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Nonlinear spatio-temporal dynamics due to transverse-mode competition in gain-switched microcavity semiconductor lasers

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Abstract

We show through numerical simulations that transverse-mode competition induced by spatial hole-burning in gain-switched vertical-cavity surface-emitting lasers gives rise to rich nonlinear dynamics, including period doubling, optical chaos, and anti-phase oscillations. The spatio-temporal nature of gain switching is explored over a wide range of modulation currents and modulation frequencies.

The most significant attributes of vertical-cavity surface-emitting lasers (VCSELs) with a microscopic cavity include a low threshold current, single-longitudinal-mode operation, a circular beam profile, and high-speed-modulation capabilities [1–3]. Large-signal modulation behavior of these devices is an important issue for their potential applications in optical communication systems. Since the laser dynamics under single-mode operation is governed by the two rate equations, introduction of a third degree of freedom through current modulation or optical feedback may cause instabilities. Indeed, edge-emitting single-mode semiconductor lasers operating in a single longitudinal mode and a single transverse mode are known to exhibit a variety of nonlinear behaviors under pulsed operation [4]. However, since VCSELs typically operate in several transverse modes, they are likely to exhibit richer nonlinear dynamics [2]. Indeed, chaotic behavior has been observed experimentally even under dc operation [5]. It has also been shown experimentally that VCSELs undergo strong transverse-mode competition under pulsed operation [6]. In spite of these experimental results, a theoretical study of multi-mode spatio-temporal nonlinear dynamics has attracted little attention.

In this Letter, we show through numerical simulations that under multi-mode operation, gain-switched VCSELs exhibit complicated nonlinear dynamics, whose origin lies in transverse-mode competition induced by spatial effects, namely spatial hole-burning and carrier diffusion. We investigate the effects of modulation frequency and modulation current and find that for certain parameter values, there exists the possibility of transition to chaos through period doubling. Our model takes into account the spatial and temporal dependence of both the optical field and the carrier density. Assuming that the VCSEL can operate in several transverse modes simultaneously, the rate equations in the cylindrical coordinates are [7]

$$\mathrm{d}E_i/\mathrm{d}t = \tfrac{1}{2}\{(1-\mathrm{i}\alpha)G_i(t) - \gamma_i\}E_i, \tag{1}$$

$$\frac{\partial N}{\partial t} = D\nabla_\mathrm{T}^2 N + \frac{J(r,\phi)}{qd} - \frac{N}{\tau_\mathrm{e}} - BN^2 - \frac{1}{d}\sum_{i=1}^{n} G_\mathrm{local}|E_i(t)|^2|\psi_i(r,\phi)|^2, \tag{2}$$

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where $E_i(t)$ is the amplitude of the ith transverse mode with the spatial distribution $\psi_i(r,\phi)$, $N(r,\phi,t)$ is the carrier density, α is the linewidth enhancement factor, and $G_i(t)$ and γ_i are the gain and cavity loss for the ith mode. In Eq. (2), τ_e is the carrier lifetime due to nonradiative recombination, D is the diffusion coefficient, B is the spontaneous recombination coefficient, d is the thickness of the active layer, and $J(r,\phi)$ is the injection current density. The local gain $G_{local} = \sigma(N - N_T)/(1 + \epsilon|E_i|^2)$ is assumed to be linearly proportional to the local carrier density $N(r,\phi,t)$, N_T is the carrier density at transparency, and ϵ is the nonlinear-gain parameter. Note that the filed E_i is normalized such that $|E_i|^2$ corresponds to the photon number in the ith mode. The modal gain $G_i(t)$ for each mode in Eq. (1) is obtained by taking into account the spatial overlap between the local-gain profile and the spatial intensity distribution $|\psi_i(r,\phi)|^2$ of that mode.

Eqs. (1) and (2) are solved numerically by using a finite-difference method with a temporal and spatial resolution of 0.1 ps and 0.1 μm. Steady-state pulse characteristics are simulated by running the code for a total time of 5 to 10 ns to eliminate the transients. For gain-switching, the injection current in Eq. (2) is taken to be

$$J(r,t) = \begin{cases} J_b + J_m \sin(2\pi f_m t) & \text{if } 0 < r < r_a \text{ and } J(r,t) > 0, \\ 0 & \text{otherwise}, \end{cases} \tag{3}$$

where J_b is the bias current density, J_m is the modulation current density, f_m is the modulation frequency, and r_a is the radius of the disc contact over which the current is injected. An index-guided VCSEL with cylindrical geometry (index guiding over 4 μm) is considered. The active region consists of three 8-nm quantum wells. A disc contact of 4-μm radius is used to excite the LP_{01} and LP_{11} transverse modes simultaneously. Other parameter values correspond to a typical GaAs VCSEL and are: $\alpha = 3$, $\gamma_i = 4 \times 10^{11}\ s^{-1}$, $D = 30\,cm^2\,s^{-1}$, $\tau_e = 5$ ns, and $B = 1 \times 10^{-10}\ cm^3\,s^{-1}$. The bias level J_b/J_{th} is set to be 0.95 for all simulations. We first set $\epsilon = 0$ to study the nonlinear dynamics for the case in which the nonlinear-gain effects are weak. Finite values of ϵ are considered later.

