Effect of Residual Dispersion in the Phase-Conjugation Fiber on Dispersion Compensation in Optical Communication Systems

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***Abstract*— A simple analytic formula is obtained for calculating the output pulse shape and spectrum in a dispersion-compensation system where the midway optical phase conjugation (OPC) is realized by nondegenerate four-wave mixing in a dispersion-shifted fiber. The nonlinearity and residual dispersion of the OPC fiber are included in our model, and their effects on the OPC process are discussed. The third-order dispersion in the OPC fiber contributes to the total third-order dispersion of the system while the second-order dispersion (group velocity dispersion) and fourth-order dispersion introduce spectral filtering. The design issues such as the bandwidth limits of the input signal, maximum frequency spacing between the signal and the pump, and optimal location of the OPC fiber are discussed.**

I. Introduction

Dispersion-induced pulse broadening is a primary limitation on optical communication systems [1]. Recently, midway spectral inversion by optical phase conjugation (OPC) has been used to compensate for dispersion broadening [2]–[5], a technique first proposed in 1979 [6]. This method can compensate for not only the effects of second-order dispersion [or called the group velocity dispersion (GVD)], but also the effects of self-phase modulation induced by fiber nonlinearity [7] provided the bit stream can be phase-conjugated in the middle of the fiber span in an ideal manner. In practice, nondegenerate four-wave mixing (FWM) in a dispersion-shifted fiber is used for OPC [2]–[5]. Even though the pump wavelength is chosen to be quite close to the zero-dispersion wavelength of the OPC fiber, some residual second-order dispersion (GVD) is often present. Moreover, even if there is no second-order dispersion (GVD), higher-order dispersive effects are likely to affect the phase-conjugation process. To our knowledge, the effects of residual dispersion on the system performance have not been fully investigated.

In this paper, we consider the OPC process by including dispersion to all orders. We also include the nonlinear effects in the OPC fiber due to the large pump power. We are able to obtain an analytic expression for the pulse spectrum at the output of fiber span. We use this result to study how the OPC process is affected by fiber dispersion and nonlinearity and how one can design the system to optimize its performance.

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II. Theory

In this paper, we consider the light fields of linear polarization so that scalar-wave treatment can be used. We also neglect the fiber loss. If l_1 and l_2 are the lengths of the fiber span before and after the phase conjugator, and l_p is the length of the OPC fiber, the total length of the system $l = l_1+l_p+l_2$. For an input signal $E_{in}(t) = A_{in}(t)\exp(-i\omega_s t)$ at the carrier frequency ω_s, its optical field spectrum can be written as $\overline{A}_{in}(\omega - \omega_s)$, where $\overline{A}_{in}(\omega) = \int_{-\infty}^{\infty} A_{in}(t)e^{i\omega t}dt$ is the Fourier transform of $A_{in}(t)$.

In the OPC fiber, pumped by a CW beam of power $P_0 = |A_0|^2$ at the frequency ω_0, phase-conjugated signal is generated through FWM. Using ω_0 as a reference frequency, the spectrum of the field entering the OPC fiber becomes,

$$\overline{A}'(\omega - \omega_0) = \overline{A}_{in}(\omega - \omega_s)\exp[i\beta_s(\omega)l_1] \tag{1}$$

where $\beta_s(\omega)$ is the modal propagation constant of the fiber of length l_1.

In the undepleted pump approximation, the spectrum of light $\overline{A}''$ at the output of the phase-conjugation fiber is given by [8]

$$\begin{aligned}\overline{A}''(\delta\omega) =& \frac{1}{1-r_+r_-}[(e^{ik_+l_p} - r_+r_-e^{ik_-l_p})\overline{A}'(\delta\omega) \\ &+ r_-(e^{ik_-l_p} - e^{ik_+l_p})\overline{A}'^*(-\delta\omega)]\end{aligned} \tag{2}$$

where $\delta\omega = \omega - \omega_0$, the propagation constants k_+ and k_- are defined as (3), shown at the bottom of the next page. Here, $r_+ = D_+/\gamma A_0^2$ and $r_- = \gamma A_0^2/D_-$, and $D_\pm = k_\pm + \beta_p(\omega_0) - \beta_p(\omega_0 + \delta\omega) - \gamma P_0$. β_p is the propagation constant of the OPC fiber, $\gamma = n_2\omega_0/(cA_{eff})$, n_2 is the nonlinear refractive index coefficient, and A_{eff} is the effective core area. The two terms on the right side of (2) correspond respectively to the original signal spectrum and the spectrum of the phase-conjugated light generated by FWM. Since the original signal spectrum is centered at $\delta\omega = \omega_s - \omega_0 \equiv \Delta$, the spectrum of the phase-conjugated light is centered at $\delta\omega = -\Delta$. In practice, there is negligible overlap of the two spectra since the signal bandwidth is usually smaller than the frequency spacing Δ. Since the original signal spectrum is blocked out by using a bandpass filter, only the second term is retained in the following.

