Mode locking in semiconductor lasers by phase-conjugate optical feedback

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We show theoretically through computer simulations that phase-conjugate optical feedback that is realized through four-wave mixing can produce mode locking in multilongitudinal-mode semiconductor lasers. Phase-conjugate feedback, in contrast to ordinary optical feedback, directly couples pairs of longitudinal modes. For certain strengths of phase-conjugate reflectivities, the phases and beat frequencies of various longitudinal modes become locked in such a way that the laser produces ultrashort mode-locked pulses, even though the laser is pumped continuously.

Mode locking of lasers and, in particular, of semiconductor lasers continues to generate intense interest owing to such potential uses as the source of ultrashort optical pulses in a soliton communication system.[1] Semiconductor lasers have been mode locked actively,[2] passively,[3] and through hybrid schemes.[4] In active mode locking, for instance, the gain or loss of the laser is modulated at the round-trip frequency of the laser.[5] The active modulator induces beat-frequency locking and phase locking, even overcoming the effects of noise, and thereby creates short pulses.

We show theoretically in this Letter that it is possible to achieve both frequency locking and phase locking in a multilongitudinal-mode laser through the use of phase-conjugate feedback (PCF), provided that four-wave mixing (FWM) is used to generate PCF. We are considering only the case in which the feedback represents a small perturbation to the laser; i.e., the output facet of the solitary laser is not antireflection coated (the weak-feedback regime). We also point out that conventional optical feedback, which occurs usually through unwanted reflections from optical fibers, optical disks, or other system components, will in general not lead to phase and frequency locking among the modes. The primary reason is that the different modes, although coupled intrinsically through the laser medium, are only self-coupled by an ordinary reflection; that is, on reflection, each mode couples back only onto itself and not directly to any other mode. Some of the effects of conventional optical feedback on multilongitudinal-mode lasers were recently reported.[6]

The schematic for PCF is shown in Fig. 1. The material used for the phase-conjugate mirror (PCM) is assumed to be a broadband (with respect to the longitudinal-mode spacing) and highly nonlinear medium with an essentially instantaneous response. The PCM is pumped by a narrow-linewidth laser operating at frequency ω_p. The phase-conjugate signal is created by a nondegenerate FWM process so that, when a frequency ω_j is incident upon the PCM, the conjugate signal is shifted to $\omega_c = 2\omega_p - \omega_j$. The conjugate signal will return directly to the laser as a result of the self-aligning nature of the PCF. However, in order for the returned signal to couple into the laser, the frequency must nearly coincide with one of the longitudinal-mode frequencies. This can only occur only if (i) ω_p nearly coincides with one one of the longitudinal modes or if (ii) ω_p lies nearly in the middle of the two neighboring modes. In either case the rate equation for the complex slowly varying amplitude of the mode oscillating at ω_j can be written as

$$\frac{dE_j}{dt} = \frac{1}{2}(1 - i\alpha)\left(G_L - \frac{1}{\tau_p}\right)E_j + \zeta_j + F_j + \kappa_j E^*_{2p-j}(t-\tau)\exp\left[-2i\delta\left(t - \frac{\tau}{2}\right)\right], \quad (1)$$

where α is the linewidth enhancement factor ($\alpha = 3$), G_L is the gain that depends linearly on the carrier density, τ_p is the photon lifetime ($\tau_p = 1.4$ ps), and F_j is a Langevin noise term to account for random spontaneous emission. ζ_j contains all the gain nonlinearities and other intrinsic mode coupling terms, including intramodal FWM. These terms arise from spatial and spectral hole burning and carrier heating. The explicit form of ζ_j can be found in Ref. 6. The last term in Eq. (1) accounts for PCF. Note that E_j now

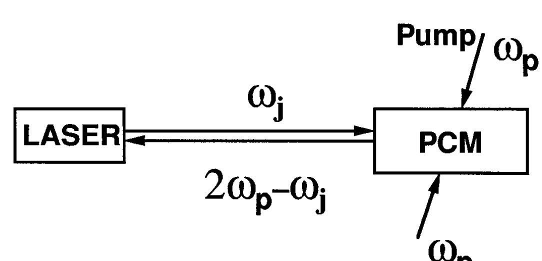

Fig. 1. Schematic arrangement of PCF. Nondegenerate FWM causes the phase-conjugate wave to be frequency shifted from the probe wave.

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depends explicitly on the field of one other longitudinal mode, namely, the mode at $2\omega_p - \omega_j$. This field component is delayed by the external cavity round-trip time τ ($\tau = 0.67$ ns), and it appears as the complex conjugate owing to the action of the PCM. The strength of the feedback is denoted by the feedback rate κ_j. The exponential term accounts for any additional phase delay caused by a small detuning δ of the pump laser frequency from the optimum frequency.[7] Since each field rate equation now depends on one other mode, the action of the PCM might be called a type of single-sideband modulator, in contrast to ordinary active modulators.

