Pulse compression and spatial phase modulation in normally dispersive nonlinear Kerr media

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Numerical simulations show that, because of the spatiotemporal coupling implied by the multidimensional nonlinear Schrödinger equation, self-focusing of ultrashort optical pulses can lead to pulse compression even in the normal-dispersion regime of a nonlinear Kerr medium. We show how this coupling can be further exploited to control the compression by use of spatial phase modulation. Both the compression factor and the position at which the minimum pulse width is realized change with the amplitude of the phase modulation.

It has been well established for many years that, to lowest order, pulse propagation in nonlinear dispersive media is described accurately by the nonlinear Schrödinger equation[1] (NSE). In the past, the NSE has helped to provide an understanding of a wide variety of effects such as beam steering,[2] soliton formation and propagation,[3,4] self-focusing,[5–14] and pulse compression.[1] Attempts to understand self-focusing began more than 25 years ago and have evolved from analytic approximations for cw beams[5–7] to the moving focus model for the self-focusing of nondispersive pulses.[8,9] However, it is only recently that the full multidimensional NSE has been employed for the investigation of self-focusing of ultrashort pulses in normally dispersive media.[10–13] In our work we model pulse propagation in waveguides with a self-focusing nonlinearity and, although they may be more accurately described as the spatial analog of soliton-effect pulse compression, we refer to the effects of the nonlinearity as self-focusing for economy of notation. The impact of the nonlinearity on both the spatial and the temporal behavior can be investigated with the multidimensional NSE. Moreover, because the nonlinearity makes the NSE inseparable, it is only in its multidimensional form that the coupling between these two behaviors can be investigated. In this Letter we present the results of numerical simulations that show how self-focusing and pulse compression are related through this spatiotemporal coupling.

The NSE is derived from Maxwell's equations for the case of an intensity-dependent (Kerr-type) index of refraction of the form $n = n_0 + n_2 I$.[1,14] We model pulse propagation with the NSE in the one-dimensional or waveguide approximation by using the well-known split-step Fourier method.[1] The waveguide approximation consists of assuming that diffraction occurs in only one transverse direction, the field behavior in the other direction being determined by the structure of the waveguide. To simplify the model and broaden the applicability of the results, we normalize all the variables including the field that is normalized so that its peak input value is unity. The coordinates are normalized as follows: transverse spatial coordinate ξ is normalized to the input beam width σ, temporal coordinate τ is normalized to the incident pulse width T_0, and propagation distance is measured in units of the diffraction length $L_d = (2\pi/\lambda)\sigma^2$, where λ is the optical wavelength. The normalized NSE then takes the form

$$i\frac{\partial u}{\partial \zeta} + \frac{1}{2}\frac{\partial^2 u}{\partial \xi^2} - \frac{s}{2}\frac{\partial^2 u}{\partial \tau^2} + N^2|u|^2 u = 0. \quad (1)$$

Here the parameter $N = (2\pi\sigma/\lambda)\sqrt{n_0 n_2 I_0}$ represents the strength of the Kerr nonlinearity. The quantity $n_2 I_0$ represents the maximum nonlinear index change for an input pulse of peak intensity I_0. The parameter $s = (2\pi/\lambda)\sigma^2\beta_2/T_0^2$ represents the relative strengths of dispersion and diffraction. Here β_2 is the group-velocity dispersion parameter, defined as in Ref. 1.

For $N = 0$ we have the linear case for which Eq. (1) is separable, and the spatial and temporal behaviors evolve independently of each other. But when $N \neq 0$ the NSE is not separable, and some spatiotemporal coupling must occur. Sometimes the coupling is weak compared with other effects. This is the case, for example, when a pulse propagates in a fiber in which the transverse spatial behavior is determined by the fiber parameters.[1] In such a situation the derivative with respect to ξ is eliminated from the NSE, and diffraction plays no role in the pulse evolution. In the anomalous dispersion regime ($\beta_2 < 0$) nonlinearity-induced self-phase modulation and group-velocity dispersion can cooperate in such a way that an intense pulse undergoes compression.[1] By contrast, in the normal-dispersion regime ($\beta_2 > 0$) the combination of self-phase modulation and group-velocity dispersion invariably broadens the pulse as it propagates. As an example, Fig. 1(a) shows pulse broadening for an $N = 3$ Gaussain pulse launched into a nonlinear medium in which diffractive effects are ignored.

