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1 November 1996

OPTICS
COMMUNICATIONS

Optics Communications 131 (1996) 274–278

Timing jitter analysis for optical communication systems using ultrashort solitons and dispersion-decreasing fibers

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Received 18 April 1996; revised version received 10 June 1996; accepted 10 June 1996

Abstract

We use adiabatic perturbation theory to calculate the timing jitter induced by fluctuations in solitons amplitude, frequency and position due to amplifiers noise when ultrashort solitons (~ 1 ps) are propagated in dispersion-decreasing fibers. The result is applied to a high-speed soliton communication system with amplifiers spacing of 50–100 km. We show that the transition from a regime where frequency fluctuations dominate the timing jitter (Gordon–Haus jitter) to the regime where amplitude fluctuations dominate (Raman-induced jitter) occurs for soliton widths of ~ 5 ps; the precise value is determined by the total distance of transmission. The contribution of third-order dispersion to the timing jitter is included in our analysis. We also provide an upper estimate of the distance where a soliton-control device (e.g., optical filters, modulators) needs to be inserted to control the timing jitter.

Keywords: Amplifier noise; Solitons; Optical fiber communication; Nonlinear optics; Optical fiber dispersion

It is well known [1] that the main limitation on the bit rate of average-soliton [2] communication systems stems from the timing jitter due to amplifier-noise-induced frequency fluctuations (the Gordon–Haus effect [3]). Several techniques such as sliding-frequency filters and synchronous modulation have been used for reducing the timing jitter. In fact, when a soliton-control method is used, bit rates are no longer limited by the timing jitter but rather by the relatively long duration (~ 15 ps) of the average solitons. To overcome this limitation on the bit rate, it has been proposed to use dispersion-decreasing fibers (DDFs) [4,5]. Such DDFs allow stable propagation of solitons shorter than those needed in the average-soliton regime if the system is appropriately designed to take into account various higher-order effects [6]. A natural question is how the amplifier noise affects such ultrashort solitons.

Previous analyses [7–9] have shown that the Raman-induced soliton self-frequency shift (SSFS) can increase the timing jitter associated with propagation of short solitons and may even dominate the total jitter for ultrashort solitons. However, these studies considered constant-dispersion fibers which are unlikely to support stable propagation of ultrashort solitons (less than 10 ps) because of fiber loss, higher-order dispersive and nonlinear effects, and a relatively short soliton period (typically 1 km). It has been recently shown that dispersion-decreasing fibers can support solitons as short as ~ 100 fs [6,10]. In this paper, we calculate the timing jitter of ultrashort solitons induced by amplifier noise in DDFs by including both the Raman effect and third-order dispersion (TOD).

The propagation of ultrashort solitons through a DDF is described by a generalized nonlinear Schrödinger equation [11],

$$i \frac{\partial u}{\partial z} + \frac{1}{2} p(z) \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = -\frac{1}{2} i \alpha u + \tau_R u \frac{\partial |u|^2}{\partial \tau} + i \delta_d \frac{\partial^3 u}{\partial \tau^3}, \quad (1)$$

where $\delta_d = \beta_3 / (6T_0 |\beta_2(0)|)$ is the normalized TOD, $p(z) = |\beta_2(z) / \beta_2(0)|$ is the normalized group-velocity dispersion (GVD), α is the fiber loss, and $\tau_R = T_R / T_0$ is the Raman parameter normalized to the soliton characteristic width T_0 . The time coordinate τ is normalized to T_0 and the propagation distance z is normalized to the dispersion length $L_D = T_0^2 / |\beta_2(0)|$.

To apply the adiabatic perturbation theory (APT) [12,13], we rescale u and z in Eq. (1) to a new amplitude v and a new distance scale η defined by

$$v = p^{-1/2} u, \quad \eta = \int_0^z p(y) dy. \quad (2)$$

Eq. (1) then takes the form of a perturbed nonlinear Schrödinger equation,

$$i \frac{\partial v}{\partial \eta} + \frac{1}{2} \frac{\partial^2 v}{\partial \tau^2} + |v|^2 v = -i \left(\frac{\alpha}{2p} + \frac{1}{2p} \frac{dp}{d\eta} \right) v + \tau_R v \frac{\partial |v|^2}{\partial \tau} + i \frac{\delta_d}{p} \frac{\partial^3 v}{\partial \tau^3}. \quad (3)$$

For a DDF with a nearly ideal dispersion profile ($p(z) \approx \exp(-\alpha z)$) and $|\beta_2^{\min}|$ larger than $\sim 0.1 \text{ ps}^2/\text{km}$, the three terms on the right side of Eq. (3) become small enough to allow for the use of APT. Here $|\beta_2^{\min}|$ is the absolute value of the minimum dispersion $|\beta_2(z_a)|$ at the fiber end, where z_a is the normalized amplifier spacing.