The equation for the carrier number remains unchanged by PCF and is given, as usual, by

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{I}{q} - \frac{N}{\tau_e} - \sum_j G_j P_j, \tag{2}$$

where N is the carrier number, I is the injection current, q is the electronic charge, τ_e is the carrier lifetime ($\tau_e = 2$ ns), and P_j is the photon number in the jth mode. P_j is related to the complex field by $E_j = \sqrt{P_j}\,\exp(-i\phi_j)$, where ϕ_j is the instantaneous phase of mode j.

The presence of mode locking can be studied in the usual way by consideration of the so-called reduced phases, defined by[8]

$$\Psi_j = (2\omega_j - \omega_{j-1} - \omega_{j+1})t + (2\phi_j - \phi_{j-1} - \phi_{j+1})\,. \tag{3}$$

Mode locking corresponds to Ψ_j going to a constant for all j. We note for completeness that the total laser output power of any multimode laser contains evidence of the longitudinal-mode beating, even if a detector is not fast enough to respond to these oscillations. All multimode lasers are not mode locked, of course, because the ω_j are not equally spaced and the phases are fluctuating randomly. The high-frequency oscillations tend to wash out in this case, so that the total power contains only slowly varying oscillations. When the Ψ_j as defined above are forced to become constant, the laser is both phase and frequency locked and is said to be mode locked. Generally, when the Ψ_j go to a multiple of 2π, then the laser is AM mode locked; when the Ψ_j go to an odd multiple of π, the laser emits an FM wave.

The delayed-feedback term in Eq. (1) and the fact that several longitudinal modes are being considered make any analytic progress difficult. Therefore we numerically integrate Eqs. (1) and (2), using a fourth-order Runge–Kutta algorithm. The strength of the PCF or, equivalently, the reflectivity of the PCM is conveniently characterized in terms of a dimensionless feedback parameter $\kappa\tau$. For simplicity we assume the reflectivity to be frequency independent, so that $\kappa\tau$ is the same for all the modes. The weak-feedback regime is explored for $\kappa\tau$ values between zero and 5. The laser behavior in the presence of PCF is generally quite complicated, and the permitted parameter space is rather large. To facilitate the discovery of regimes in which mode locking occurs, we employ bifurcation diagrams, which permit immediate identification of a stable, periodic, or chaotic regime.[9] The bifurcation diagrams are constructed with the noise sources turned off, so that only deterministic rather than stochastic dynamics are involved.

We have investigated the effect of PCF on lasers with three, four, and five modes. In all cases, stable regimes of mode locking are found. For simplicity, when the number of modes is odd, we assume the PCM pump frequency to be near the central mode; when the number of modes is even, we take the PCM pump to lie between the two central modes. In the three-mode case, Ψ_2 is constant and equal to zero for $0.6 < \kappa\tau < 2.0$, indicating AM mode locking. The locking is stable even in the presence of spontaneous emission noise.

A representative bifurcation diagram for the slowly varying total power versus $\kappa\tau$ is shown in Fig. 2 for the four-mode case. Also shown on the bifurcation diagram is the total standard deviation (square root of the sum of the two variances) of the Ψ (Ψ_2 and Ψ_3 for four modes) with and without the presence of noise. The PCM pump laser frequency is tuned exactly between the two central modes (modes 2 and 3). This explicitly couples modes 2 and 3 together and modes 1 and 4 together. The relative injection current is $I/I_{\mathrm{th}} = 1.4$. The nonlinear gain terms are adjusted so that the modes are weakly coupled; i.e., with no feedback, all modes oscillate simultaneously with a certain power distribution rather than in the bistable fashion that is characteristic of strong coupling.[10] In these simulations the intrinsic FWM terms are neglected.

The bifurcation diagram indicates that the laser in the presence of PCF exhibits complex dynamical behavior. For different values of $\kappa\tau$ the laser exhibits quasi-periodicity, period-doubling bifurcations, and chaos. Of particular interest is the region corresponding to $\kappa\tau$ values between ~1.6 and 2.4. In this region the average power output is relatively well behaved, exhibiting small-amplitude quasi-periodic behavior. However, over this entire region the total standard deviation of the two Ψ values is practically zero without noise and $\sim \pi/10$ even with noise; that is, the Ψ's are constant in this range, and the laser is

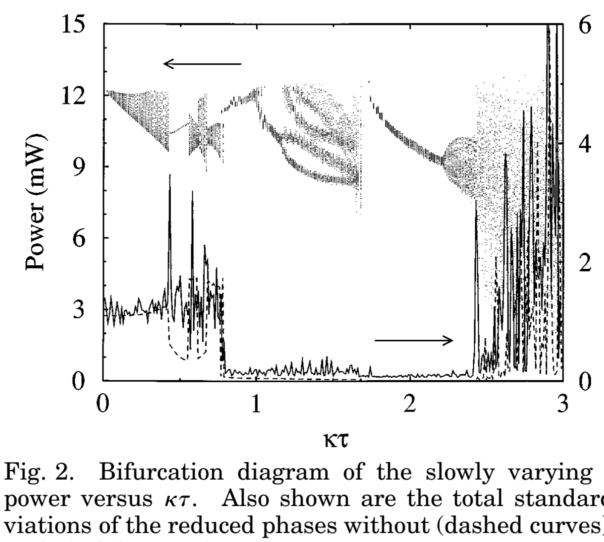

Fig. 2. Bifurcation diagram of the slowly varying total power versus $\kappa\tau$. Also shown are the total standard deviations of the reduced phases without (dashed curves) and with (solid curves) noise.

