# Importance of prechirping in constant-dispersion fiber links with a large amplifier spacing

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We use the variational method to find the optimum launch conditions that can sustain path-averaged solitons in a periodically amplified, constant-dispersion, optical communication system even when amplifier spacing is comparable to or larger than the dispersion length. We determine the amount of prechirping and the initial peak power required and show that both the pulse width and the chirp recover their initial values at each amplifier. The prechirped solitons are different from the standard solitons in constant dispersion since their width and chirp are allowed to vary over each amplifier section. This feature results in an interesting regime in which amplifier spacing can exceed the dispersion length. Numerical solutions of the nonlinear Schrödinger equation show that the use of prechirped solitons improves stability in comparison with guiding-center solitons in constant-dispersion fiber links. © 2000 Optical Society of America [S0740-3224(00)00904-8]

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## 1. INTRODUCTION

Soliton communication systems are leading candidates for long-haul lightwave transmission links because they offer the possibility of a dynamic balance between group-velocity dispersion and self-phase modulation, the two effects that severely limit the performance of nonsoliton systems.[1,2] Most system experiments employ the technique of lumped amplification and place fiber amplifiers periodically along the transmission line for compensating the fiber loss. The principal concept that has emerged in the context of lumped amplification is the path-average or guiding-center soliton.[3–5] Its use allows propagation of solitons through lossy fibers provided that the amplifier spacing $L_A$ is short compared with the dispersion length $L_D$. The soliton is launched with enough energy that the path-averaged peak power over one amplifier spacing is equal to the peak power needed in the lossless case. However, the limitation that $L_A \ll L_D$ results in unreasonably short amplifier spacings (<10 km) at high bit rates. This limitation comes from the fact that the system is not perfectly periodic when $L_A$ becomes comparable to or exceeds $L_D$. As a result, large perturbations generate spectral side bands and dispersive radiation that degrade the system performance.[6–8] Several techniques have been proposed to design soliton communication systems that can operate beyond the average-soliton regime.[9–13] However, their use often requires optical elements such as a fast saturable absorber.

A question one may ask is whether the periodicity of solitons can be restored in constant-dispersion fiber links, even when $L_A > L_D$, by modifying the system design in an appropriate way. For example, the guiding-center soliton is launched with a unique peak power obtained by averaging the soliton energy over one amplifier spacing. However, the soliton is assumed to remain unchirped.[3] With the advent of dispersion management, the importance of prechirping of solitons has been realized.[14–19] In this paper we use the prechirping concept for constant-dispersion fibers and allow both the width and the chirp of the soliton to vary in each fiber section between two amplifiers. We use variational analysis to determine the optimal launch conditions for the path-averaged soliton. We require the pulse width and chirp to be periodic and determine the exact prechirp and peak power needed to maintain periodicity of the soliton in constant-dispersion fiber links. The use of prechirping provides an interesting operating regime for systems in which $L_A$ can be comparable to and even exceed $L_D$. This regime is especially useful at high bit rates ($B > 10$ Gb/s) for which the dispersion length becomes comparable to or shorter than 10 km.

## 2. VARIATIONAL ANALYSIS

The propagation of soliton pulses in each fiber section between two consecutive amplifiers is described by the nonlinear Schrödinger equation,[1] (NSE)

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$$i\frac{\partial A}{\partial z} - \frac{1}{2}\beta_2\frac{\partial^2 A}{\partial t^2} + \gamma_0|A|^2 A = -\frac{i}{2}\alpha A, \tag{1}$$

where $A$ is the amplitude of the electric field, $\beta_2$ is the group velocity dispersion parameter, $\gamma_0$ is the nonlinear parameter responsible for self-phase modulation, and $\alpha$ accounts for the fiber loss. The loss term can be eliminated with the following change of variables,

$$A = B\exp(-\alpha z/2), \tag{2}$$

resulting in the following form for the NSE:

$$i\frac{\partial B}{\partial z} - \frac{1}{2}\beta_2\frac{\partial^2 B}{\partial t^2} + \gamma(z)|B|^2 B = 0, \tag{3}$$

where $\gamma(z) = \gamma_0\exp(-\alpha z)$. The effects of fiber loss are now included through the $z$ dependence of $\gamma$.

