

Absolute Instabilities in Lasers with Host-Induced Nonlinearities and Dispersion

Guido H. M. van Tartwijk and Govind P. Agrawal, *Fellow, IEEE*

Abstract—We analyze the occurrence of absolute instabilities in lasers that contain a dispersive host material with third-order nonlinearities. Starting from the Maxwell–Bloch equations, we derive general multimode equations to distinguish between convective and absolute instabilities. We find that both self-phase modulation and intensity-dependent absorption can dramatically affect the absolute stability of such lasers. In particular, the self-pulsing threshold (the so-called second laser threshold) can occur at few times the first laser threshold even in good-cavity lasers for which no self-pulsing occurs in the absence of intensity-dependent absorption.

Index Terms—Laser stability, nonlinear optics, optical fiber lasers, optical Kerr effect, optical pulse generation, optical propagation in dispersive media.

I. INTRODUCTION

ALMOST immediately after the advent of the laser, it was recognized that laser output can become unstable, resulting in irregular power spikes even at a constant pumping level [1]. Over the last 30 years or so, laser instabilities have been studied extensively both from the fundamental and applied viewpoints [2], [3]. The fundamental studies have led to the flourishing field of optical chaos. On the applied side, the development of techniques for controlling chaos are being used to make lasers tailored for specific applications (high power, short pulses, clean far field, etc.).

Since deterministic chaos is studied in a wide variety of disciplines, the understanding of laser instabilities can be improved by referring to plasma and fluid instabilities that have been studied for a long time. A famous example is provided by the Lorenz–Haken equations which are named after the fluid dynamicist Lorenz and the laser theorist Haken [3], [4]. In fluid dynamics, instabilities are categorized into two types: convective and absolute [5]. Convective instabilities are characterized by the growth of localized perturbations upon propagation inside a nonlinear medium, while absolute instabilities exhibit purely temporal dynamics. Absolute laser instabilities have been studied for more than 30 years. The Lorenz–Haken equations describe the dynamics of a homo-

geneously broadened gain medium in a unidirectional ring-cavity. Although rarely stated explicitly, the Lorenz–Haken equations can only show absolute instabilities. The fundamental concepts such as second laser threshold, self-pulsing, Hopf bifurcation, and different routes to chaos are all formulated within the context of absolute laser instabilities [3].

In the last 15 years or so, new laser systems have been designed that are not easily modeled by the Lorenz–Haken equations. Examples of such lasers are fiber lasers and solid-state (e.g., Ti:sapphire) lasers, which are capable of producing ultrashort optical pulses through passive mode locking while operating at a constant pump power. What these lasers have in common is that the gain is provided by atoms or ions doped inside a host material. As a result, the cavity contains not only a gain element but also other nonlinear elements, which are responsible for nonlinear processes such as self-phase modulation (SPM) and intensity-dependent absorption (IDA) [6]. Also, group-velocity dispersion (GVD) of the host medium plays a nonnegligible role. Because of the dispersive and nonlinear effects, evolution of the optical field over a single round trip must be considered, contrary to the Lorenz–Haken model in which such effects are ignored. This means that the convective nature of any instability must be considered while discussing instabilities for such lasers.

A well-known example of a convective instability occurs in nonlinear fiber optics [6]. Optical fibers, without any gain element and without any longitudinal resonances (no cavity), show a convective instability known as the modulation instability. When the power of a CW optical beam becomes sufficiently large, the combination of SPM and anomalous GVD causes the CW beam to break up spontaneously into a pulse train (and eventually into optical solitons) whose repetition rate depends on the fiber parameters. Mathematically, a linear stability analysis shows that perturbations of the form $\exp[-i(\Omega t - Kz)]$ grow exponentially as $\exp(gz)$ with a growth rate $g = -\text{Im}(K)$ that depends on the frequency of perturbation. The repetition rate of the resulting pulses corresponds to the frequency Ω for which the growth rate g is maximum.

Adding gain to the system, e.g., by doping the fiber with rare-earth ions and pumping it optically, can affect considerably the conditions under which modulation instability arises [7]. The instability, however, remains convective in nature. When such a host material (with or without gain) is put into a cavity, the resulting boundary conditions at the cavity mirrors can change the nature of the instability from convective to

Manuscript received October 31, 1997; revised June 12, 1998. This work was supported in part by the NASA Ames Research Center and the National Science Foundation under Grant PHY94-15583.

G. H. M. van Tartwijk was with the Institute of Optics, University of Rochester, Rochester, NY 14627 USA. He is now with Uniphase Netherlands B.V., 5656 AA Eindhoven, The Netherlands.

G. P. Agrawal is with the Institute of Optics and Rochester Theory Center for Optical Science and Engineering, University of Rochester, Rochester, NY 14627 USA.

Publisher Item Identifier S 0018-9197(98)07176-0.

absolute. Feedback is a necessary ingredient for absolute instabilities to occur. A well-known example is the Ikeda instability [8], which arises when a Kerr medium is placed in a unidirectional ring cavity. Even without gain and dispersion, the feedback mechanism provided by the cavity results in an absolute instability.