We assume a fundamental soliton of the form

$$v_s(B, q, \phi, \omega; \tau) = B \operatorname{sech}[B(\tau - q)] \exp(i\phi - i\omega\tau), \quad (4)$$

where the parameters B , q , ϕ , and ω represent the rescaled soliton amplitude, position, phase, and frequency, respectively, and are slowly varying functions of η . By using the standard APT [13], their evolution is governed by

$$\frac{dB}{d\eta} = - \left(\frac{\alpha}{p} + \frac{1}{p} \frac{dp}{d\eta} \right) B, \quad (5)$$

$$\frac{d\omega}{d\eta} = -\frac{8}{15} \tau_R B^4, \quad (6)$$

$$\frac{dq}{d\eta} = -\omega + \frac{\delta_d}{p} B^2 + 3 \frac{\delta_d}{p} \omega^2, \quad (7)$$

where the evolution equation of $\phi(\eta)$ was omitted because the timing jitter does not depend on the soliton phase when soliton interaction is negligible.

According to Eq. (2), the normalized and rescaled solitons differ only by their different amplitudes related by $A = Bp^{-1/2}$. Using Eq. (2) and assuming a loss-matched dispersion profile, $p(z) = \exp(-\alpha z)$, Eqs. (5)–(7) can be solved to yield

$$A(z) = A_0 \exp(-\alpha z/2), \quad (8)$$

$$\omega(z) = -\frac{8}{15} \tau_R A_0^4 z_1(z) + \omega_0, \quad (9)$$

$$q(z) = q_{GH}(z) \omega_0 + q_R(z) A_0^4 + q_{TOD}^A(z) A_0^2 + q_{TOD}^\omega(z) \omega_0^2 + q_{C1}(z) A_0^8 + q_{C2}(z) A_0^4 \omega_0 + q_0, \quad (10)$$

where A_0 , ω_0 , and q_0 are the initial values of the soliton amplitude, frequency and position, respectively, and

$$\begin{aligned}
q_{GH}(z) &= -z_1(z), & q_R(z) &= \frac{8}{15} \frac{\tau_R}{\alpha} [z_1(z) - z_2(z)], \\
q_{TOD}^A(z) &= \delta_d z, & q_{TOD}^\omega(z) &= 3\delta_d z, \\
q_{C1}(z) &= \frac{192}{225} \frac{\delta_d \tau_R^2}{\alpha^2} [z - 2z_1(z) + z_2(z)], & q_{C2}(z) &= -\frac{48}{15} \frac{\delta_d \tau_R}{\alpha} [z - z_1(z)], \\
z_1(z) &= \frac{1 - \exp(-\alpha z)}{\alpha}, & z_2(z) &= \frac{1 - \exp(-2\alpha z)}{2\alpha}.
\end{aligned} \tag{11}$$

Here, the parameters $z_1(z)$ and $z_2(z)$ have been introduced for convenience, $q_{GH}(z)$ is the term leading to the Gordon–Haus effect, $q_R(z)$ governs the soliton displacement due to the Raman effect, the two terms $q_{TOD}^A(z)$ and $q_{TOD}^\omega(z)$ are associated with the direct effect of TOD on the soliton position, while the last two terms $q_{C1}(z)$ and $q_{C2}(z)$ represent cross-coupling effects of TOD and the Raman effect. Full numerical simulations of Eq. (1) for various combinations of parameters have shown that Eq. (10) gives the evolution of the soliton position with an accuracy better than 1% for nearly all the values of the parameters used in this paper.

The displacement $\delta q(z_a)$ of a soliton at the end of one DDF segment resulting from variations δA_0 , $\delta \omega_0$, and δq_0 in the soliton amplitude, mean frequency, and position at the fiber input is obtained by differentiating Eq. (10) and is given by

$$\begin{aligned}
\delta q(z_a) &= q_{GH}(z_a) \delta \omega_0 + 4q_R(z_a) A_0^3 \delta A_0 + 2q_{TOD}^A(z_a) A_0 \delta A_0 \\
&+ q_{C2}(z_a) A_0^4 \delta \omega_0 + 8q_{C1}(z_a) A_0^7 \delta A_0 + \delta q_0.
\end{aligned} \tag{12}$$