The variational analysis based on the Lagrangian provides approximate analytical results for features such as pulse compression, maximal pulse amplitude, and induced frequency chirp.[20] Since this method is well known and details are available in books,[2] we only outline the procedure briefly. We choose the following ansatz for the soliton amplitude, shape, and phase:

$$B(z,t) = a\,\mathrm{sech}\left(\frac{t}{\tau}\right)\exp\left(i\phi - \frac{iCt^2}{2\tau^2}\right), \tag{4}$$

where $a$ is the amplitude, $\phi$ is the phase, $C$ is the chirp, and $\tau$ is the pulse width. Note that a similar ansatz is made for dispersion-managed solitons, but the pulse shape is often taken to be Gaussian. For constant-dispersion fibers, "sech" pulse shape is assumed and the possibility of chirping is allowed. All of the soliton parameters (except $\phi$) remain constant for a lossless fiber but vary along $z$ when solitons are amplified periodically to compensate for fiber losses. With the standard procedure,[2] a set of four ordinary differential equations governing variations of soliton parameters along the fiber link is obtained through the variational analysis:

$$\frac{\mathrm{d}}{\mathrm{d}z}(a^2\tau) = 0, \tag{5}$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}z} = \frac{\beta_2 C}{\tau}, \tag{6}$$

$$\frac{\mathrm{d}C}{\mathrm{d}z} = \frac{4}{\pi^2}\gamma(z)a^2 + \frac{\beta_2}{\tau^2}\left(\frac{4}{\pi^2} + C^2\right), \tag{7}$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}z} = \frac{\beta_2}{3\tau^2} + \frac{5}{6}\gamma(z)a^2. \tag{8}$$

These ordinary differential equations are equivalent to solving the NSE within the variational approximation. Note, however, that this approach is only approximate and does not account for characteristics such as energy loss to continuum radiation,[21] damping of the amplitude oscillations, and changing of soliton shape.[20] It should be stressed that Eqs. (5)–(8) can also be applied for dispersion-managed solitons if $\beta_2$ is made explicitly $z$ dependent. In this paper we consider the case of constant-dispersion fibers only.

## 3. PRECHIRP AND PEAK POWER

Equation (5) shows the conservation of pulse energy $E_p = \int_{-\infty}^{\infty}|B|^2\mathrm{d}t$ and relates the amplitude $a$ of the pulse to its width $\tau$. We can write $a^2 = a_0^2\tau_0/\tau$, where $a_0$ and $\tau_0$ are the initial amplitude and the pulse width, respectively. As a result, $\phi$ is strictly determined by $\tau$, and the variational analysis is reduced to solving a pair of coupled ordinary differential equations for $C$ and $\tau$ only [Eqs. (6) and (7)]. Furthermore, it is useful to introduce the normalized length $\xi = z/L_A$ and the normalized pulse width $W = \tau/\tau_0$. If we introduce the dispersion length $L_D = \tau_0^2/|\beta_2|$ and assume that $\beta_2 < 0$ (anomalous group-velocity dispersion), Eqs. (6) and (7) become

$$\frac{\mathrm{d}W}{\mathrm{d}\xi} = -\frac{z_A C}{W}, \tag{9}$$

$$\frac{\mathrm{d}C}{\mathrm{d}\xi} = \frac{4z_A P_0\exp(-\Gamma\xi)}{\pi^2 W} - \frac{z_A}{W^2}\left(\frac{4}{\pi^2} + C^2\right), \tag{10}$$

where $z_A = L_A/L_D$, $\Gamma = \alpha L_A$, and $P_0 = \gamma_0 a_0^2 L_D$ is the normalized initial peak power. Our objective is to find a periodic solution of Eqs. (9) and (10) such that all soliton parameters (except $\phi$) recover their initial values after one amplifier spacing. This periodicity condition can be met only under certain launch conditions. The optimal launch conditions are determined by solving Eqs. (9) and (10) with the boundary conditions