In this paper, we discuss under what conditions a convective instability becomes absolute in a laser. In Section II, we derive, starting from the Maxwell–Bloch equations, a set of Lorenz–Haken-type multimode equations capable of describing the temporal evolution of a laser whose cavity includes optical elements exhibiting dispersion and nonlinearities. The usefulness of this new set of equations is illustrated in Section III by considering a relatively simple case of a single-mode laser. We discuss the stability of that mode as a function of host nonlinearities.

II. MAXWELL–BLOCH EQUATIONS

For definiteness, we focus on a fiber laser although the analysis can be applied to any solid-state laser with some modifications. Our starting point is a set of Maxwell–Bloch equations describing the propagation of optical fields in an optical fiber, doped with rare-earth ions. We write the optical field $\vec{\mathcal{E}}$ and the dopant-induced polarization $\vec{\mathcal{P}}$ as

$$\vec{\mathcal{E}}(x, y, z, t) = \frac{1}{2} \hat{x} F(x, y) A(z, t) \exp[i(K_0 z - \omega_0 t)] + \text{c.c.} \quad (1)$$

$$\vec{\mathcal{P}}(x, y, z, t) = \frac{1}{2} \hat{x} F(x, y) B(z, t) \exp[i(K_0 z - \omega_0 t)] + \text{c.c.} \quad (2)$$

where \hat{x} is the polarization unit vector of light assumed to be linearly polarized along the x axis, $F(x, y)$ is the transverse profile of the fundamental fiber mode, and K_0 is the wavenumber corresponding to the optical frequency ω_0 . We assume that the field-polarization direction is preserved upon propagation. After substituting (1) and (2) in Maxwell's equations, modeling dopants as a homogeneously broadened two-level system, and making use of the slowly varying envelope and rotating-wave approximations, we obtain the following equations for the slowly varying complex amplitudes A and B [6]:

$$\frac{\partial A}{\partial z} + \frac{1}{v_g} \frac{\partial A}{\partial t} = \frac{i}{2} B - \frac{1}{2} \alpha A - \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + i\gamma |A|^2 A \quad (3)$$

$$T_2 \frac{dB}{dt} = -(1 - i\hat{\delta})B - iAg \quad (4)$$

$$T_1 \frac{dg}{dt} = g_0 - g + \text{Im}(A^* B) / P_s \quad (5)$$

where g is the gain realized by pumping the dopants, α is the optical loss of the host fiber, T_1 is the population lifetime of the dopants, T_2 is the dipole-dephasing time, v_g is the group velocity, β_2 is the GVD coefficient of the host fiber, the complex parameter γ accounts for the host nonlinearities responsible for SPM and IDA, $\hat{\delta} = (\omega_0 - \omega_A)T_2$ is the scaled

detuning between the optical frequency ω_0 and the atomic resonance frequency ω_A , g_0 is the unsaturated gain, and P_s is the saturation power for the dopants. We have written (3)–(5) in such a way that A has units of \sqrt{W} , B has units of $\sqrt{W} \cdot \text{m}^{-1}$, and g has units of m^{-1} .

The main assumptions in our model are the use of a homogeneously broadened gain medium and the neglect of the stochastic nature of spontaneous emission. The former is not valid for all dopants but is a reasonable assumption for many types of dopants [6]. The latter can be justified if one is interested only in deterministic instabilities.

There are two distinct origins of the nonlinear effects in (3)–(5). The host nonlinearity $\gamma \equiv \gamma' + i\gamma''$ accounts for SPM and IDA effects induced by the silica fiber. The SPM effects are governed by $\gamma' = n_2 \omega_0 / c A_{\text{eff}}$, where n_2 is the nonlinear-index coefficient, c is the speed of light in vacuum, and A_{eff} is the effective mode area [6]. The effects of IDA are accounted for by γ'' . When $\gamma'' > 0$, the loss in the cavity increases with intensity, modeling processes such as two-photon absorption [6]. In contrast, negative values for γ'' imply a decrease in cavity losses with increasing intensity and model fast saturable absorption. The dopant-induced nonlinear effects are governed by the saturation power $P_s \equiv \hbar^2 c n \epsilon_0 A_{\text{eff}} / (2\mu^2 T_1 T_2)$, where \hbar is Planck's constant divided by 2π , μ is the dipole moment of the atomic transition, and n is the background refractive index.

The Maxwell–Bloch equations, together with the boundary conditions imposed by the laser cavity, provide the most general framework for studying laser instabilities. They are capable of handling both convective and absolute instabilities and can show transitions between them. However, their solutions require a numerical approach. Without host nonlinearities ($\gamma = 0$) and without GVD ($\beta_2 = 0$), the steady-state solutions can be obtained, and their linear stability properties have been studied [9]. However, such an approach is quite cumbersome, and it is not easy to carry out the analysis after the inclusion of host nonlinearities and GVD. If one is interested only in absolute instabilities, an analytic approach can be developed, as discussed in the next section.

