Influence of the Raman effect on dispersion-managed solitons and their interchannel collisions

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We calculate the self-frequency shift experienced by a soliton in a dispersion-managed fiber that is due to the Raman effect, as well as the energy and frequency shifts that result from a collision of such solitons with different wavelengths. We find that dispersion management suppresses both types of frequency shift but does not significantly affect the energy shift that is accumulated over a large propagation distance. The latter shift may represent a potential problem for wavelength-division-multiplexed systems with several gigabits per second in a single channel.

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As the demand for an increased bit rate in optical fiber communications grows, pulses as short as 5 ps (or even shorter) must be used. For such pulses the Raman effect represents a small but nonnegligible perturbation. Here we use a recently developed technique[1] to study the effect of that perturbation on (i) a single dispersion-managed (DM) soliton and (ii) the collision between two DM solitons in different wavelength channels.

The equation for an optical pulse in the presence of the Raman effect is

$$iA_Z - \frac{1}{2}\kappa'' A_{TT} + \gamma|A|^2 A = \gamma\tau_R A|A|_T{}^2, \tag{1}$$

where the notation is the same as in Ref. 2. Equation (1) is for the case of an idealized lossless fiber. We show below that periodically compensated fiber loss changes our main results only in an insignificant way.

By use of the following nondimensionalization of variables, $z = Z/L_{\mathrm{map}}$, $\epsilon = \gamma P_0 L_{\mathrm{map}}$, $\tau = t/(|\kappa_2'' - \kappa_1''|L_1L_2/L_{\mathrm{map}})^{1/2}$, and $u = A/\sqrt{P_0}$, $\mu = \tau_R/(|\kappa_2'' - \kappa_1''|L_1L_2L_{\mathrm{map}})^{1/2}$, Eq. (1) is cast into the form

$$iu_z + \frac{D(z)}{2}u_{\tau\tau} + \epsilon\left(\frac{D_0}{2}u_{\tau\tau} + |u|^2u\right) = \mu\epsilon u|u|_\tau{}^2. \tag{2}$$

Here L_{map} is the period of the dispersion map, $\kappa_{1,2}''$ and $L_{1,2}$ are, respectively, the dispersion coefficients and the lengths of the two sections in one cell of the map ($L_1 + L_2 = L_{\mathrm{map}}$), P_0 is the input peak power, the average dispersion is $\epsilon D_0 = -(\kappa_1''L_1 + \kappa_2''L_2)L_{\mathrm{map}}/(|\kappa_1'' - \kappa_2''|L_1L_2)$, and the local dispersion, $D(z)$, is defined accordingly to have zero average over the map cell: $\int_0^1 D(z)\mathrm{d}z = 0$ (cf. Ref. 1). The local dispersion is assumed to be much greater than both the average dispersion and the nonlinearity; hence $\epsilon \ll 1$. In addition, the Raman effect is an even smaller perturbation for a 5-ps pulse; hence $\mu \ll 1$.

When $\mu = 0$, Eq. (2) has a pulse solution, referred to as a DM soliton, that is expressable as an infinite sum of Hermite–Gaussian harmonics.[3] When $0 < \mu \ll 1$, with an accuracy of ~5%, the evolution of the perturbed DM soliton could be approximated by a chirped Gaussian[1]:

$$u_0 = \frac{a_0}{\left(1 + i\dfrac{\Delta}{T_0{}^2}\right)^{1/2}}\exp(-\xi^2/2 + i\Psi), \tag{3}$$

where $\Delta \equiv \Delta(z_0) = -\mathrm{sgn}(\kappa_2'' - \kappa_1'')/2 + \int_0^z D(z')\mathrm{d}z'$, $\xi = [\tau - \tau_c(z)]/[T_0(1 + \Delta^2/T_0{}^4)^{1/2}]$, $\tau_c(z) = -\omega_0(\Delta + D_0z_1)$, and $\Psi = \xi^2\Delta/(2T_0{}^2) - \omega_0[\tau - \tau_c(z)]$. We use the notation $z_0 \equiv z$ and $z_1 \equiv \epsilon z$ for the fast and the slow variables, respectively.[1] Thus any function $f(z)$ is to be considered as $f(z_0, z_1, \ldots)$. We ignore the soliton's center coordinate and the z-dependent phase as inessential to this analysis.