$$C(0) = C(1), \qquad W(0) = W(1) \equiv 1. \tag{11}$$

In general, one should solve Eqs. (9)–(11) numerically by considering different input values for the peak power $P_0$, pulse width $\tau_0$, and initial chirp $C(0)$. Because of the multidimensional nature of the parameter space, an exhaustive search for periodic solutions is quite time consuming. However, we can solve Eqs. (9) and (10) approximately by using a perturbation method in the regime $z_A \ll 1$. The natural parameter for perturbation expansion is $z_A$ since $C$ and $W$ vary little along the fiber length for $z_A \ll 1$. Expanding $C$ and $W$ up to second order in $z_A$, we can write

$$W = W_0 + W_1 z_A + W_2 z_A^2, \tag{12}$$

$$C = C_0 + C_1 z_A + C_2 z_A^2. \tag{13}$$

Since $C_0 = 0$ and $W_0 = 1$ (the lossless case), we obtain the following two equations:

$$\frac{\mathrm{d}W_2}{\mathrm{d}\xi} = -C_1, \tag{14}$$

$$\frac{\mathrm{d}C_1}{\mathrm{d}\xi} = \frac{4P_0\exp(-\Gamma\xi)}{\pi^2} - \frac{4}{\pi^2}. \tag{15}$$

The width parameter $W$ has no first-order corrections. These equations can be solved by direct integration to obtain $C_1(\xi)$ and $W_2(\xi)$.

Applying the boundary condition $C_1(0) = C_1(1)$ gives the following launch condition for the soliton peak power,

$$P_0 = \frac{\Gamma}{1 - \exp(-\Gamma)} = \frac{G\ln G}{G - 1}, \tag{16}$$

where $G = \exp(\alpha L_A) = \exp(\Gamma)$ is the amplifier gain. Similarly, applying the boundary condition $W_2(0) = W_2(1)$ provides the input chirp

$$C_1(0) = \frac{2}{\pi^2} - \left(\frac{4}{\pi^2}\right)\frac{\exp(-\Gamma) + \Gamma - 1}{\Gamma[1 - \exp(-\Gamma)]}$$
$$= \frac{4}{\pi^2}\left[\frac{1}{2} + \frac{(G - 1) - G\ln G}{\ln G(G - 1)}\right]. \qquad (17)$$

These conditions can also be obtained by using the Lie transform as discussed in Ref. 3 and applied to dispersion-managed solitons given in Ref. 15. We use them in the next section to show how initial chirping can be useful in the regime $z_A > 1$.

## 4. NUMERICAL RESULTS

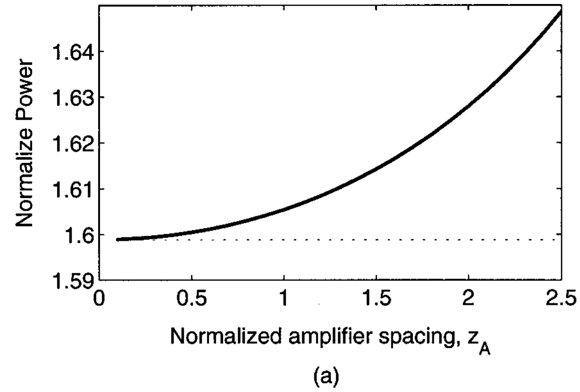
In this section we discuss the new operating regime of chirped solitons in constant-dispersion fibers and compare it with the standard regime in which unchirped solitons are launched at the input end. The perturbation analysis of Section 3 provides an estimate of the chirp only for $z_A \ll 1$. However, we expect, on physical grounds, chirped solitons to be useful for designing high-speed periodically amplified fiber links even when $z_A$ exceeds 1. The operating region in which the amplifier spacing is comparable to or larger than the dispersion length ($z_A > 1$) can be studied by solving Eqs. (9) and (10) numerically.