The root-mean-square (RMS) displacement σ_q , obtained by summing the individual displacements over a chain of N amplifiers composing the link, is given by

$$\begin{aligned}
\sigma_q^2 &= \left(\frac{1}{3} N^3 q_{GH}^2 + \frac{5}{12} N^4 q_{GH} q_{C2} A_0^4 + \frac{2}{13} N^5 q_{C2}^2 A_0^8 \right) \sigma_\omega^2 + \left(2N^4 q_R q_{TOD}^A A_0^4 + \frac{4}{3} N^5 q_R^2 A_0^6 \right. \\
&\left. + \frac{104}{45} N^6 q_R q_{C1} A_0^{10} + \frac{4}{3} N^3 (q_{TOD}^A)^2 A_0^2 + \frac{44}{13} N^5 q_{TOD}^A q_{C1} A_0^8 + \frac{176}{105} N^7 q_{C1}^2 A_0^{14} \right) \sigma_A^2 + N \sigma_{q_0}^2,
\end{aligned} \tag{13}$$

where we have replaced the summation over the N sections by an integration and assumed that the noise of different amplifiers are uncorrelated [3]. The quantities σ_A , σ_ω , and σ_q are defined by using $\sigma_P \equiv \langle \delta P^2 \rangle^{1/2}$, where σ_P is the RMS deviation of the variable P . The expression above for the timing jitter σ_q includes the contributions of the Raman effect and TOD. An expression similar to Eq. (13) has been derived in Ref. [9] for the case of constant-dispersion fibers but without considering the q_C terms which become important for femtosecond solitons. Moreover, its application is limited by the difficulty to fulfill the average-soliton condition for ultrashort solitons in constant-dispersion fibers.

Fluctuations of the soliton frequency and position at the output of an amplifier due to the noise added during the amplification process has been derived in Ref. [14] and generalized to the soliton amplitude and phase in Ref. [13]. The three fluctuations needed here are given by

$$\sigma_A^2 = A_0 \epsilon_0 n_{sp} F(G), \quad \sigma_\omega^2 = \frac{1}{3} \sigma_A^2, \quad \sigma_q^2 = \frac{\pi^2}{12 A_0^2} \sigma_A^2, \tag{14}$$

where n_{sp} is the spontaneous emission factor (set to 2), $F(G) = G - 1$ with the total gain $G = \exp(\alpha z_a)$ for each amplifier, and $\epsilon_0 = 2\hbar\omega/E_s$ is the ratio of the energy of a photon to the average energy per bit of information. The average energy per bit corresponds to half the soliton energy when the same number of “zeros” and “ones” are transmitted. Eq. (13), with variances of the soliton parameters given by Eqs. (14), represents the total RMS timing jitter of a soliton as short as few hundreds of femtoseconds in dispersion-tailored fibers and is the main result of this paper.

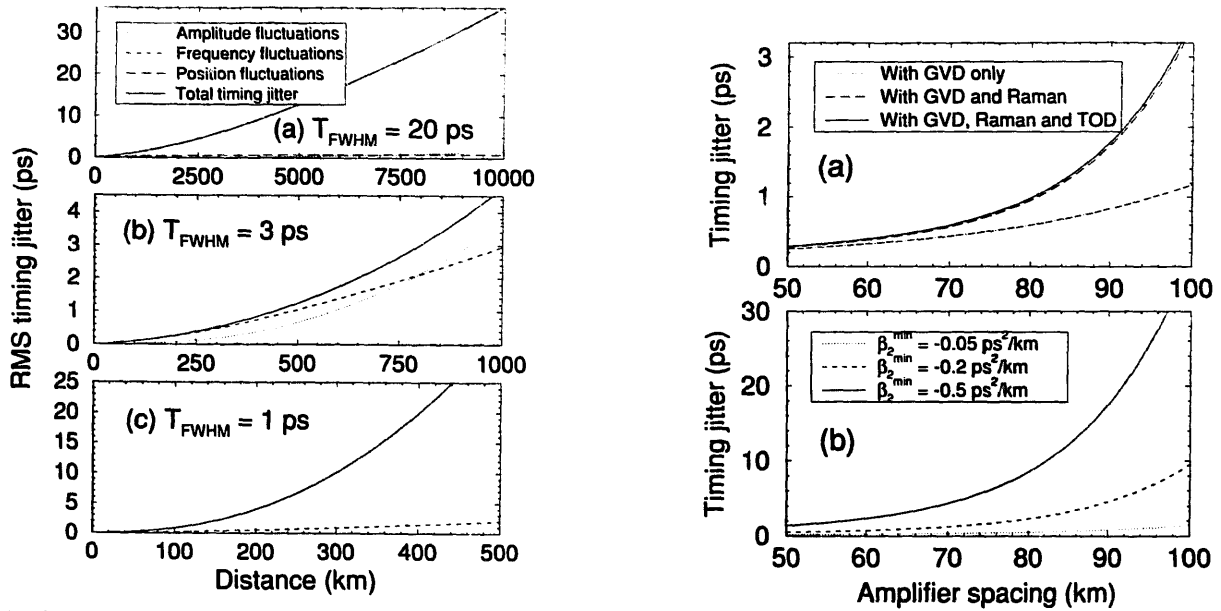


Fig. 1. Timing jitter as a function of transmission distance: (a) $T_{FWHM} = 20$ ps; (b) $T_{FWHM} = 3$ ps and (c) $T_{FWHM} = 1$ ps. Amplifier spacing is 80 km and β_2^{min} is -0.1 ps²/km. Timing jitter is dominated by frequency fluctuations for $T_{FWHM} > 3$ ps and by amplitude fluctuations for 1 ps solitons.