III. MULTIMODE LASER EQUATIONS

Rather than solving (3)–(5) numerically, we make use of the fact that any cavity supports a set of longitudinal modes whose field distribution $f_m(z, t)$ reproduces itself after each round trip inside the cavity. These modes can be obtained by solving (3) with $B = 0$ (no gain in the fiber cavity) and using the appropriate boundary conditions at the cavity mirrors. For a high- Q laser cavity, one can distribute the mirror losses throughout the cavity and replace the fiber loss α in (3) with $\alpha_T = \alpha + \alpha_M$, where α_M is the distributed mirror loss. The boundary condition then simply becomes $A(L, t) = A(0, t)$, where L is the cavity length. For a Fabry–Perot cavity with mirror reflectivities R_1 and R_2 , α_M is given by

$$\alpha_M = \frac{1}{2L} \ln \left(\frac{1}{R_1 R_2} \right). \quad (6)$$

The approximation that the localized mirror loss can be replaced by a distributed loss only holds for a high- Q laser cavity [2].

When dealing with a unidirectional ring laser without host dispersion and nonlinearities, the form of $f_m(z, t)$ becomes simply $\exp[-i\omega_m(t - z/v_g)]$, where ω_m are the mode frequencies and the loss term has been ignored. In the presence of GVD and SPM, $f_m(z, t)$ may depend on time in a complicated manner, but the temporal patterns remain invariant from round trip to round trip. However, the amplitude of such “temporal modes” can grow from round trip to round trip if an absolute instability occurs. It is thus important to consider time scales describing short-term and long-term dynamics, distinguished on the basis of the round-trip time $T_r = L/v_g$. The field-propagation equation (3) describes both the long-term and short-term effects. By making the coordinate transformation [10]

$$\tau = t - z/v_g, \quad T = z/v_g \quad (7)$$

we distinguish between the propagation variable τ describing evolution over a single round trip, and the long-term timescale T measured in units of T_r . The steady-state solutions of (3)–(5) are found by putting the derivative with respect to T equal to zero and applying the boundary conditions at the cavity mirrors.

In general, a laser can operate in several longitudinal modes simultaneously. Thus, the laser field can be written as $A_s(\tau) = \sum_m a_m f_m(\tau)$. If this steady state is perturbed, the expansion coefficients a_m become a function of T . We can study the long-term stability of the steady state by expanding the field A and the polarization B into a set of longitudinal modes

$$A(T, \tau) = \sum_m a_m(T) f_m(\tau) \quad (8)$$

$$B(T, \tau) = \sum_m b_m(T) f_m(\tau). \quad (9)$$

When the set of functions $\{f_m(\tau)\}$ is complete, the modal expansion in (8) and (9) is exact. However, it is hard to prove completeness of $\{f_m(\tau)\}$ under all operating conditions. As a practical matter, the infinite sum in (8) and (9) is always truncated. This is justified since only a finite number of longitudinal modes are excited in a laser at a given pump power.

After expansions (8) and (9) are substituted in the Maxwell–Bloch equations (3) and (4) using the coordinate transformation (7), we multiply the equations for a_m and b_m by f_q^* , and integrate them over one round-trip time T_r . We then obtain

$$\begin{aligned} & \frac{1}{v_g} \sum_m \frac{da_m}{dT} G_{mq} \\ &= \frac{i}{2} \sum_m b_m G_{mq} - \frac{1}{2} \alpha_T \sum_m a_m G_{mq} \\ & \quad - \frac{i}{2} \beta_2 \sum_m a_m D_{mq} + i\gamma \sum_{m,n,p} a_m a_n^* a_p C_{mnpq} \end{aligned} \quad (10)$$

$$\begin{aligned} & T_2 \sum_m \frac{db_m}{dT} G_{mq} \\ &= -(1 - i\hat{\delta}) \sum_m b_m G_{mq} - i \sum_m a_m g_{mq} \end{aligned} \quad (11)$$

where the coefficients G_{mq} , D_{mq} , and C_{mnpq} are defined as

$$G_{mq} = \int_0^{T_r} f_m f_q^* d\tau \quad (12)$$

$$D_{mq} = \int_0^{T_r} \frac{d^2 f_m}{d\tau^2} f_q^* d\tau \quad (13)$$

$$C_{mnpq} = \int_0^{T_r} f_m f_n^* f_p f_q^* d\tau. \quad (14)$$

The values of these coefficients determine how many modes should be considered, i.e., at what point the infinite sum in (8) and (9) can be truncated.

The complex gain coefficient g_{mq} in (11) is given by

$$g_{mq} = \int_0^{T_r} g(T, \tau) f_m(\tau) f_q^*(\tau) d\tau \quad (15)$$

and should be interpreted as follows. When $m = q$, g_{mq} represents the modal gain for the mode $f_{m=q}$, while for $m \neq q$ g_{mq} represents gain modulation because of mode beating (sometimes referred to as population pulsations). From (5), we readily obtain the rate equation for the coefficients g_{mq} :

$$T_1 \frac{dg_{pq}}{dT} = g_0 G_{pq} - g_{pq} - \frac{i}{2P_s} \sum_{m,n} (a_m^* b_n - a_n^* b_m) C_{mnpq}. \quad (16)$$

Note that we have not made any assumptions about the orthogonality of the longitudinal modes. The nonorthogonality of longitudinal modes has attracted considerable attention in recent years [11]–[13]. In general, the more “open” a resonator is (the larger the mirror loss), the less orthogonal the longitudinal modes are. For fiber lasers, longitudinal modes are expected to be nearly orthogonal since mirror losses are often kept to a minimum.