One may ask whether spatiotemporal coupling occurring when both diffraction and dispersion occur simultaneously in a nonlinear medium can lead to pulse compression even for normal group-velocity dispersion. The answer turns out to be yes.

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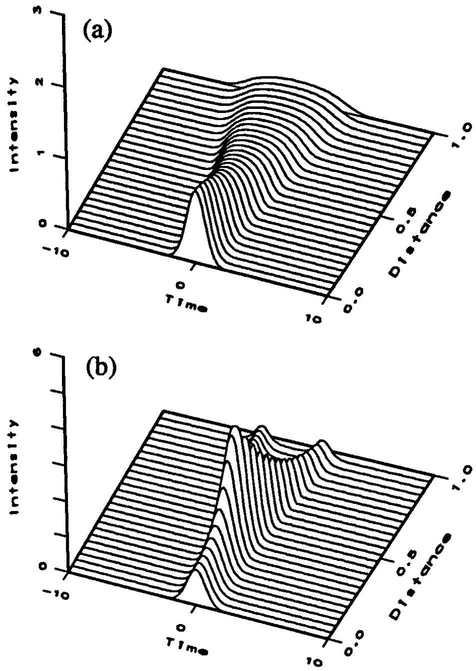

Fig. 1. Input Gaussian pulse evolution in a normally dispersive nonlinear medium for $N = 3$ nonlinearity and the pulse traveling in (a) a fiber and (b) a waveguide such that $s = 0.5$ [see Eq. (1)].

Figure 1(b) shows pulse evolution under conditions identical to those of Fig. 1(a), except that diffraction has been included and $s = 0.5$. Clearly, self-focusing can lead to a modest amount of pulse compression, as indicated in Fig. 1(b). This compression must be due to the spatiotemporal coupling implied by the inseparability of the NSE because it occurs only when the diffractive term is included in the simulation. Although the diffractive term is necessary, the dispersive term is not, because it is the self-focusing that leads to the pulse compression.[8] What we will see is that under some conditions the coupling is strong enough to override the pulse-spreading influence of the normal dispersion.

Because spatiotemporal coupling is due to the nonlinearity, we first investigate the dependence of this effect on the strength of the nonlinearity for a range of s parameters. In Fig. 2 we have plotted the normalized FWHM of the pulse as a function of propagation distance for several different N's and $s = 0.5$. The linear case shows a monotonic increase, as we expected. For nonzero N we see the appearance of a secondary minimum that gets deeper and closer to $\zeta = 0$ as N is increased. We expect that, because in the absence of diffractive (spatial) effects the compression does not occur, the stronger the diffraction is relative to dispersion, the greater the compression will be, i.e., the maximum pulse compression should increase as s decreases. As seen in Fig. 3(a), where we have plotted the minimum pulse width as a function of N for $s = 0.1$, 0.5, 1.0, this is in fact the case. Another interesting feature of this plot is the threshold behavior seen for $s = 1.0$ and 0.5. We see that in both cases for a large range of N values there is no compression. What happens in these cases is that the secondary minima seen in Fig. 2 are larger than the initial pulse width. If we look at Fig. 3(b), which plots the position of the minimum pulse width as a function of N, we can again observe this threshold behavior. We also see that the position of the minimum decreases with N, whereas it is nearly independent of s once threshold is reached. This gives an insight into the mechanism at work here. What we are observing is a result of the power dependence of the self-focusing distance for a cw beam. The secondary minima of Fig. 2 always occur at a distance equal to the point of maximum focus associated with the peak of the input pulse. This conclusion is borne out by the dashed curve in Fig. 3(b), which plots the position of the minimum beam width of spatial solitons as a function of the strength of the nonlinearity for the cw case.

Because the observed compression is a spatiotemporal effect, it is reasonable to expect that it can be controlled to some extent through spatial phase modulation. To determine the impact of spatial phase modulation, we solve the NSE for the case of a modulated input field of the form

$$u(\xi, \tau, 0) = \exp\left(-\frac{\xi^2}{2} - \frac{\tau^2}{2}\right)\exp[i\phi(\xi)]. \quad (2)$$

For the case of sinusoidal phase modulation, $\phi(\xi)$ has the form

$$\phi(\xi) = \phi_0 \sin(2\pi p\xi + \delta), \quad (3)$$

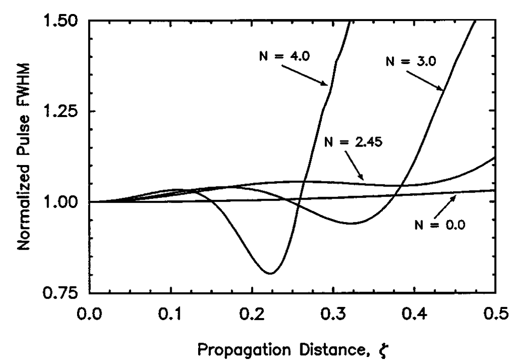

Fig. 2. Normalized pulse FWHM at beam center for fields initially Gaussian in space and time, traveling in a dispersive nonlinear waveguide such that $s = 0.5$ and with input nonlinearities as indicated.

