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Spectrum-induced changes in diffraction of pulsed optical beams

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Abstract

Propagation of ultrashort optical pulses in a linear homogeneous medium is studied using the standard diffraction theory of electromagnetic fields. Diffraction pattern of a Gaussian beam consisting of such short pulses depends on the pulse shape. In general, the pulsed beam does not remain Gaussian on propagation and its spot size is enhanced. In the visible region, changes in the diffracted intensity become noticeable only for pulse widths below 10 fs and are larger for chirped pulses and for pulses whose spectrum has long tails. We also show that the diffraction characteristics of coherent but pulsed and CW but partially coherent Gaussian beams are identical if the two have identical optical spectra.

1. Introduction

Propagation-induced changes in the spectrum of partially coherent light have attracted considerable attention recently [1]. It has also been realized that the broad spectrum of partially coherent light can lead to propagation-induced changes in the shape of an optical beam [2,3]. Indeed, the conditions under which the beam shape remains invariant on propagation were investigated as early as 1984 [2]. It was shown in 1990 that a partially coherent, continuous-wave (CW), Gaussian beam whose spot size at the beam waist is frequency independent does not remain Gaussian on propagation in a linear medium (such as free space) if its spectrum is relatively broad [3]. One may ask if that would also happen for a coherent Gaussian beam consisting of ultrashort optical pulses such that the field spectrum is quite broad. Surprisingly, a definite answer to this simple question is not provided in the literature. Of course, propagation of ultrashort optical pulses in a linear optical medium has been studied extensively in recent years [4–11]. However, most studies assume a Gaussian shape for the ultrashort pulse. In practice, pulses emitted from passively mode-locked lasers often have a non-Gaussian shape, a ‘‘sech’’ profile being the common one [12]. Kaplan has recently considered pulses of arbitrary shape [13] but focused mostly on the temporal evolution of the on-axis intensity.

In this paper, we consider the spatial aspects of diffracted pulses of arbitrary shapes assuming that the transverse intensity distribution of the beam is initially Gaussian. Propagation of such pulsed beams in a linear homogeneous medium can be studied by using the standard diffraction theory of electromagnetic fields [14]. We show that, in general, the pulsed optical beam does not remain Gaussian on propagation and that the diffracted intensity profile depends on the pulse spectrum. We focus on the spectrum-induced changes and study how the shape and size of the diffracted beam depend on the initial pulse shape and on the frequency chirp, when the input pulse is not transform-limited.

2. Diffracted field

In a linear and homogeneous dielectric medium, Maxwell's equations for the electromagnetic field $E(x,y,z,t)$ can be solved in the frequency domain using the Helmholtz equation [14]

$$(\nabla^2 + k^2)\tilde{E}(x,y,z,\omega) = 0, \tag{1}$$

where $\tilde{E}$ is the Fourier component of E at the frequency

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ω, $k(\omega) = n\omega/c$, n is the refractive index of the linear medium assumed to be nondispersive, and c is the speed of light in vacuum. The vector nature of the electromagnetic field is ignored here assuming that the plane of polarization does not change during propagation.

We consider the case in which $\tilde{E}$ is known at the plane $z=0$ and Eq. (1) is solved in the region $z>0$. For a pulsed Gaussian beam having its beam waist at the input plane $z=0$, $\tilde{E}(x,y,0,\omega)$ can be written as

$$\tilde{E}(x,y,0,\omega) = \exp\left[-(x^2+y^2)/2a^2\right] S(\omega), \tag{2}$$

where a is related to the spot size and is assumed to be frequency independent. This assumption holds if the input field can be factored as $E(x,y,0,t) = F(x,y)A_0(t)$, where $F(x,y)$ and $A_0(t)$ govern the beam shape and the pulse shape, respectively. It may not hold within a stable laser cavity whose Hermite–Gaussian modes are characterized by a frequency-dependent spot size at the beam waist [2]. $S(\omega)$ is the complex spectral amplitude of the pulse obtained by taking the Fourier transform of $A_0(t)$.