If we assume that the functions $\{f_m\}$ are orthonormal, the coefficients G_{mq} reduce to δ_{mq} . The multimode equations (10), (11), and (16) then transform into the following set of Lorenz–Haken-type equations:

$$\begin{aligned} \frac{1}{v_g} \frac{da_q}{dT} &= \frac{i}{2} b_q - \frac{1}{2} \alpha_T a_q - \frac{i}{2} \beta_2 \sum_m a_m D_{mq} \\ & \quad + i\gamma \sum_{m,n,p} a_m a_n^* a_p C_{mnpq} \end{aligned} \quad (17)$$

$$T_2 \frac{db_q}{dT} = -(1 - i\hat{\delta}) b_q - i \sum_m a_m g_{mq} \quad (18)$$

$$\begin{aligned} T_1 \frac{dg_{pq}}{dT} &= g_0 \delta_{pq} - g_{pq} \\ & \quad - \frac{i}{2P_s} \sum_{m,n} (a_m^* b_n - a_n^* b_m) C_{mnpq}. \end{aligned} \quad (19)$$

Equations (17)–(19) form a complete set of equations describing the dynamics of various longitudinal modes of the laser. They include the effects of host dispersion and nonlinearity through the terms containing β_2 and γ in (17). What constitutes SPM in a propagation-based description now splits into several different kinds of nonlinearities in the modal description. The triple sum in (10) describes the phenomenon of SPM when $m = n = p = q$, cross-phase modulation

for $m = q$ and $n = p$, and four-wave mixing for other combinations of m , n , and p [6].

Equations (17)–(19) constitute the main result of this paper. In principle, they can be used to study the phenomenon of passive mode locking by including a larger and larger number of longitudinal modes as the mode-locked pulses become shorter. When the mode-locked pulse pattern becomes unstable, the laser may switch to a higher harmonic mode-locked operation [14] or even enter a chaotic regime [15]. Equations (17)–(19) can be used to describe such behavior. When the steady-state pattern $A_s(\tau)$ does not vary too rapidly, one can restrict the analysis to a small number of modes, although such a restriction will exclude passive mode-locking. We illustrate the usefulness of (17)–(19) for this case by using a simple example.

IV. A SIMPLE EXAMPLE

The simplest case, analogous to the single-mode Lorenz–Haken model, is obtained from (17)–(19) by setting $m = n = p = q = 1$. The resulting equations become

$$\frac{1}{v_g} \frac{da_1}{dT} = \frac{i}{2} b_1 - \frac{\alpha_T}{2} a_1 + i\tilde{\gamma} |a_1|^2 a_1 \quad (20)$$

$$T_2 \frac{db_1}{dT} = -(1 - i\delta) b_1 - i a_1 g_{11} \quad (21)$$

$$T_1 \frac{dg_{11}}{dT} = g_0 - g_{11} + \frac{1}{P_s} \text{Im}(a_1^* b_1) \quad (22)$$

where the SPM coefficient is defined as $\tilde{\gamma} = \gamma C_{1111}$. The effect of GVD on a single longitudinal mode merely involves a frequency shift of magnitude $\omega_D = \beta_2 v_g D_{11}/2$, which has been scaled out of the problem by changing the carrier frequency ω_0 to $\omega_0 + \omega_D$. This is equivalent to resetting the atomic detuning by $\delta = \delta + \omega_D$. We normalize these equations using the standard Lorenz–Haken notation and rewrite them in the following way:

$$\frac{dx}{dt} = -\sigma(x - y) + iqx|x|^2 \quad (23)$$

$$\frac{dy}{dt} = -(1 - i\delta)y + (r - z)x \quad (24)$$

$$\frac{dz}{dt} = -bz + \text{Re}(x^*y) \quad (25)$$

where the normalized time $t = T/T_2$ is measured in units of T_2 . The parameters σ , b , r , and q , and the variables x , y , and z are related to quantities appearing in (3)–(5) as

$$\sigma = \alpha_T v_g T_2/2, \quad b = T_2/T_1, \quad (26)$$

$$r = g_0/\alpha_T, \quad q = \tilde{\gamma} P_s T_1 v_g,$$

$$x = \sqrt{b/P_s} a_1, \quad y = (ib_1/\alpha_T) \sqrt{b/P_s},$$

$$z = r - g_{11}/\alpha_T. \quad (27)$$

These equations are identical to the standard Lorenz–Haken equations [3] except for the last term in (23) that is responsible for SPM and IDA occurring because of the host nonlinearities [3].

We now employ the standard linear stability analysis to investigate the stability properties of (23)–(25). First, we look

for CW solutions of the form

$$x_s(t) = X \exp[-i(\Delta\omega_s t + \varphi_s)] \quad (28)$$

$$y_s(t) = Y \exp(-i\Delta\omega_s t) \quad (29)$$

$$z_s(t) = z_s \quad (30)$$

where $P = X^2$ is the (scaled) optical field intensity, Y is the (scaled) polarization amplitude, $\Delta\omega_s$ is the frequency shift with respect to the optical frequency ω_0 , and φ_s is the phase lag between electric field and polarization. We stress that all these quantities are referring to the single longitudinal mode under consideration.

