx \left\{ 1 - \frac{4I_1I_2}{(I_1 + I_2)} \left[1 - |\gamma_0|^2 \frac{(1 + 4\sigma_I^2/\sigma_a^2)^2}{(1 + 4\sigma_I^2/\sigma_c^2)^2} \right] \right\}^{1/2}. \tag{23}$$

Relation (23) shows that degree of polarization along the axis in the far zone is independent of both z and δ. The individual components of the polarization matrix are enhanced by a factor $1 + \delta^2$, but the enhancement factor is relatively small; there is an enhancement of 4% for $\delta = 0.2$. In the region $z \sim L_d$, the degree of polarization P depends both on z and δ, as seen in Fig. 4, but it changes by only a few percent for $\delta = 0.2$. Larger changes can occur when the source spectrum is asymmetric.

5. CONCLUSIONS

In this paper we have studied propagation of a partially coherent optical beam inside a linear, homogeneous, nondispersive, dielectric medium, taking into account the vector nature of electromagnetic fields. We have considered the propagation-induced polarization changes by using a generalization of the well-known Gaussian–Schell scalar model for the cross-spectral density of the source. In this model, the diagonal and off-diagonal components of the cross-spectral density tensor have, in general, different coherence radii. The use of the model makes it possible to obtain expressions for the four components of the polarization matrix as an integral involving the source spectrum.

In general, propagation-induced polarization changes depend on the source spectrum. We consider separately the cases of narrow-band as well as broadband spectra. In both cases, the degree of polarization changes with the propagation distance, and it also becomes nonuniform across the cross section of the beam. The extent of variations depends on the coherence radius associated with the cross-correlation function. When the spectrum is symmetric, the bandwidth of the source spectrum plays a relatively minor role.

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