By using the angular-spectrum representation or the Green-function approach, one can solve Eq. (1) easily [14]. If we make the paraxial approximation, we obtain the standard result for each spectral component of the diffracted Gaussian beam [15]:

$$\tilde{E}(x,y,z,\omega) = \frac{S(\omega)}{1+id}\exp\left(-\frac{x^2+y^2}{2a^2(1+id)}\right)\exp(ikz), \tag{3}$$

where d is the normalized propagation distance defined as

$$d(\omega) = \frac{z}{ka^2} = \frac{zc}{na^2\omega}, \tag{4}$$

The propagated field is obtained by taking the inverse Fourier transform of Eq. (3) and is given by

$$E(x,y,z,t) = \int_{-\infty}^{\infty} \frac{S(\omega)}{1+id}\exp\left(-\frac{x^2+y^2}{2a^2(1+id)}\right) \times\exp[-i\omega(t-z/v)]\,d\omega, \tag{5}$$

where $v = c/n$ is the phase velocity in the medium. For a CW beam at the frequency ω_0, $S(\omega) = \delta(\omega-\omega_0)$, and we recover the standard result showing that a Gaussian beam remains Gaussian on propagation in free space [15]. For a pulsed beam, the spectrum can become broad enough for ultrashort optical pulses that one must consider the ω dependence of d in Eq. (5). In that case, a pulsed beam may not remain Gaussian on propagation.

The integral in Eq. (5) can be evaluated in a closed form only in a few special cases. For a Gaussian-shaped pulse, $S(\omega)$ is also Gaussian. If we assume $d \gg 1$ for all frequencies, the far-field distribution can be obtained in a closed form [9]. In fact, if we replace $1+id$ by id in Eq. (5), we obtain the general result valid for all pulse shapes

$$E(x,y,z,t) = \frac{a^2}{zv}\frac{\partial A_0}{\partial \tau}, \tag{6}$$

where $\tau = t - [z + (x^2+y^2)/2z]/v$ is the reduced time and $A_0(t)$ is the input pulse shape. The pulse shape in the far field is then related to the first derivative of the input pulse [9,13].

3. Intensity distribution

Eq. (5) can be used to obtain the spatial distribution of the instantaneous intensity. However, practical photodetectors cannot respond at femtosecond time scales, making it hard to verify the results. It is more useful to integrate the instantaneous intensity over time and consider the spatial distribution of the pulse energy (or the time-averaged intensity in the case of a pulse train) defined as

$$I(x,y,z) = \int_{-\infty}^{\infty} |E(x,y,z,t)|^2\,dt. \tag{7}$$

The spatial distributions of $I(x,y,z)$ and $|E(x,y,z,t)|^2$ are generally different [9]. We focus on $I(x,y,z)$ since this quantity is easier to measure experimentally and refer to it as the intensity distribution assuming that the pulsed beam consists of a pulse train. The repetition rate is small enough (<100 MHz) for most lasers that $S(\omega)$ can be treated as a smooth function even for a pulse train.

By using E from Eq. (5) in Eq. (7) and performing the integration over time, we obtain the simple result

$$I(x,y,z) = \int_{-\infty}^{\infty} \frac{|S(\omega)|^2}{1+d^2}\exp\left(-\frac{x^2+y^2}{a^2(1+d^2)}\right)d\omega, \tag{8}$$

where $d(\omega)$ [given in Eq. (4)] varies inversely with ω and the pulse spectrum is normalized such that $\int_{-\infty}^{\infty}|S(\omega)|^2\,d\omega = I_0$, I_0 being the on-axis intensity at $z=0$. Eq. (8) is our main result and can be used to study the diffraction characteristics of a pulsed beam for arbitrary pulse shapes. Interestingly, this equation is identical to the one obtained in Ref. [3] for partially coherent CW Gaussian beams. As a result, the diffraction characteristics of coherent but pulsed and CW but partially coherent Gaussian beams are identical if the two have the same optical spectra. Of course, the origin of spectral broadening is quite distinct in the two cases. The role of pulse width is played by the coherence time in the case of partially coherent CW beams. Note also that the field spectrum $|S(\omega)|^2$ in Eq. (8) is replaced by the power spectrum related to the Fourier transform of the autocorrelation function associated with the partially coherent CW beam [14]. We refer to both of them as ‘‘optical spectrum’’ in this paper.