After substituting (28)–(30) in the evolution equations (23)–(25) and denoting $q \equiv q' + iq''$, the field intensity $P \equiv X^2$ is a positive real solution of the quartic equation

$$\sum_{n=0}^4 c_n P^n = 0 \quad (31)$$

where the coefficients are given by

$$c_4 = q'^3/\sigma \quad (32)$$

$$c_3 = [1 + 2(1 + \sigma^{-1})]q''^2 + (q''/\sigma)|q|^2 \quad (33)$$

$$c_2 = q''(\sigma + 1)^2/\sigma + 2q''(1 + \sigma^{-1})(\sigma + q') - 2b\delta q'q''/\sigma - brq''^2 + b|q|^2 \quad (34)$$

$$c_1 = [1 + (q''/\sigma)](\sigma + 1)^2 + b\delta^2 q''/\sigma - 2bq''(\sigma + 1)(r - 1) - 2b\delta q' \quad (35)$$

$$c_0 = b\delta^2 - b(r - 1)(\sigma + 1)^2. \quad (36)$$

For each solution P , the accompanying frequency shift $\Delta\omega_s$, inversion z_s , phase lag φ_s , and polarization Y are determined by the following relations:

$$\tan \varphi_s = \Delta\omega_s + \delta = \frac{\delta - q'P}{\sigma + 1 + q''P} \quad (37)$$

$$z_s = r - \left(1 + \frac{q''P}{\sigma}\right)(1 + \tan^2 \varphi_s) \quad (38)$$

$$Y = \frac{bz_s}{P} \cos \varphi_s. \quad (39)$$

The presence of SPM affects the CW characteristics of the laser through the frequency shift $\Delta\omega_s$. At resonance, i.e., $\delta = 0$, the laser operates at the gain peak in the absence of SPM. In the presence of SPM, this CW state will be detuned from the gain peak, thus producing less output power. Thus, the effects of detuning δ and the SPM parameter q' either counteract or re-enforce each other. In Fig. 1 we show how the laser power varies with pump at fixed SPM parameter $q' = 1$ for three different detunings δ . In general, the detuning δ increases the first laser threshold, but for $\delta = 5$ and $r > 4$ the laser output power exceeds that obtained for $\delta = 0$. In Fig. 1, we choose $\sigma = 3$ and $b = 1$, typical values used in the context of the Lorenz–Haken model.

The stability of the CW solutions is investigated by performing a linear stability analysis. After writing the perturbed steady states as

$$x(t) = [X + u(t)] \exp\{-i[\Delta\omega_s t + \varphi_s + \psi_A(t)]\} \quad (40)$$

$$y(t) = [Y + v(t)] \exp\{-i[\Delta\omega_s t + \psi_B(t)]\} \quad (41)$$

$$z(t) = z_s + w(t) \quad (42)$$

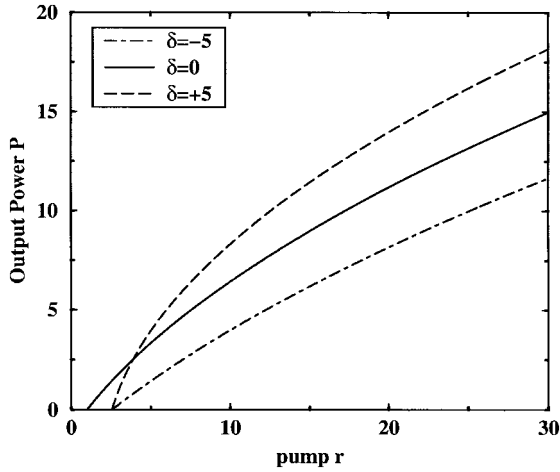


Fig. 1. The laser power P as a function of pump level r in the presence of SPM ($q' = 1$) for three different values of atomic detuning δ (indicated in the figure). Other parameters are $\sigma = 3$ and $b = 1$.

linearizing the evolution equations for the perturbations u, v, w, ψ (where ψ is the phase difference $\psi_A - \psi_B$), and assuming a solution of the form $\exp(st)$ for u, v, w, ψ , the characteristic eigenvalue equation for s is found to be a quartic polynomial obtained from the determinant equation as shown in (43), shown at the bottom of the page. When any of the four complex roots of (43) has a positive real part, the steady state under consideration is unstable.

Because of the complexity of the determinant in (43), we do not attempt to find an analytical expression for the second threshold. Instead, we study two cases in detail. One is characterized by the combination $\sigma = 3$ and $b = 1$, and is called the “bad-cavity” laser, while the other (“good-cavity” laser) has the combination $\sigma = b = 1$. The bad-cavity laser has a finite second threshold even in the absence of host nonlinearities, while the good-cavity laser has no second threshold in the Lorenz–Haken limit obtained by setting $q = 0$ [3]. For the resonant case ($\delta = 0$), the second threshold is simply given by $r_{th,2} = \sigma(\sigma + b + 3)/(\sigma - b - 1)$ [3]. A necessary condition, the so-called bad-cavity condition, for the second threshold to exist is $\sigma > b + 1$. An analytical expression for the second threshold for arbitrary detuning values was given in [16].