We use a root-finding algorithm to satisfy the boundary conditions imposed by Eq. (11). For definiteness, we choose $L_A = 40$ km and $G = 10$ ($\Gamma = 2.3$) and find the optimum values of $P_0$ and $C(0)$ numerically for $z_A$ in the range 0–2.5. Figure 1(a) compares the peak power $P_0$ needed for launching chirped (solid curve) and unchirped (dotted curve) solitons as $z_A$ is increased from 0 to 2.5. In the regime $z_A \ll 1$ the launch power is virtually the same for both chirped and unchirped solitons; this result agrees with our perturbation analysis. As $z_A$ increases, chirped solitons require more power. However, the increase in peak power is less than 2% even for $z_A = 2.5$. Figure 1(b) shows the amount of prechirping required as a function of $z_A$. The input solitons need to be prechirped more and more as amplifier spacing increases. The need for negative prechirping can be understood by examining Eq. (10), which shows that $\mathrm{d}C/\mathrm{d}\xi$ contains a negative term (since $\beta_2 < 0$ for anomalous dispersion) and an exponentially decreasing positive term. Initially the positive term dominates, and the chirp increases with propagation. However, the nonlinear term is reduced because of fiber loss, and $\mathrm{d}C/\mathrm{d}\xi$ becomes negative, resulting in a downward concave trajectory. In addition, the boundary condition [Eq. (11)] requires that

$$\int_0^1 C(\xi)\mathrm{d}\xi = 0. \qquad (18)$$

For a concave-down trajectory this integral relation can be satisfied only for negatively prechirped pulses [$C(0) < 0$].

Since both the soliton width and the chirp are allowed to vary along $z$ periodically in the new operating regime



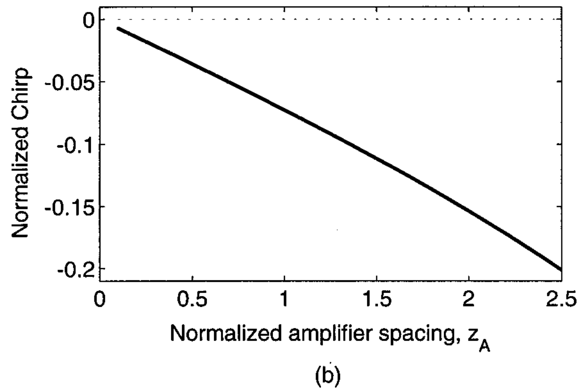




Fig. 1. Comparison of (a) launching peak power and (b) initial chirp for chirped (solid curves) and unchirped (dotted curves) solitons as a function of normalized amplifier spacing when amplifiers with 10-dB gain ($\Gamma = 2.3$) are placed 40 km apart.

proposed here, it is important to consider the extent of their variation in each fiber section between two amplifiers. Figure 2 shows variation of pulse width and chirp along the fiber length for $z_A = 0.4$ (top row) and $z_A = 2.1$ (bottom row) under launch conditions corresponding to a chirped (solid curve) and an unchirped (dotted curve) soliton. In the $z_A \ll 1$ regime the chirp is fairly periodic and recovers its initial value in both cases. But since the unchirped soliton does not impose periodicity of the pulse width, soliton width is reduced by 1% after one amplifier spacing. In contrast, the width recovers its initial value for the chirped soliton. In the $z_A > 1$ regime, however, the perturbation becomes too great for the unchirped soliton to maintain the periodic nature of the pulse width and chirp. As seen in Fig. 2(c), the soliton width can vary by as much as 20% (dotted curve) and is smaller by 10% after one amplifier spacing. In contrast, the chirped soliton recovers both pulse width and chirp after each amplifier. Also, width variations are much smaller (<5%) for chirped solitons showing clearly that such solitons are not perturbed significantly even when $z_A > 1$.

To check the validity of variational analysis, Fig. 3 is obtained with the same parameters as those used in Fig. 2 except that the NSE is solved numerically over 20 amplification stages (a total transmission distance of 800 km). The root-mean-square (RMS) width[1] and chirp of the pulse are calculated numerically. We decided to estimate the RMS width since the shape of the pulse is not guaranteed to remain preserved even though variational analysis requires it. We estimate the chirp parameter by fitting a parabola to the phase profile in the vicinity $\tau = 0$ and noting from Eq. (4) that the quadratic term var-