The gain-switched pulses for modulation frequencies of 2.5, 5 and 10 GHz by choosing $J_m/J_{th} = 15$ are shown in Fig. 1. Evidently, the details of nonlinear dynamics are strongly dependent on the modulation frequency. The device exhibits period-two oscillations for $f_m = 2.5$ and 10 GHz, and period-six oscillations for $f_m = 5$ GHz. The two transverse modes show quite different dynamics for $f_m = 2.5$ and 5 GHz. The origin of this surprising behavior lies in spatial hole-burning and carrier diffusion. The fundamental LP_{01} mode reaches threshold when the carrier density has recovered fully. However, the spatial hole burnt by it does not get filled completely over the modulation cycle of 0.4 ns at 2.5 GHz, resulting in the excitation of the LP_{11} mode. Since the LP_{11} mode peaks near $r = 2$ μm, its hole burning helps to recover the carrier density near $r = 0$. As a result, the LP_{01} mode is excited during the next modulation cycle, and the process repeats. However, for $f_m = 10$ GHz, output pulses for the two modes exhibit similar period-two dynamics, although the oscillations are 180 degrees out of phase. This anti-phase nature can be understood by noting that each mode burns its own hole in the carrier-density profile which cannot be refilled over a short modulation period of 100 ps. Moreover, the hole burnt by one mode helps to fill the hole burnt by the other mode in the previous modulation cycle, resulting in period-two oscillations.

This physical explanation is supported by Fig. 2, which shows spatio-temporal variations of the carrier density for $f_m = 10$ GHz and $J_m/J_{th} = 15$, values corresponding to the output pulses shown by the upper trace of Fig. 1. Valleys and hills in this figure correspond to pulse peak and pulse tails, respectively. When the fundamental mode dominates, a deep hole is burnt at $r = 0$. It is partially filled during the next cycle, resulting in period-doubling behavior.

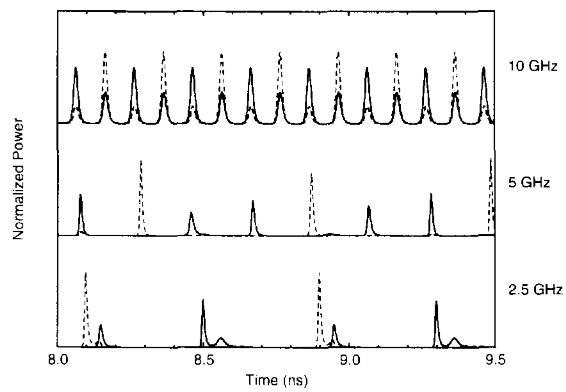

Fig. 1. Gain-switched pulse trains at modulation frequencies of 2.5, 5 and 10 GHz for a VCSEL operating simultaneously in the LP_{01} (solid trace) and LP_{11} (dashed trace) transverse modes. In all cases, $J_b/J_{th} = 0.95$ and $J_m/J_{th} = 15$. Values of other parameters are given in the text.

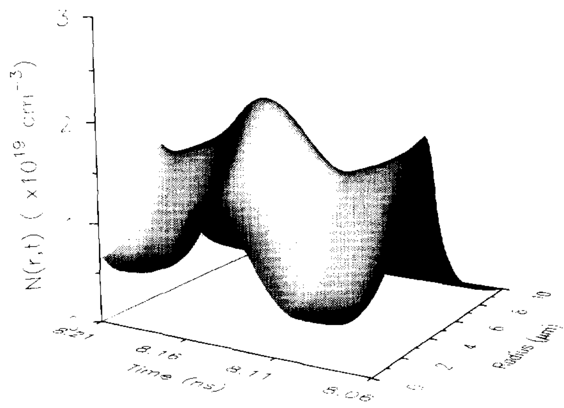

Fig. 2. Spatio-temporal evolution of the carrier density under gain switching at 10 GHz.

To investigate the role of transverse-mode competition, we have constructed the bifurcation diagram for the case of 10-GHz gain switching under two-mode operation using J_m/J_{th} as a bifurcation parameter, and the results are shown in Fig. 3. The dots (LP_{01}) and crosses (LP_{11}) represent the sampled output peak powers respectively for the two modes at a given value of J_m/J_{th} (Poincoré map). Pulse trains for specific values of J_m/J_{th} are shown in Fig. 4.