The analysis is quite general at this point as dispersion to all orders is included through $\beta_p(\omega)$ in (3). To make further progress, we expand $\beta_p(\omega)$ in a Taylor series around the pump frequency ω_0 and retain terms up to fourth order in $\omega - \omega_0$. After some algebra, the phase-conjugation term in (2) can be written as

$$\overline{A}''(\delta\omega) = i\gamma P_0 l_p H(\delta\omega) e^{i\phi_H} \overline{A}'^{*}(-\delta\omega)] \tag{4}$$

where,

$$H(\delta\omega) = \mathrm{sinc}[l_p\sqrt{(\beta_{p2}\delta\omega^2/2+\beta_{p4}\delta\omega^4/24+\gamma P_0)^2-(\gamma P_0)^2}] \tag{5}$$

and $\phi_H = \beta_{p3}(\delta\omega)^3 l_p/6$. In (5), $\mathrm{sinc}(x)$ denotes $\sin(x)/x$, and $\beta_{p2} = \beta_p''(\omega_0)$, $\beta_{p3} = \beta_p'''(\omega_0)$ and $\beta_{p4} = \beta_p''''(\omega_0)$ are the second- (GVD), third-, and fourth-order dispersion coefficients of the OPC fiber evaluated at the pump frequency. We have ignored the constant term and the term linear in $\delta\omega$ in ϕ_H since they do not distort the pulse.

The output optical spectrum $\overline{A}''_{out}(\delta\omega) = \overline{A}''(\delta\omega)\exp[i\beta_s(\delta\omega+\omega_0)l_2]$ is obtained by considering wave propagation in the second section of the communication fiber of length l_2. By using (1) and (4), the output spectrum is

$$\overline{A}''_{out}(\delta\omega) = i\gamma P_0 l_p H(\delta\omega)\exp(i\phi'')\overline{A}^{*}_{in}(-\delta\omega-\Delta) \tag{6}$$

where $\beta_s(\omega)$ is also expanded in a Taylor series up to the fourth-order in $\delta\omega$ such that $\phi'' \simeq \beta_{s2}(l_2-l_1)\delta\omega^2/2 + \overline{\beta_3} l\delta\omega^3/6 + \beta_{s4}(l_2-l_1)\delta\omega^4/24$. Here, $\overline{\beta_3} \equiv (\beta_{s3}l_1 + \beta_{p3}l_p + \beta_{s3}l_2)/l$ is the average third-order dispersion coefficient. Although the second-order dispersion (GVD) and fourth-order dispersion terms can be reduced to zero by choosing $l_1 = l_2$, the third-order term adds up in the three fiber sections.

Since the output spectrum is centered at $\delta\omega = -\Delta$, it is useful to shift the frequency origin to its central frequency $2\omega_0 - \omega_s$. Thus, the output optical field can be written as $E_{out}(t) = A_{out}(t)\exp[-i(2\omega_0-\omega_s)t]$. The Fourier transform of the output signal $A_{out}(t)$ is related to the Fourier transform of the input signal $A_{in}(t)$ as,

$$\overline{A}_{out}(\omega) = i\gamma P_0 l_p H(\omega-\Delta)\exp(i\phi)\overline{A}^{*}_{in}(-\omega) \tag{7}$$

where

$$\begin{aligned}\phi =&[\beta_{s2}(l_2-l_1) - \overline{\beta_3}l\Delta + \frac{1}{2}\beta_{s4}(l_2-l_1)\Delta^2]\omega^2/2+\\ &[\overline{\beta_3}l - \beta_{s4}(l_2-l_1)\Delta]\omega^3/6 + \beta_{s4}(l_2-l_1)\omega^4/24.\end{aligned} \tag{8}$$

Equations (7) and (8) are the main result of our paper. (7) can be used to calculate the output pulse shape by taking its Fourier transform. Notice that β_{s2}, β_{s3}, and β_{s4} are the values of the dispersion coefficients at the pump frequency. Their values at the input and output carrier frequencies can be easily calculated.