When $\mu = 0$, the parameters T_0, a_0, and ω_0 are, respectively, the DM soliton's minimum width, maximum amplitude, and frequency. They are independent of both z_0 and z_1; in addition, a_0 may be taken to be real. We define the so-called strength S of the dispersion map as $S = 1/(2T_0{}^2)$. In physical units,

$$S = \ln 2\,\frac{|(\kappa_2'' - \kappa_{\mathrm{av}}'')L_2 - (\kappa_1'' - \kappa_{\mathrm{av}}'')L_1|}{\tau_{\mathrm{FWHM}}^2}, \tag{4}$$

where τ_{FWHM} is the pulse width at half-maximum and $\kappa_{\mathrm{av}}'' = (\kappa_1''L_1 + \kappa_2''L_2)/L_{\mathrm{map}}$. In the Gaussian approximation the average dispersion is related to the soliton parameters by[3]

$$D_0 = a_0{}^2T_0{}^2I(S)/\sqrt{2}, \tag{5}$$

where $I(S) = 2/(1 + S^2)^{1/2} - [\ln\{(1 + S^2)^{1/2} + S\}]/S$. Note that $I(S) = 0$ for $S \approx 3.3$.

When $0 < \mu \ll 1$, the following slow evolutions of the soliton parameters occur[1]:

$$\frac{a_0}{2(a_0{}^2T_0)}\frac{\mathrm{d}(a_0{}^2T_0)}{\mathrm{d}z_1} = \mu\,\mathrm{Im}\,R_0,$$

$$\frac{a_0T_0}{2}\frac{\mathrm{d}\omega_0}{\mathrm{d}z_1} = \mu\,\mathrm{Re}\,R_1. \tag{6}$$

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Here $\epsilon\mu R$ denotes the perturbation that appears on the right-hand side of Eq. (2). The quantities R_n, $n = 0, 1, \ldots$, are the expansion coefficients of R over the Hermite–Gaussian basis[1]:

$$R_n(z_1) = \int_0^1 \frac{\mathrm{d}z_0}{2^n n! \sqrt{\pi}} \frac{(1 + i\Delta/T_0{}^2)^{n/2+1}}{(1 - i\Delta/T_0{}^2)^{n/2}} \times \int_{-\infty}^{\infty} \mathrm{d}\xi R(\xi, z) H_n(\xi) \exp(-\xi^2/2 - i\Psi), \quad (7)$$

where $H_n(\xi)$ are the Hermite polynomials and the phase, Ψ, is given after Eq. (3). The first of Eqs. (6) gives the slow evolution of the soliton's energy, $E \equiv (a_0{}^2 T_0)$, which is the physical quantity measured by the optical receiver. Using Eq. (7), we obtain $R_0 = 0$ and R_1 is purely real; thus the frequency is the only DM soliton parameter that is affected by the Raman effect:

$$\mathrm{d}\omega_0/\mathrm{d}z_1 = -\mu\sqrt{2}\, a_0{}^2 S/(1 + S^2)^{1/2}. \quad (8)$$

In dimensional units, Eq. (8) is rewritten as follows:

$$\frac{\mathrm{d}\nu_0}{\mathrm{d}Z} = -\frac{\tau_R}{\tau_{\mathrm{FWHM}}^2}\left(\frac{\sqrt{2}\ln 2}{\pi}\right)\frac{\gamma P_0}{(1 + S^2)^{1/2}}\,. \quad (9)$$

In the limit of the uniform fiber ($S \rightarrow 0$), Eq. (9) reduces (within the above-noted 5% accuracy) to the well-known expression for the self-frequency shift of the nonlinear Schrödinger soliton.[4] As the DM strength increases, the frequency shift is suppressed owing to the increase of the pulse-stretching factor, $(1 + S^2)^{1/2}$.[1] In other words, the Raman effect operates most effectively in those spans in which the soliton is narrower. These spans are shorter for stronger maps, and hence the Raman self-frequency shift is suppressed by dispersion management.

Now let us turn to a collision of solitons u_1 and u_2 that propagate in different channels. We assume that their widths (and hence the resulting DM strengths) are the same and that their respective central frequencies, $-\omega_0$ and ω_0, are separated by $\Delta\omega \equiv 2\omega_0 \gg 1$. In general, because of third-order dispersion, solitons u_1 and u_2 will see different average dispersions, ϵD_{01} and ϵD_{02}, respectively. Hence, from Eq. (5), their amplitudes, a_{01} and a_{02}, will also be different. Below we calculate the energy and frequency shifts of each soliton that result from the collisional Raman effect. It should be noted that the analogous problem for the nonlinear Schrödinger solitons has already been studied.[5,6] There it was found that the energy shift of each soliton was independent of the frequency separation $\Delta\omega$, whereas the frequency shift scales as $\Delta\omega^{-1}$. In this Letter we arrive at similar conclusions for DM solitons.