It is evident from Eq. (8) that, in general, a pulsed Gaussian beam does not remain Gaussian on propagation and that the shape and width of the diffracted beam depend on the pulse spectrum. However, if $d(\omega)$ varies little over the pulse spectrum and can be replaced by a constant, the spectrum-induced changes become small enough to be negligible, and the diffracted beam retains its Gaussian character. Thus, in practice, deviations from the Gaussian nature will be noticeable only for ultrashort optical pulses having a broad spectrum. As an example, Fig. 1 shows changes in the diffracted intensity occurring for a ''sech'' pulse with $A_0(t) = \mathrm{sech}(t/T_0)$ when its full width at half maximum (FWHM) is reduced from 5 to 2 fs. The quantity plotted is the relative deviation

$$\Delta(r,z) = [I(r,z) - I_{cw}(r,z)]/I(r,z), \tag{9}$$

where $r = \sqrt{x^2 + y^2}$ is the radial distance and I_{cw} is the intensity of a CW Gaussian beam obtained by using a narrow spectrum in Eq. (8). The propagation distance $z = 2L_d$ in Fig. 1, where L_d is the diffraction length at the center wavelength of the pulse spectrum taken to be 800 nm (e.g., a mode-locked Ti:sapphire laser). For pulses wider than 10 fs, $\Delta \approx 0$, and the diffraction properties of the pulsed beam reduce to those of a CW beam. However, Δ varies by a few percent even for a 5 fs pulse, and deviations begin to exceed 10% for a 2 fs pulse. Thus, such effects should be observable in practice as pulse width is reduced to below 5 fs.

We have considered several different pulse shapes. The results for a Gaussian pulse are similar to those shown in Fig. 1. However, as seen in Fig. 2, deviations become considerably larger for an exponential pulse for which $A_0(t) = \exp(-|t|/T_0)$. This is not surprising if we note

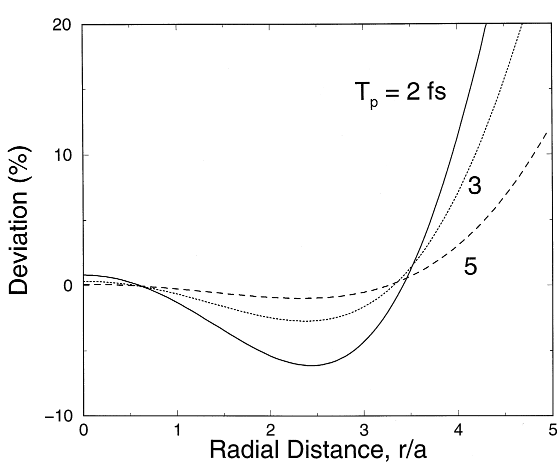

Fig. 1. Spectrum-induced changes in the diffracted intensity as a function of radial distance for ''sech'' pulses of three different pulse widths (FWHM). Deviation in the intensity of the pulsed optical beam from that of a CW beam is plotted at a distance $z = 2L_d$. The pulse spectrum is centered at a wavelength of 800 nm.