III. Discussion

Equation (7) shows that the residual dispersion in the OPC fiber acts as a bandpass filter whose spectral characteristics are determined by $H(\omega)$, which is given by (5). Indeed, if β_{p2} and β_{p4} are negligible, $H(\omega) \simeq 1$. Note, however, that the phase ϕ in (8) still depends on β_{p3} the third-order dispersion coefficient of the OPC fiber. In fact, the choice $l_2 = l_1$, in which OPC takes place exactly midway, does not even cancel the effect of second-order dispersion (GVD) represented by the term proportional to ω^2 in ϕ. This has been pointed out by several authors [3]. A better strategy would be to ensure that the ω^2 term in (8) vanishes by choosing

$$l_2 - l_1 = \overline{\beta_3}l\Delta/(\beta_{s2}+\beta_{s4}\Delta^2/2). \tag{9}$$

We now turn our attention to the effects of β_{p2} and β_{p4} of the OPC fiber. Regardless of the phase ϕ discussed above, (7) shows that the output signal is equivalent to the input signal first going through an ideal OPC, and then passing through a filter with a response function $H(\omega)$. An important feature of the spectrum $\overline{A}^{*}_{in}(-\omega)$ passing through the filter is that its peak is shifted away from the center frequency of the filter by $-\Delta$, and it falls down to zero at the center frequency of the filter. If the full bandwidth of H is $\Delta\nu_H = \Delta\omega_H/(2\pi)$, the maximum bandwidth of the input signal A_{in} is limited by $\Delta\nu_H/2$. This maximum bandwidth can be realized only by choosing $|\Delta| = \Delta\nu_H/4$ to fully use the effective filter window. If the bandwidth of A_{in} is smaller than $\Delta\nu_H/2$, then $|\Delta|$ is allowed to vary within a small range. When the bandwidth of A_{in} is much smaller than $\Delta\nu_H/2$, the maximum allowed $|\Delta|$ is $\Delta\nu_H/2$.

Two cases should be considered separately depending on whether or not $\beta_{p2} = 0$ in (5). If $\beta_{p2} \neq 0$, then the effects of fourth-order dispersion are negligible, and one can set $\beta_{p4} = 0$ in (5). Fig. 1(a) and (b) shows the spectral characteristics of $H(\omega)$ for normal ($\beta_{p2} > 0$) and anomalous ($\beta_{p2} < 0$) dispersion, respectively, for three values of the OPC fiber length. The nonlinear length, defined as $L_N = (\gamma P_0)^{-1}$ plays an important role. Typically, $\gamma \sim 5$ W^{-1}/km and $P_0 \sim 100$ mW, resulting in $L_N \sim 2$ km. The frequency scales in Fig. 1(a) and (b) are normalized to $(\gamma P_0/|\beta_{p2}|)^{1/2}$, with values ~ 0.1 THz for $|\beta_{p2}| = 1$ ps^2/km. This gives $\Delta\nu_H \sim 0.4$ THz for $l_p \sim L_N$. According to the preceding discussion, the maximum bandwidth of the input signal A_{in} is ~ 0.2 THz, and can be realized by choosing $|\Delta| \sim 0.1$ THz. In most systems, the bandwidth of A_{in} is only a few GHz, which is much smaller than the $\Delta\nu_H/2 = 0.2$ THz in this case, therefore, the OPC filter will limit $|\Delta|$ to below ~ 0.2 THz or ~ 1.6 nm for $|\beta_{p2}| = 1$ ps^2/km. In practice, $|\Delta| \sim 4$ nm or ~ 0.5 THz. Thus, $\Delta\nu_H$ must exceed $2|\Delta| \sim 1$ THz for distortionless phase-conjugation. From Fig. 1(a) and (b), this objective can be realized only if $|\beta_{p2}| < 0.01$ ps^2/km, indicating the need to choose the pump wavelength extremely close to the zero-dispersion wavelength of the OPC fiber.

$$k_{\pm}(\delta\omega) = \frac{1}{2}[\beta_p(\omega_0+\delta\omega) - \beta_p(\omega_0-\delta\omega)] \pm \frac{1}{2}\sqrt{[2\beta_p(\omega_0) - \beta_p(\omega_0+\delta\omega) - \beta_p(\omega_0-\delta\omega) - 2\gamma P_0]^2 - (2\gamma P_0)^2}. \tag{3}$$