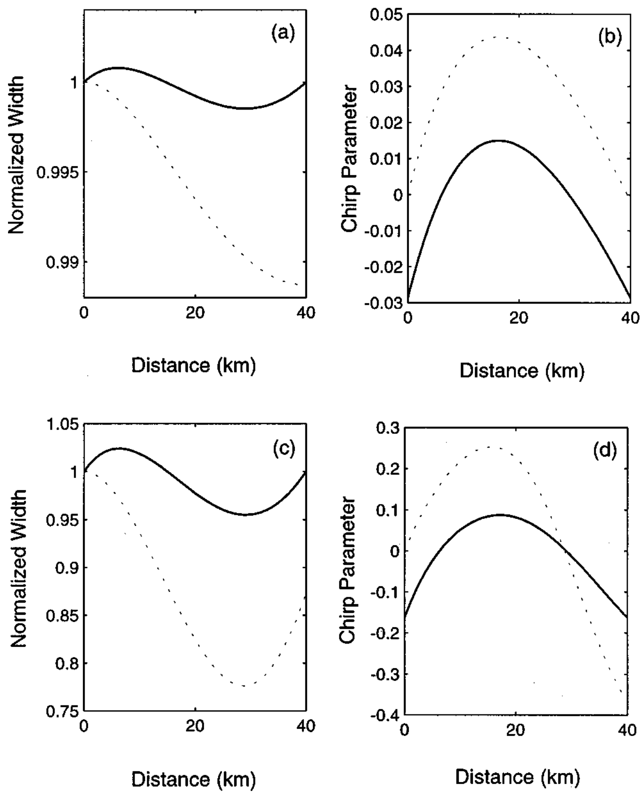



Fig. 2. Evolution of pulse width and chirp over one amplifier stage for chirped (solid curves) and unchirped (dotted curves) solitons as predicted by variational analysis. Normalized amplifier spacing $z_A = 0.4$ for top row and 2.1 for bottom row.

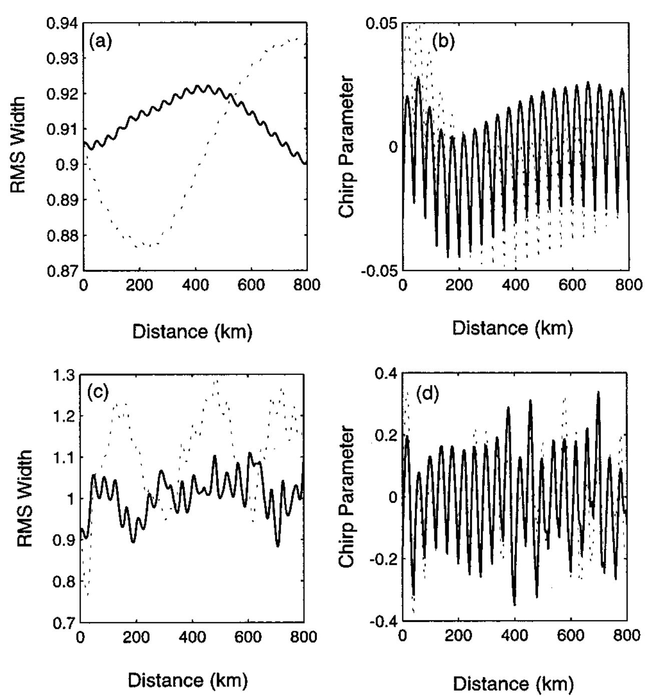



Fig. 3. Same as in Fig. 2 except that soliton evolution over 20 amplification stages (total distance of 800 km) is shown by solving the NSE numerically.