We first consider a laser without IDA and set $q'' = 0$. SPM is then the only host-induced nonlinearity. In Fig. 2, we show the pump parameter at the second threshold as a function of the SPM parameter q' for the bad-cavity laser operating on-resonance ($\delta = 0$). In absence of SPM ($q' = 0$), the second threshold is located at $r = 21$. Quite surprisingly,

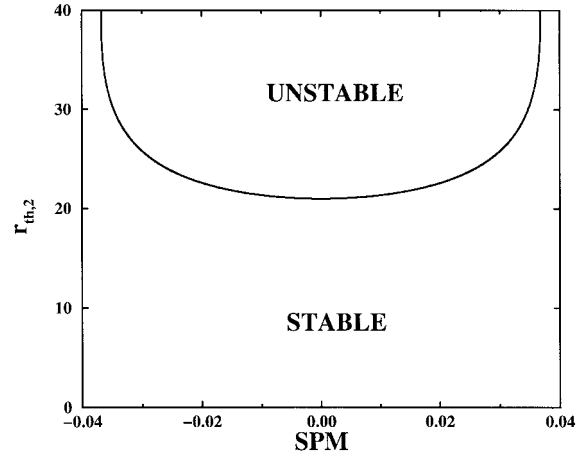


Fig. 2. The effect of SPM on the location of the second threshold at atomic resonance ($\delta = 0$), for the “bad-cavity” laser with $\sigma = 3$ and $b = 1$.

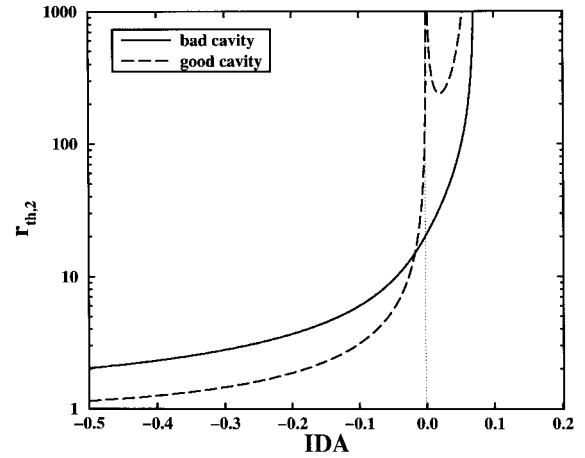


Fig. 3. The effect of IDA on the location of the second threshold at atomic resonance ($\delta = 0$). Solid line: bad-cavity laser with $\sigma = 1$ and $b = 1$. Dashed line: good-cavity laser with $\sigma = 3$ and $b = 1$.

the global effect of SPM is to stabilize the system: already at $|\gamma'| = 0.04$ the second threshold ceases to exist. This result is surprising since SPM is essential for convective instabilities to occur in passive fibers and fiber amplifiers. From the steady-state analysis above we know that SPM causes a shift in operating frequency away from the gain peak, thus lowering the output power. From a stability point of view this shift can be interpreted as a shift away from the second threshold. The good-cavity laser ($\sigma = b = 1$) shows no second threshold with or without SPM.

Quite a different picture emerges when IDA is included. For both good- and bad-cavity lasers, IDA can reduce the

$$\begin{vmatrix} s + \sigma + 3q''P & -\sigma \cos \varphi_s & 0 & \sigma Y \sin \varphi_s \\ -(r - z_s) \cos \varphi_s & s + 1 & X \cos \varphi_s & (r - z_s)X \sin \varphi_s \\ -Y \cos \varphi_s & -X \cos \varphi_s & s + b & XY \sin \varphi_s \\ \frac{\Delta\omega_s}{X} + 3q'X + (r - z_s) \frac{\sin \varphi_s}{Y} & \frac{-(\Delta\omega_s + \delta)}{Y} + \frac{\sigma \sin \varphi_s}{X} & -\frac{X}{Y} \sin \varphi_s & s + \left[\sigma \frac{Y}{X} + (r - z_s) \frac{X}{Y} \right] \cos \varphi_s \end{vmatrix} = 0 \quad (43)$$

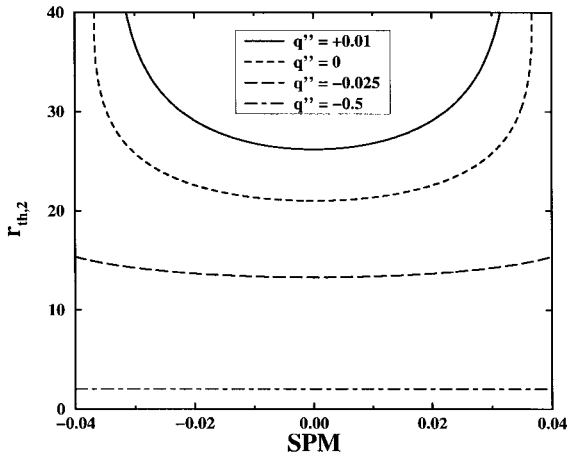


Fig. 4. The location of the second threshold for the bad-cavity laser of Fig. 3, as a function of SPM for four different values of IDA (indicated in the figure).

second threshold dramatically, as shown in Fig. 3. In both cases, positive values of q'' account for two-photon absorption while negative values of q'' can be used to describe the effect of a fast saturable absorber. Clearly, fast saturable absorption (negative q'') reduces the second laser threshold for both good- and bad-cavity lasers. In the presence of a fast saturable absorber, self-pulsing can therefore occur at pump levels only a few times above the first threshold. We stress that this self-pulsing is not related to passive mode locking because of our assumption of a single longitudinal mode. In fact, the frequency of self-pulsing is related to the relaxation oscillations and is a fraction of the longitudinal-mode spacing.