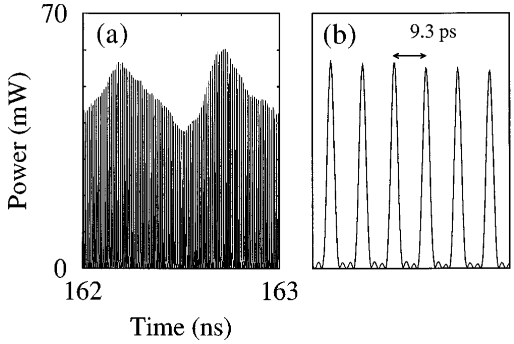

Fig. 3. (a) Representative mode-locked pulse train for $\kappa\tau = 1.8$. (b) Zoomed-in version, revealing the spacing between pulses as well as the secondary pulse structure.

mode locked. Interestingly, the Ψ values are nearly constant even in the feedback range $0.8 < \kappa\tau < 1.6$, although the modes exhibit a somewhat complicated (period 4) quasi-periodic behavior. The best mode locking, however, occurs in the range $1.6 < \kappa\tau < 2.4$. We make the following points regarding the mode locking: (1) Usually, mode-locked solutions are restricted such that the mode amplitudes as well as the reduced phases are constant. In the presence of PCF the mode amplitudes are rarely constant, even in the single-mode case,[11] yet this only weakly affects the purity of the mode-locked pulses, as shown below. (2) Not only are the mode amplitudes not constant but their behavior can be rather complex, as indicated by the bifurcation diagram in the $\kappa\tau$ range from 1 to 2.4. Although the bifurcation diagram shows only total power, the individual mode powers behave in a similar fashion. For $\kappa\tau = 1.3$, for example, the slowly varying laser output is quasi-periodic with a period-4 behavior, yet the phases are locked to a very good degree, the total standard deviation being $\sim 0.2\pi$. (3) As Eq. (3) shows, the fact that both Ψ_2 and Ψ_3 are constant implies that all three beat frequencies are the same. The particular values of Ψ_j dictate the quality of the mode-locked pulses. For example, it can be shown analytically that, although $\Psi_j = 0$ for all j yields the best pulses, good pulses are obtained as long as Ψ_j remain near zero, for example, $|\Psi_j| < \pi/4$. In our four-mode case the constants Ψ_2 and Ψ_3 are nearly equal and opposite, both with and without noise, through the entire locking range. The particular value of Ψ_j depends on the feedback strength.

Finally, the mode-locked pulses themselves, with the noise sources turned on, are shown in Fig. 3. The quasi-periodic behavior of the mode amplitudes mentioned above is responsible for the slow variation of the pulse heights. This variation occurs at a frequency of $\sim$3 GHz and is close to the relaxation-oscillation frequency of the solitary laser. The spontaneous emission noise has a minimal effect. Also shown in Fig. 3 is a zoomed-in version of the mode-locked pulses. The pulses are spaced, as usual, by the laser round-trip time, or the reciprocal of the longitudinal-mode spacing [9.3 ps = $(107\ \text{GHz})^{-1}$]. A simulation time step of 0.1 ps permits the well-known secondary maxima to be clearly discernible.

Why does PCF lead to mode locking? A possible interpretation is that PCF provides a frequency shift that can counteract the mode pulling and pushing (frequency shifts induced by nonlinear gain) that ordinarily render the various beat frequencies unequal. For very weak feedback the PCF-induced frequency shift is too small to accomplish the locking. Beyond a certain feedback strength, however, locking occurs over a wide region. At large values of feedback, mode locking is lost because of the onset of chaos. We note that some nonlinear gain is actually beneficial since it helps damp the relaxation oscillations and therefore delay the onset of chaos.

We comment briefly on the assumption of an extremely fast-responding PCM. In reality, of course, any material has a finite response time, leading to a frequency-dependent response. However, our assumption is reasonably valid as long as the PCM bandwidth is much larger than the longitudinal-mode spacing. In our simulations we assumed a mode spacing of $\sim$100 GHz, which is a typical value for solitary semiconductor lasers. For this case a semiconductor laser amplifier could provide frequency-independent PCF since its bandwidth has been measured to be greater than 1 THz. More generally, a PCM based on a Kerr-type nonlinearity would have an adequately wide bandwidth. Alternatively, to use a PCM with a smaller bandwidth one could construct an external cavity laser, with any desired longitudinal-mode spacing and number of modes.

In summary, we have shown theoretically that it is possible to produce mode locking by using phase-conjugate feedback. Although PCF can produce chaotic output as well, stable mode-locked regions have been found for three, four, and five modes, even in the presence of spontaneous emission noise.

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