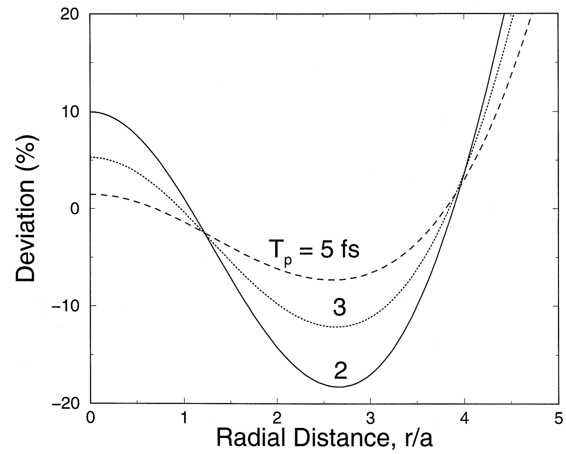

Fig. 2. Same as in Fig. 1 except that the input pulse has a Lorentzian spectrum. Deviations are larger for such a pulse because of long spectral tails.

that the Lorentzian spectrum of such pulses has long tails that decay as ω^{-2} rather than exponentially.

So far, we have assumed that input pulses have a constant phase and are thus transform-limited. In practice, optical pulses often acquire a time-dependent phase, resulting in a frequency that varies with time. Such pulses are referred to as being chirped. We have found that the frequency chirp has a dramatic effect on the diffracted beam. As an example, Fig. 3 shows the intensity profiles at $z = 2L_d$ for 5 fs (FWHM) chirped Gaussian pulses ($T_0 = 3$ fs) using $A_0(t) = \exp[-(1 + iC)t^2/2T_0^2]$, where C is the chirp parameter [16]. The diffracted beam is nearly Gaussian when pulses are not chirped ($C = 0$) but acquires a

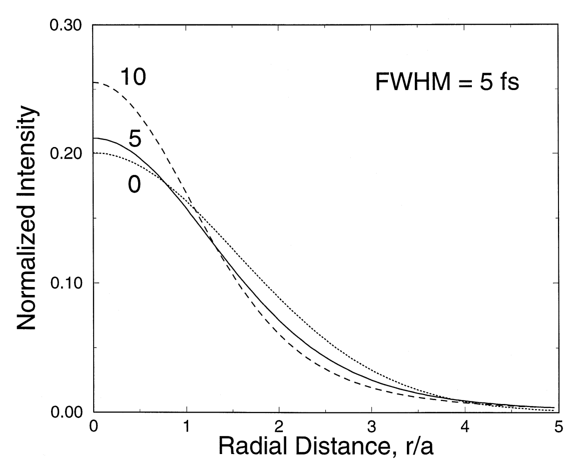

Fig. 3. Radial intensity distribution at a distance $z = 2L_d$ for a pulsed optical beam consisting of chirped Gaussian pulses for three values of the chirp parameter C. The pulse spectrum is centered at a wavelength of 800 nm.

non-Gaussian shape for even relatively small values of the chirp parameter ($C \sim 1$). Notice the considerable enhancement of the on-axis intensity for chirped pulses. As a result, the beam appears to have a smaller spot size (on the basis of FWHM) even though it spreads over a larger radial region. These features can be understood by noting that the spectrum of a linearly chirped Gaussian pulse, having a quadratic variation of phase across the pulse, broadens by a factor of $(1+C^2)^{1/2}$ compared with that of the unchirped pulse [16].

4. Spot size

It is seen in Figs. 1–3 that a pulsed diffracted beam has a higher intensity at the beam center but, at the same time, it spreads more than a CW Gaussian beam. To quantify such effects, we estimate the spot size through the root-mean-square (RMS) width σ defined using

$$\sigma^2(z) = \frac{\int_0^\infty r^2 I(x,y,z)\, r\, \mathrm{d}r}{\int_0^\infty I(x,y,z)\, r\, \mathrm{d}r}, \tag{10}$$

where $r = (x^2+y^2)^{1/2}$ is the radial distance. Using Eq. (8), the integration over r can be carried out analytically, resulting in the following simple expression

$$\sigma^2(z) = 1 + (z/L_\mathrm{d})^2 F_p^2, \tag{11}$$

where

$$F_p^2 = \int_{-\infty}^{\infty} \frac{\omega_0^2}{\omega^2} |S(\omega)|^2 \,\mathrm{d}\omega \tag{12}$$

and $L_\mathrm{d} = \omega_0 a^2/v$ is the diffraction length at the frequency ω_0 at which the pulse spectrum is centered. For a relatively wide pulse (or for a CW beam) $F_p = 1$, and we recover the standard result well known for CW Gaussian beams [15]. The factor F_p depends on the pulse shape only and is a rough measure of the change in the spot size induced by the pulse spectrum.