Fig. 4 shows the combined effect of SPM and IDA for the bad-cavity laser. For four different values of q'' , the second laser threshold is shown as a function of the SPM parameter q' . The effects of SPM and negative IDA have opposite effects on the second threshold: SPM stabilizes the laser by increasing the second threshold, while negative IDA destabilizes the laser and reduces the pumping level at which the second threshold occurs. For the good-cavity laser, a qualitatively similar figure is obtained, although in that case no second threshold exists when $q'' = 0$.

Fiber ring lasers usually oscillate in many longitudinal modes simultaneously because their mode spacing (~ 10 MHz) is a fraction of the gain bandwidth (> 1 THz). By using a grating, they can be forced to operate in a single longitudinal mode. When we use typical values for a neodymium-doped fiber laser, $\alpha_T = 46 \text{ km}^{-1}$, $T_2 = 1.75 \text{ ps}$, $T_1 = 0.1 \text{ ms}$, $\gamma = 2.3 \text{ W}^{-1}\text{km}^{-1}$, and $P_s = 11.4 \text{ } \mu\text{W}$, the dimensionless parameters are $\sigma = 8.84 \times 10^{-6}$, $b = 1.77 \times 10^{-8}$, and $q' = 5.23 \times 10^{-4}$. These values make the single-longitudinal-mode fiber ring laser a good-cavity laser with no second threshold by itself. Fig. 5 shows the effect of fast saturable absorption on the second threshold of such a laser. Even relatively small values of q'' cause a second threshold to exist at quite low pump values. For example, when $q'' = -3.65 \times 10^{-8}$, the second laser threshold exists at a pump level only twice the first laser threshold. This value of q'' corresponds to $\text{Im}(\gamma) = 1.6 \times 10^{-4} \text{ W}^{-1}\text{km}^{-1}$ if we use $C_{1111} = 1$. This is a weak saturable absorption indeed, since it only reduces the losses

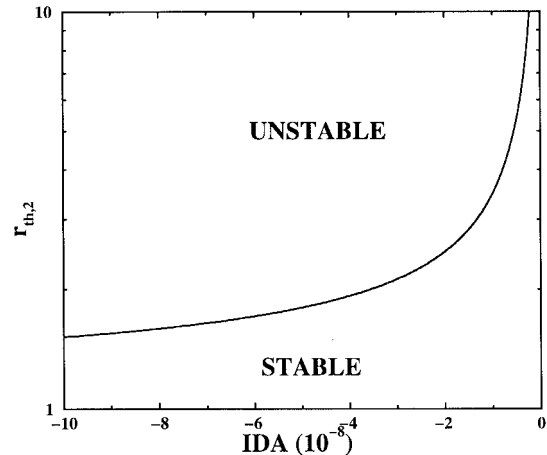


Fig. 5. The location of the second threshold for a fiber ring laser which is forced to oscillate in only one longitudinal mode, as a function of (negative) IDA.

in the cavity by 0.001% for an intracavity power of 3 W. At the second laser threshold, the relaxation-oscillation frequency is 50 kHz, and self-pulsing will occur with a repetition rate close to that value.

V. CONCLUSIONS

In this paper, we have analyzed the effects of GVD and host nonlinearities (SPM and IDA) on the absolute stability of lasers. Starting from the Maxwell–Bloch equations, we derive a set of multimode laser equations, which govern the absolute dynamics of the laser on a timescale longer than the cavity round-trip time. The presence of a resonator gives rise to longitudinal modes that can differ from their conventional form in the presence of GVD and host-nonlinearities. Our analysis shows the link between the propagation-based (convective) modulation instability and the purely temporal (absolute) instabilities of these spatio-temporal patterns. The latter can occur only in a resonator as absolute instabilities require optical feedback.

To illustrate our formalism, we consider the case of a fiber ring laser forced to oscillate in a single longitudinal mode. The nonlinear dynamics of the slowly-varying amplitude of that mode is governed by the Lorenz–Haken equations except that a nonlinear term is added to the field equation. This nonlinear term contains a complex parameter q whose real and imaginary parts account for nonlinear effects such as SPM and IDA, respectively. SPM is found to stabilize the laser, while IDA, in general, destabilizes the laser. Thus, we show that SPM can have a double role as far as stability issues are concerned. On the one hand, it can cause convective instabilities in combination with GVD, which can lead to exponential growth of a localized perturbation. However, once the perturbation fills the entire cavity, its subsequent growth is dictated by the cavity modes. SPM is found to suppress such subsequent growth. IDA induced by fast saturable absorbers, however, quickly gives rise to a second laser threshold at relatively low pump levels. In our single-mode laser, the laser begins to self-pulse at a repetition rate close to the relaxation–oscillation frequency.

ACKNOWLEDGMENT

The authors would like to acknowledge helpful discussions with Dr. P. Goorjian.