We substitute $u = (u_1 + u_2)$ (thus neglecting any radiation) into Eq. (2) and obtain for u_n ($n = 1$ or 2) an equation of similar form on the left-hand side, and the right-hand side is replaced with

$$\epsilon[-2u_n|u_{3-n}|^2 + \mu u_n(|u_n|^2)_\tau + \mu(u_n|u_{3-n}|^2)_\tau + \mu u_n u_{3-n}(u_{3-n}^*)_\tau]. \quad (10)$$

The first term is the interchannel collisional term.[7–9] The second term is the self-Raman effect considered above. The last two terms describe how the Raman effect influences the soliton parameters during a collision. The explicit form of the relevant part of this perturbation, for the first soliton is

$$R = \frac{\mu a_{01} a_{02}{}^2 \exp(-\xi_2{}^2 - \xi_1{}^2/2 + i\Psi_1)}{T_0(1 + \Delta^2/T_0{}^4)(1 + i\Delta/T_0{}^2)^{1/2}} \times \left[\delta\xi(1 + i\Delta/T_0{}^2) - 4\xi_+ + 2i\omega_0 T_0(1 + \Delta^2/T_0{}^4)^{1/2}\right], \quad (11)$$

where the subscripts 1 and 2 denote the quantities pertaining to the first and the second solitons, respectively, $\xi_+ = (\xi_1 + \xi_2)/2$, and $\delta\xi = (\xi_1 - \xi_2)$. Using the expression for ξ that is found below Eq. (3), we find that $\delta\xi = -[2\omega_0\Delta(z_0) + Vz_1]/\{T_0[1 + \Delta^2(z_0)/T_0{}^4]^{1/2}\}$, where $V = \omega_0(D_{01} + D_{02})$ and we have explicitly indicated the dependence of $\delta\xi$ on both the fast and the slow evolution variables. Now since we are interested only in the total shifts, $\delta E/E$ and $\delta\omega_0$, during the collision, we need to integrate Eqs. (6) over z_1. Thus the final answer will involve two z integrations, the inner one over z_0 [cf. Eq. (7)] and the outer one over z_1. Since z_0 and z_1 are treated as independent variables by the method of multiple scales, we can reverse the order of the integrations.[9] Then all integrals can be evaluated explicitly. Assuming a complete collision (i.e., where the pulses are well separated before and after the collision, and hence the integration over z_1 is from $-\infty$ to ∞), we obtain for the first soliton

$$\frac{\delta E}{E} \equiv \int_{-\infty}^{\infty} \frac{\mathrm{d}(\ln E)}{\mathrm{d}z_1}\mathrm{d}z_1 = -\mu\frac{8\sqrt{\pi}D_{02}S^{1/2}}{(D_{01} + D_{02})I(S)}\,, \quad (12)$$

$$\delta\omega_0 \equiv \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega_0}{\mathrm{d}z_1}\mathrm{d}z_1 = -\mu\frac{4\sqrt{\pi}D_{02}S^{3/2}}{\omega_0(D_{01} + D_{02})I(S)}\,, \quad (13)$$

where we have used Eq. (5). For the second soliton, we simply change the sign in Eq. (12) and replace D_{02} with D_{01} in the numerators of both equations. Equations (12) and (13) reduce to the results for the uniform fiber[5,6] when $S \rightarrow 0$.

It is easy to understand the increase of the energy and frequency shifts in Eqs. (12) and (13) that occurs with the increase of the DM strength [recall that $I(S) = 0$ for $S \approx 3.3$]. As is well established,[7] owing to the periodically varying dispersion the centers of the DM solitons zigzag about one another, resulting in multiple crossings during a complete collision. The Raman energy and the frequency shifts from the individual crossings accumulate and add up (in contrast with the non-Raman interchannel collisional frequency shifts,[7,9] which alternate in sign and thus tend to cancel). Thus the total shifts from a complete collision are enhanced compared with those for a single crossing. We can estimate this enhancement factor by the number of such crossings in a complete collision. The latter number is of the order of the ratio of the DM soliton's instantaneous and average velocities, $(2\omega_0)/V \sim S/[\epsilon I(S)]$, which gives the $I(S)$ dependence in Eqs. (12) and (13). (The factor $1/\epsilon$ is absent there owing to our definition of μ.) When the average dispersion is very low $[I(S) \approx 0]$ the upper limit of the integration in Eqs. (12) and (13) is not infinity but rather depends on z. In that case the energy and frequency shifts both grow linearly (on average) with z.