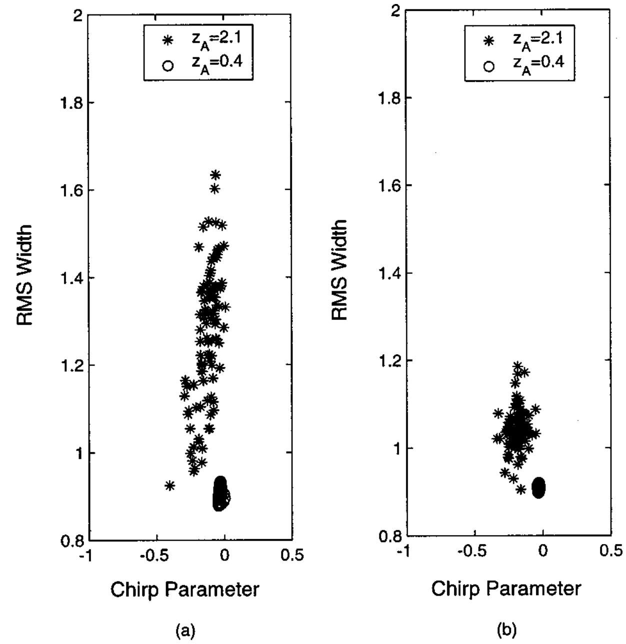



Fig. 4. Poincaré map obtained by plotting soliton width and chirp at the end of each amplifier section for 100 amplification stages (4000 km) for unchirped (left graph) and chirped (right graph) solitons. For $z_A = 0.4$, a nearly circular compact region shows the quasi-periodic nature of soliton evolution. For $z_A = 2.1$, soliton width and chirp vary over a wider region.

ies as $Ct^2/2\tau^2$. Figure 3 shows that the periodicity in $C$ and $\tau$ is maintained only approximately over multiple amplifiers. For example, RMS pulse width varies 1% from amplifier to amplifier when $z_A = 0.4$, and variations become as large as 10% when $z_A = 2.1$. This is not surprising and indicates that the sech pulse shape is not true pulse shape for the periodic solution of the NSE. As we noted earlier, variational analysis cannot accurately predict the soliton parameters once the shape of the soliton is no longer preserved. Figure 3(a) and 3(c) show that the RMS width varies less when a chirped soliton is launched. For instance, in the case $z_A = 2.1$, the widths of unchirped solitons exhibit more than 20% variation, whereas chirped solitons exhibit a maximum of 10% variation in width. This feature suggests that, in general, the use of prechirped solitons is likely to provide better system performance compared with unchirped solitons.

To explore the soliton-stability issue, we have plotted the chirp and width variations in the two-dimensional phase space as a Poincaré map, since such a map shows the phase-space region over which width and chirp vary along the fiber length. Figure 4 shows the Poincaré map for chirped (right side) and unchirped (left side) solitons over 100 amplifier spacing (4000 km). Ideally, if the system is perfectly periodic, we would expect all points to coincide, resulting in a single dot in the plot. Our numerical results show that for both $z_A = 0.4$ and $z_A = 2.1$, the chirped soliton is more localized, implying that both the soliton width and the chirp vary over a smaller range from one amplifier to the next. This behavior confirms our variational result that prechirping is necessary for stable propagation.

Finally, we compare our results with those of Forysiak *et al.*,[11] who also studied the average-soliton dynamics in the regime $z_A > 1$. They used an operator-splitting technique to find the optimum distance at which an unchirped pulse can be launched, whereas our analysis predicts the initial chirp required at the beginning of the fiber section. These two points of view are formally equivalent if we note from Fig. 2 that the chirp indeed becomes zero at a certain distance. However, the two approaches are so different that a direct comparison is difficult. The function $F(z)$ in Ref. 11 is a measure of the deviation of the system performance from the ideal NSE case, and its variation with $z$ appears to be similar to the chirp variation $-C(z)$ seen in Fig. 2 for $z_A = 2.1$. However, we were not able to relate $F(z)$ and $C(z)$ analytically.

## 5. CONCLUSION

In conclusion, we have found a new operating regime for soliton transmission in constant-dispersion lightwave systems. This regime requires launching of an initially chirped soliton. Our variational analysis recovers the guiding-center soliton result in the regime $z_A \ll 1$. By allowing both the pulse width and the chirp to vary over each amplifier section, we find that prechirping of the pulse is necessary to sustain path-averaged solitons in the regime $z_A \sim 1$ in a periodically amplified optical communication system. We use the results of variational analysis to determine the amount of prechirping and initial peak power required to recover initial launch values at each amplifier. Numerical solutions of the NSE show that the use of prechirped solitons improves stability since variations of pulse width and chirp over a large transmission distance are much smaller than with guiding-center solitons. The new operating regime should be useful at high bit rates (>20 Gb/s) because it permits amplifier spacing to become larger than the dispersion length.