Fig. 2. Timing jitter as a function of amplifier spacing for different fiber parameters: (a) influence of GVD, TOD, and Raman effect; (b) influence of the minimum dispersion β_2^{min} . In both cases the soliton width T_{FWHM} is 2 ps and the total distance of propagation is 300 km.

Fig. 1 shows the RMS timing jitter as a function of transmission distance for three soliton widths by using Eq. (13) for a DDF with a $\beta_2^{min} = -0.1$ ps²/km, $\beta_3 = 0.05$ ps³/km, $T_R = 6$ fs, $L_A = 80$ km and $\alpha = 0.22$ dB/km. The solid line represents the total timing jitter, while the broken lines represent the timing jitter resulting from amplitude, frequency, and position fluctuations. For 20 ps solitons (Fig. 1a), the timing jitter is dominated by frequency fluctuations and reduces to the well-known Gordon–Haus jitter. When 3 ps solitons are used (Fig. 1b), the contribution of amplitude fluctuations on the timing jitter becomes of the same order as the Gordon–Haus jitter, indicating that the Raman-induced SSFS becomes important. For a transoceanic distance of 10000 km and amplifier spacing of 80 km, equal contribution of Raman and Gordon–Haus jitter is observed for 7 ps solitons. For 1 ps solitons (Fig. 1c), the latter effect dominates to the extent that the Gordon–Haus jitter can be ignored for all practical purposes.

Instead of analyzing the total timing jitter in terms of soliton parameters fluctuations as in Fig. 1, one can consider how various fiber properties contribute to the timing jitter. Fig. 2a shows how higher-order effects contribute to the total timing jitter for 2 ps solitons propagating over 300 km as a function of the amplifier spacing. For 100 km amplifier spacing, the timing jitter is relatively low when only GVD is considered. It increases significantly when the Raman effect is included, and is further increased by the TOD. Even if the relative contribution of TOD to the total timing jitter is relatively small for 2 ps solitons it can reach 20% if 1 ps solitons are used. Fig. 2b shows the timing jitter for various values of a different fiber parameter, β_2^{min} , as a function of the amplifier spacing. The strong dependence of the timing jitter on β_2^{min} is due to the relatively high dispersion at the fiber input for a high value of $|\beta_2^{min}|$ for long amplifier spacings.

From a practical standpoint, the jitter needs to be reduced through some form of soliton control. The question is how often such a control must be applied. We provide an estimate of the soliton-control distance by plotting in Fig. 3 the distance where the RMS timing jitter becomes 20% of the soliton width. As also observed experimentally, broad solitons require synchronous modulation on a scale of thousands of kilometers (or more frequent sliding-frequency filters) while very short solitons will require control at every amplification stage.

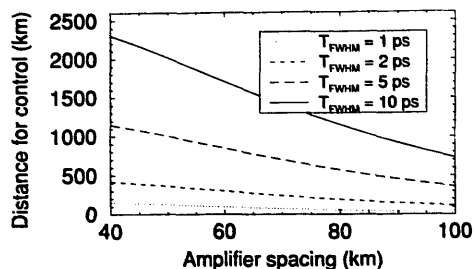


Fig. 3. Estimate of the soliton control distance as a function of the amplifier spacing for different widths in the range 1–10 ps. $\beta_2^{\min} = -0.1 \text{ ps}^2/\text{km}$.

The rapid increase of the timing jitter with decreasing soliton width suggests that soliton communication systems using subpicosecond solitons may represent a serious challenge. However, even though timing jitter for 2–10 ps solitons is increased by higher-order effects, it stays to a level where its control does not seem out of reach.

It is interesting to note that extensive wavelength-division multiplexing using solitons already makes use of DDFs to ensure symmetric collisions between solitons of different channels [15]. This makes the analysis presented in this paper particularly relevant for describing the part of the timing jitter originating from soliton–noise interactions (other sources of jitter need to be considered too), especially if one wants to increase the bit rate of individual channels by reducing the soliton duration.

In summary, we have derived a general expression for the timing jitter in dispersion-decreasing fibers by including the higher-order dispersive and nonlinear effects. The higher-order effects increase the timing jitter beyond that of the Gordon–Haus value for solitons shorter than ~ 10 ps. We show that amplifier-induced amplitude fluctuations dominate the timing jitter for short solitons. We also provide an upper estimate of the distance where a soliton-control device such as a modulator must be inserted to control the timing jitter.

This research was supported by the “Fonds pour la Formation de Chercheurs et l’Aide à la Recherche” of the Québec’s government (Canada) and by the US Army Research Office.

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