We also verified the validity of Eqs. (8) and (12) by numerically solving Eq. (2) with $\epsilon = 0.2$, $L_1 = L_2 = 1/2$, $a_0 = 1$, and $D_{01} = D_{02}$ determined from Eq. (5). We considered four sets of parameters: $(S = 0.5, \mu = 0.05)$, $(S = 1, \mu = 0.0125)$, $(S = 1.5, \mu = 0.008)$, and $(S = 2, \mu = 0.003)$. As we increased S, we decreased μ to keep the ratio $\delta E/E$ from becoming too large. For sets 1, 2, 3, and 4 of S and μ, Eq. (12) appears to overestimate the numerical values by 3%, 2%, 5%, and 6.5%, respectively. Equation (8) for the self-frequency shift underestimates the numerical values by approximately 5–6% in all four cases. The frequency shifts given by Eq. (13) could not be resolved in our numerical simulations, because their magnitudes were below our spectral resolution, which was $2\pi/(800\, T_0)$.

Let us now discuss how a periodically compensated fiber loss would affect these results. Its effect can be included in Eq. (2) by multiplication of the nonlinear terms by a periodic function $G(z)$,[1] with $\int_0^1 G(z)\mathrm{d}z = 1$. Calculations analogous to those presented above yield the following: First, Eq. (8) is modified by only a numerical factor of order 1. Second, Eqs. (12) and (13) do not change their form, although the explicit expression for $I(S)$ changes slightly.[1] We can explain this invariance of the energy and frequency shifts with respect to the form of $G(z)$ by simply noting that a collision of DM solitons occurs over many amplifications stages, and therefore the loss-induced periodic variations of the pulse power are smeared out.

Now, in a DM transmission line of length z, there would occur approximately $N = z/z_{bc}$ complete collisions between any two channels, where z_{bc} is the minimum distance between interchannel collisions. Obviously, $z_{\mathrm{bc}} \sim 8T_0/(\epsilon V)$, where we assume the timing window to be ~ 8 $(\approx 5 \times 2\sqrt{\ln 2}\,)$ Gaussian widths of each pulse and their relative velocity to be $V \approx 2\epsilon\omega_0 D_0$. Using Eq. (5), we obtain the following estimate for the number of collisions: $N \sim \epsilon z I(S)(2\omega_0 T_0)/(8\sqrt{2}\,)$. Assuming that the spectral separation between two neighboring channels is ~ 5 spectral widths of each pulse, whence $2\omega_0 T_0 = 5$,[2] and summing over $M - 1$ channels (if the total number of channels is M), we find that a DM soliton in the channel with the lowest or the highest wavelength has approximately $(1/2)\epsilon z I(S) M(M - 1)/4$ collisions (the factor 1/2 in front of ϵ accounts for the probability of having a soliton in each time slot). Multiplying this number by the right-hand side of Eq. (12), we find that the relative shift of the pulse energy induced by the Raman effect in all the collisions can be as large as

$$\left(\frac{\delta E}{E}\right)_{\mathrm{total}} \sim \gamma P_0 Z M(M - 1)\frac{\tau_R}{\tau_{\mathrm{FWHM}}}, \tag{14}$$

in dimensional units. This result does not depend on the DM strength because neither does the total number of interchannel soliton crossings. As an example, for a pulse with the peak power $P_0 = 2$ mW, $\tau_{\mathrm{FWHM}} = 5$ ps, and for typical values[2] $\gamma = 2\ \mathrm{W}^{-1}\,\mathrm{km}^{-1}$, $\tau_R = 5$ fs, relation (14) yields an energy shift that is close to 50% for $Z = 10^4$ km and $M = 4$ channels.

For the frequency shift it is more important to estimate the relative timing jitter between two neighboring solitons than the total frequency shift of a single soliton.[10] Thus a critical difference arises between the DM case and that of a uniform, periodically amplified fiber. In the latter a relative frequency shift of two consecutive solitons could differ considerably since, in principle, each soliton can experience most of its collisions either just before or just after the amplifiers. Hence the maximum timing jitter is roughly proportional to the frequency shift accumulated over distance z. In a DM system, as noted above, the collision is smeared out over many amplification stages, and hence the relative frequency shift between two consecutive solitons can differ, at most, by the frequency shift that occurs over one collision.[10] The resulting timing jitter is then estimated as $\delta\omega_0 z_{\mathrm{bc}}$. For $L_{\mathrm{map}} = 40$ km, $S = 2$, and with all the other parameters the same as above, this estimate yields timing jitter of less than 10% of the timing window. As this value is independent of the transmission distance [provided that $z \gg z_{\mathrm{bc}}$, which holds unless the average dispersion is too close to zero], we conclude that the collisional Raman frequency shift does not present a serious problem for (unfiltered) DM systems with many wavelength channels. However, the corresponding energy shifts can indeed present a problem for such systems.

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