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## REFERENCES


1. G. P. Agrawal, *Fiber-Optic Communication Systems*, 2nd ed. (Wiley, New York, 1997).
2. A. Hasegawa and Y. Kodama, *Solitons in Optical Communications* (Clarendon, Oxford, UK, 1995).
3. A. Hasegawa and Y. Kodama, “Guiding-center soliton in optical fibers,” Opt. Lett. **15**, 1443–1445 (1990); “Guiding-center soliton,” Phys. Rev. Lett. **66**, 161–164 (1991).
4. L. F. Mollenauer, S. G. Evangelides, and H. A. Haus, “Long-distance soliton propagation using lumped amplifiers and dispersion shifted fiber,” J. Lightwave Technol. **9**, 194–196 (1991).
5. K. J. Blow and N. J. Doran, “Average soliton dynamics and the operation of soliton systems with lumped amplifiers,” IEEE Photonics Technol. Lett. **3**, 369–371 (1991).
6. S. M. J. Kelly, “Characteristic sideband instability of periodically amplified average soliton,” Electron. Lett. **28**, 806–807 (1992).
7. N. J. Smith, K. J. Blow, and I. Andonovic, “Sideband generation through perturbations to the average soliton model,” J. Lightwave Technol. **10**, 1329–1333 (1992).
8. J. P. Gordan, “Dispersive perturbations of solitons of the nonlinear Schrödinger equation,” J. Opt. Soc. Am. B **9**, 91–97 (1992).
9. N. J. Smith and N. J. Doran, “Picosecond soliton transmission using concatenated nonlinear optical loop-mirror intensity filters,” J. Opt. Soc. Am. B **12**, 1117–1125 (1995).
10. M. Matsumoto, H. Ikeda, T. Uda, and A. Hasegawa, “Stable soliton transmission in the system with nonlinear gain,” J. Lightwave Technol. **13**, 658–665 (1995).
11. W. Forysiak, N. J. Doran, F. M. Knox, and K. J. Blow, “Average soliton dynamics in strongly perturbed systems,” Opt. Commun. **117**, 65–70 (1995).
12. R. J. Essiambre and G. P. Agrawal, “Soliton communication beyond the average-soliton regime,” J. Opt. Soc. Am. B **12**, 2420–2425 (1995).
13. Z. M. Liao and G. P. Agrawal, “High-bit-rate soliton transmission using distributed amplification and dispersion management,” IEEE Photonics Technol. Lett. **11**, 818–820 (1999).
14. T. Okamawari, Y. Ueda, A. Maruta, Y. Kodama, and A. Hasegawa, “Interaction between guiding centre solitons in a periodically compensated optical transmission line,” Electron. Lett. **33**, 1063–1065 (1997).
15. Y. Kodama and A. Maruta, “Optimal design of dispersion management for a soliton-wavelength-division-multiplexed system,” Opt. Lett. **22**, 1692–1694 (1997).
16. I. Morita, M. Suzuki, N. Edagawa, K. Tanaka, S. Yamamoto, and S. Akiba, “Performance improvement by initial phase modulation in 20 Gbit/s soliton-based RZ transmission with periodic dispersion compensation,” Electron. Lett. **33**, 1021–1022 (1997).
17. H. Sugahara, T. Inoue, A. Maruta, and Y. Kodama, “Optimal dispersion management for wavelength-division-multiplexed RZ optical pulse transmission,” Electron. Lett. **34**, 902–904 (1998).
18. T. Georges and B. Charbonnier, “Reduction of the dispersive wave in periodically amplified links with initially chirped solitons,” IEEE Photonics Technol. Lett. **9**, 127–129 (1997).
19. F. Favre, D. Guen, and T. Georges, “Experimental evidence of pseudoperiodical soliton propagation in dispersion-managed links,” J. Lightwave Technol. **17**, 1032–1036 (1999).
20. D. Anderson, “Variational approach to nonlinear pulse propagation in optical fibers,” Phys. Rev. A **27**, 3135–3145 (1983).
21. W. L. Kath and N. F. Smyth, “Soliton evolution and radiation loss for the nonlinear Schrödinger equation,” Phys. Rev. E **51**, 1484–1492 (1995).