For $J_m/J_{th} < 9$, the relaxation-oscillation frequency is smaller than the modulation frequency, resulting in period-doubling behavior and chaos, which are known to occur under such conditions [4]. In the range $9 < J_m/J_{th} < 14$, the two modes oscillate independently with a regular periodic pattern, an example of which is shown for $J_m/J_{th} = 10$ in Fig. 4. This regime of operation may find applications in optical data recording, since the two pulse trains can be focused to different spots because of their different spatial patterns. This regime is also useful for optical communications since the two modes are often orthogonally polarized, and therefore can be easily discriminated at the detector. As the modulation current is increased beyond $J_m/J_{th} = 14$, both modes undergo period doubling, as seen in Fig. 4 for $J_m/J_{th} = 20$. However, the two modes do not oscillate independently and exhibit anti-phase oscillations because of strong mode competition. Eventually, mode competition becomes so strong that chaotic behavior is observed following a period-doubling route. We also observe period-three oscillations, an example of which is shown in Fig. 4 for $J_m/J_{th} = 21$. To confirm that the origin of chaos is

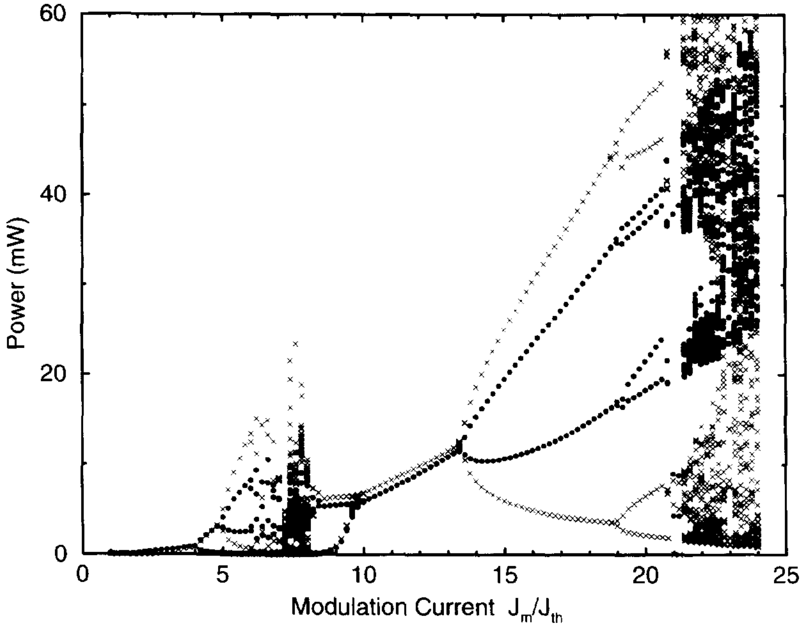

Fig. 3. Bifurcation diagram as a function of modulation current J_m/J_{th} for 10-GHz gain switching of a VCSEL operating simultaneously in two transverse modes. Dots and crosses correspond to the LP_{01} and LP_{11} modes, respectively.

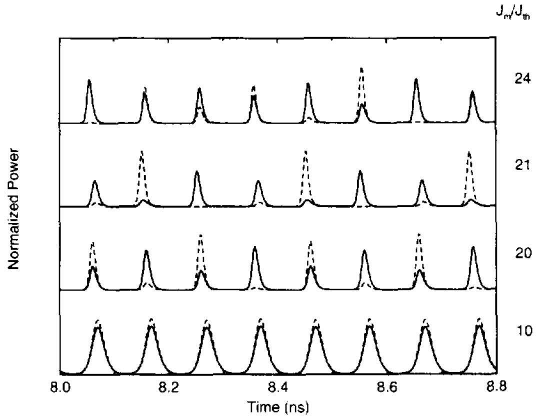

Fig. 4. Gain-switched pulse trains for four different values of J_m / J_{th} (shown on right margin). Solid and dashed traces correspond to the LP_{01} and LP_{11} modes, respectively.

indeed transverse-mode competition, numerical simulations were repeated using the same set of parameters except for a disc-contact radius of 2 μm, which selectively excites the fundamental LP_{01} mode only. No chaos were observed under identical operating conditions when the VCSEL operates in a single transverse mode.

Since it is known that the nonlinear gain can affect the nonlinear dynamics significantly [8], simulations have been performed for several different values of the nonlinear-gain parameter ϵ. For relatively small values of $\epsilon < 1 \times 10^{-7}$, the bifurcation diagram (Fig. 3) remains qualitatively the same although quantitative changes do occur. For larger values of $\epsilon > 3 \times 10^{-7}$, both the period-doubling and chaotic behaviors disappear.

In conclusion, we have shown that transverse-mode competition in gain-switched VCSELs under multi-mode operation can lead to interesting nonlinear dynamics. Temporal effects are coupled to spatial effects through spatial hole-burning and carrier diffusion. Nonlinear dynamics is found to be strongly dependent on both the modulation frequency and injection current. At high modulation frequencies, transverse-mode competition leads to chaos through a period-doubling route.

Acknowledgements

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