REFERENCES

- [1] G. Makhov, C. Kikuchi, J. Lambe, and R. W. Terhune, "Maser action in ruby," *Phys. Rev.*, vol. 109, pp. 1399–1400, 1958.
- [2] N. B. Abraham, P. Mandel, and L. M. Narducci, "Dynamical instabilities and pulsations in lasers," in *Progress in Optics XXV*, E. Wolf, Ed. Amsterdam, The Netherlands: North-Holland, 1988.
- [3] C. O. Weiss and R. Vilaseca, *Dynamics of Lasers*. New York: Weinheim, 1991.
- [4] H. Haken, "Analogy between higher instabilities in fluids and lasers," *Phys. Lett.*, vol. 53A, pp. 77–78, 1975.
- [5] A. Bers, *Handbook of Plasma Physics*, R. Z. Sagdeev and M. N. Rosenbluth, Eds., Vol. 1, *Basic Plasma Physics*, A. A. Galeev and R. N. Sudan, Eds. Amsterdam, The Netherlands: North-Holland, 1983.
- [6] G. P. Agrawal, *Nonlinear Fiber Optics*, 2nd ed. New York: Academic, 1995.
- [7] G. H. M. van Tartwijk and G. P. Agrawal, "Maxwell–Bloch dynamics and modulation instability in fiber lasers and amplifiers," *J. Opt. Soc. Amer. B*, vol. 14, pp. 2618–2627, 1997.
- [8] K. Ikeda, H. Daido, and O. Akimoto, "Optical turbulence: Chaotic behavior of transmitted light from a ring-cavity," *Phys. Rev. Lett.*, vol. 45, pp. 709–712, 1980.
- [9] L. A. Lugiato, L. M. Narducci, and M. F. Squicciarini, "Exact linear stability analysis of the plane-wave Maxwell–Bloch equations for a ring laser," *Phys. Rev. A*, vol. 34, pp. 3101–3108, 1986.
- [10] H. A. Haus, J. G. Fujimoto, and E. P. Ippen, "Structures for additive pulse mode locking," *J. Opt. Soc. Amer. B*, vol. 8, pp. 2068–2076, 1991.
- [11] A. E. Siegman, *Lasers*. Mill Valley, CA: University Science, 1986.
- [12] J. Arnaud, "Natural linewidth of anisotropic lasers," *Opt. Quantum Electron.*, vol. 18, pp. 335–343, 1986.
- [13] D. Lenstra, "Linewidth modifications due to nonorthogonality, inhomogeneity, and directionality of longitudinal laser modes," *IEEE J. Quantum Electron.*, vol. 29, pp. 954–960, 1993.
- [14] S. Arahira, Y. Matsui, and Y. Ogawa, "Mode-locking at very high repetition rates more than terahertz in passively mode-locked distributed-Bragg-reflector laser diodes," *IEEE J. Quantum Electron.*, vol. 32, pp. 1211–1224, 1996.
- [15] G. Sucha, S. R. Bolton, S. Weiss, and D. S. Chemla, "Period doubling and quasiperiodicity in additive-pulse mode-locked lasers," *Opt. Lett.*, vol. 20, pp. 1794–1796, 1995.
- [16] C. Ning and H. Haken, "Detuned lasers and the complex Lorenz equations: subcritical and supercritical Hopf bifurcations," *Phys. Rev. A*, vol. 41, pp. 3826–3837, 1990.

Guido H. M. van Tartwijk was born in Veghel, The Netherlands, in 1967. He received the M.Sc. degree in theoretical physics from the Eindhoven University of Technology, The Netherlands, in 1990, studying coherent magneto-tunneling in double-barrier structures. He received the Ph.D. degree from the Vrije Universiteit, Amsterdam, The Netherlands, in 1994. His dissertation was on semiconductor laser dynamics with optical injection and feedback.

After a post-doctoral stay at the Universitat de les Illes Balears, Palma, Spain, he received a TALENT-stipend from the Netherlands Organization for Scientific Research in 1995, allowing him to spend one year at The Institute of Optics, Rochester, NY. After working one additional year at the same institute, he is presently with Uniphase Netherlands B.V., Eindhoven, The Netherlands, where he works as a Research Scientist on new semiconductor lasers and amplifiers for optical communication.



Govind P. Agrawal (M'83–SM'86–F'96) received the B.S. degree from the University of Lucknow in 1969 and the M.S. and Ph.D. degrees from the Indian Institute of Technology, New Delhi, in 1971 and 1974, respectively.

After holding positions at the Ecole Polytechnique, France, the City University of New York, NY, and AT&T Bell Laboratories, Murray Hill, NJ, in 1989 he joined the faculty of the Institute of Optics at the University of Rochester, Rochester, NY, where he is a Professor of Optics. His research

interests focus on quantum electronics, nonlinear optics, and laser physics. In particular, he has contributed significantly to the fields of semiconductor lasers, nonlinear fiber optics, and optical communications. He is an author or co-author of more than 250 research papers, several book chapters and review articles, and three books entitled *Semiconductor Lasers* (New York: Van Nostrand Reinhold, 1993), *Nonlinear Fiber Optics* (New York: Academic, 1995), and *Fiber-Optic Communication Systems* (New York: Wiley, 1997). He has also edited the books *Contemporary Nonlinear Optics* (New York: Academic, 1992) and *Semiconductor Lasers: Past, Present and Future* (Woodbury, NY: AIP Press, 1995).

Dr. Agrawal is a fellow of the Optical Society of America.