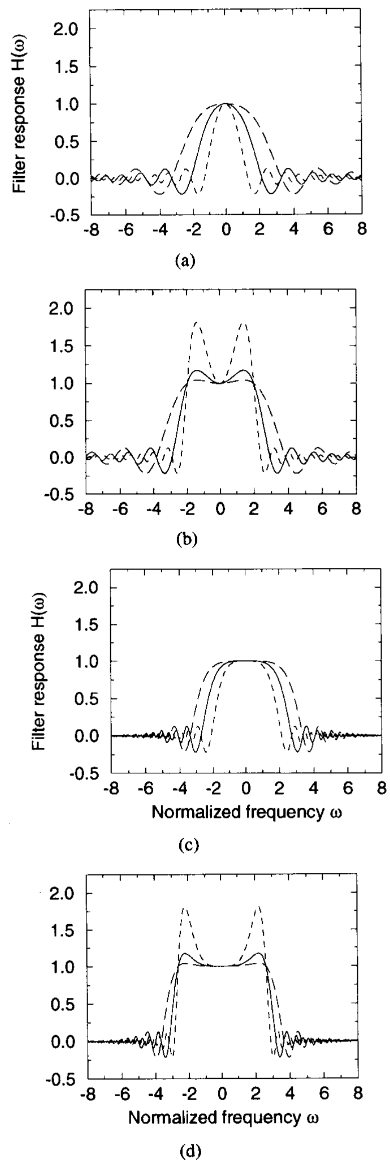

Fig. 1. (a) Spectral filter response for $\beta_{p2} > 0$ and $\beta_{p4} = 0$, at $l_p = 0.5L_N$ (long dashed line), $l_p = L_N$ (solid line), and $l_p = 2L_N$ (short dashed line), respectively. The frequency is normalized to $\sqrt{\gamma P_0/|\beta_{p2}|}$. (b) Same as (a), except for $\beta_{p2} < 0$. (c) Same as (a) except for $\beta_{p4} > 0$ and $\beta_{p2} = 0$, and the frequency is normalized to $(\gamma P_0/\beta_{p4})^{1/4}$. (d) Same as (c) except for $\beta_{p4} < 0$.

Note that the modulation instability occurring for $\beta_{p2} < 0$ (anomalous dispersion) can affect the OPC process since H develops off-center peaks if $l_p > L_N$, as shown in Fig. 1(b). The signal is simply amplified if its bandwidth is much smaller than the width of the peak, otherwise the signal will be distorted.

In the ideal situation in which β_{p2} is exactly zero, $H(\omega)$ still has a finite bandwidth because of the fourth-order dispersion coefficient β_{p4}. Fig. 1(c) and (d) shows the spectral characteristics in such cases. The frequency scale is now normalized to $(\gamma P_0/|\beta_{p4}|)^{1/4}$ whose value is $\sim$ 0.5 THz if we use $|\beta_{p4}| = 5 \times 10^{-3}$ ps^4/km as a representative value. The discussion is similar to the discussion of the effect of β_{p2}. If we choose $l_p \sim L_N$, the bandwidth of A_{in} is limited to $\Delta\nu_H/2 \sim$ 1 THz at the maximum. This can be realized when $|\Delta| \sim$ 0.5 THz. For system operating below several GHz, $|\Delta|$ should not exceed $\sim$ 1 THz, translating into the requirement that the pump and signal wavelength should not differ by more than $\sim$ 8 nm for systems operating near 1.5 μm.

Note that the filter effect depends on the dispersion coefficients, the pump power, and the length of the OPC fiber. For fixed l_p/L_N, larger pump power and smaller dispersion coefficients produce a wider filter whose bandwidth is proportional to $(\gamma P_0/|\beta_{p2}|)^{1/2}$ and $(\gamma P_0/|\beta_{p4}|)^{1/4}$ for second-order (GVD) and fourth-order dispersion limited cases, respectively. Longer l_p provides larger signal [from (7)] but narrower filter bandwidth. Generally, $l_p \sim L_N$ will give a phase-conjugated signal with amplitude comparable to that of the signal entering the OPC fiber if its spectrum falls within the effective filter window.

In conclusion, we have obtained a simple analytic formula for calculating the output pulse spectrum in a dispersion-managed system where the midway OPC is realized by nondegenerate FWM in a dispersion-shifted fiber. The effects of fiber nonlinearity and residual dispersion on the OPC process are considered. The third-order residual dispersion in OPC fiber contributes to the total third-order dispersion of the system while the second- (GVD) and fourth-order residual dispersions introduce spectral filtering. The optimal position of the OPC fiber around the midpoint of the communication fiber was determined. The bandwidth limits of the input signal and maximal frequency spacing between the carrier and the pump due to the filter effects are discussed.

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