The factor F_p can be evaluated in a closed form only in a few special cases. However, it can be expressed in the form of a convergent series by using the transformation $f = (\omega - \omega_0)T_0$, where T_0 is related to the pulse width. Eq. (12) then becomes

$$F_p^2 = \int_{-\infty}^{\infty} (1+\delta f)^{-2} |S(f)|^2 \,\mathrm{d}f, \tag{13}$$

where $\delta = (\omega_0 T_0)^{-1}$ is a small parameter and indicates the spectral width of the pulse normalized to the carrier frequency. By expanding $(1+\delta f)^{-2}$ in a Taylor series and assuming a symmetric pulse spectrum, we obtain

$$F_p^2 = \sum_{m=0}^{\infty} (2m+1)\delta^{2m} \langle f^{2m} \rangle, \tag{14}$$

where $\langle f^n \rangle = \int_{-\infty}^{\infty} f^n |S(f)|^2 \,\mathrm{d}f$ denotes the nth moment of the pulse spectrum. The spectral moments can be expressed in a closed form for several common pulse shapes including the Gaussian and ‘‘sech’’ pulses. In practice, only a few terms in the series need to be included if $\delta \ll 1$. As an example, consider a chirped Gaussian pulse with $A_0(t) = \exp[-(1+iC)t^2/2T_0^2]$. Retaining the terms up to $m=2$ in Eq. (14), we obtain the following expression for F_p:

$$F_p \approx 1 + \tfrac{3}{4}\delta^2(1+C^2) + \tfrac{51}{32}\delta^4(1+C^2)^2, \tag{15}$$

where $\delta = (\omega_0 T_0)^{-1}$ and C is the chirp parameter.

It is worthwhile to consider a simple practical example. Shortest optical pulses (<5 fs wide) have been generated from Ti:sapphire lasers operating in the spectral region near 800 nm [17]. For 5 fs pulses ($T_0 = 3$ fs) with a spectrum centered at 800 nm, $\delta = 0.085$. If pulses are unchirped, there is only a 0.5% enhancement in the RMS spot size compared with the CW case. However, the enhancement becomes 20% for chirped Gaussian pulses with $C = 5$ and increases rapidly for larger values of C. Such changes should be observable with the existing mode-locked lasers. If input pulses are transform-limited, they can be chirped easily by using the nonlinear phenomenon of self-phase modulation in a nonlinear medium such as an optical fiber [18] when the pulsed beam is intense enough.

5. Concluding remarks

In this paper we have considered propagation of a pulsed optical beam in a linear, homogeneous, nondispersive medium with focus on the spectrum-induced changes in the diffracted beam. The input beam is initially Gaussian (in transverse directions) but consists of a train of ultrashort optical pulses of arbitrary shape. The propagation problem is solved analytically by using the diffraction theory of electromagnetic fields. In general, a pulsed beam does not remain Gaussian on propagation, and the diffracted intensity depends on the pulse shape. In the visible region, changes in the diffracted intensity become noticeable only for pulse widths below 10 fs and are larger for pulses whose spectrum has long tails. We also studied the effect of frequency chirp and found that it affects considerably the shape and size of the diffracted beam. The RMS spot size is generally enhanced for pulsed optical beams and can double in size for chirped pulses. The predicted effects are large enough that they can be observed with the existing mode-locked lasers.

As a side remark, we have shown that the diffraction characteristics of coherent but pulsed and CW but partially coherent Gaussian beams are identical if the two have identical optical spectra (in the sense of Section 3). Of course, the origin of spectral broadening is quite distinct in the two cases. The role of pulse width is played by the coherence time in the case of partially coherent CW beams.

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