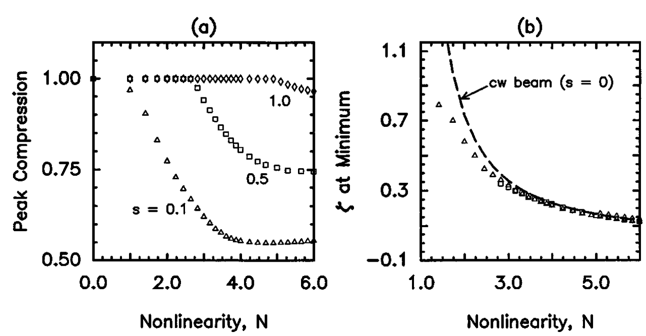

Fig. 3. (a) Minimum normalized pulse FWHM and (b) the position of the minimum as a function of the nonlinearity at input for $s = 1.0$ (diamonds), $s = 0.5$ (squares), and $s = 0.1$ (triangles). The dashed curve in (b) is the position of the minimum beam width of a (cw) spatial soliton of order N.

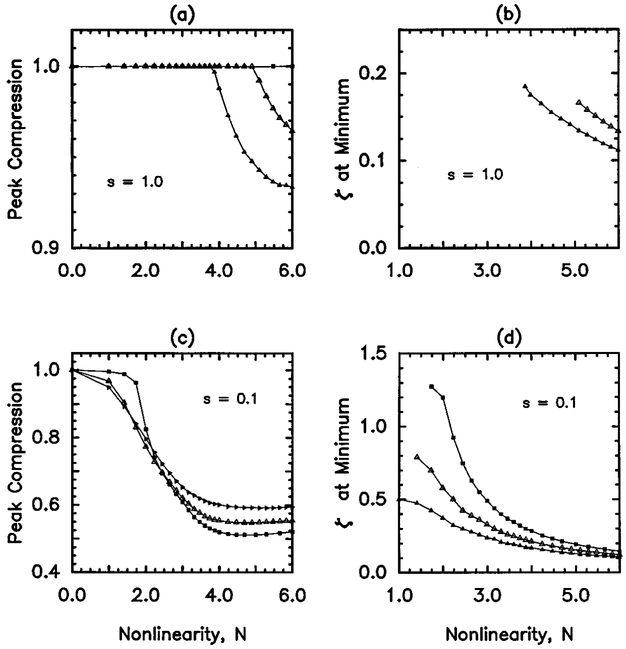

Fig. 4. Effects of spatial phase modulation on (a) the minimum normalized pulse FWHM and (b) the position of the minimum for $s = 1.0$; (c) the minimum FWHM and (d) the position of the minimum for $s = 0.1$. The phase is modulated at input according to Eq. (3) with $p = 0.2$, $\delta = \pi/2$, and $\phi_0 = 1.0$ (filled triangles), $\phi_0 = -1.0$ (filled squares), and unmodulated or $\phi_0 = 0.0$ (open triangles). There are only two curves in (b) because there is no compression for the case of defocusing phase modulation and $s = 1.0$.

where ϕ_0 is the amplitude of the modulation, p is the modulation frequency, and δ is a constant phase shift. For typical modulation frequencies in the range $p = 0.1-0.3$ and the phase shift $\delta = \pi/2$, the phase modulation described by Eq. (2) is nearly quadratic or lenslike across the width of the input field, such that positive and negative ϕ_0 correspond to convex and concave lenses, respectively.

Because the unmodulated compression is due to self-focusing, if we want to enhance (suppress) the pulse compression we should choose $\phi_0 > 0$ ($\phi_0 < 0$) because it will enhance (suppress) the self-focusing. In Fig. 4 we see the effects of the modulation. For $s = 1.0$ [Fig. 4(a)] focusing ($\phi_0 > 0$) modulation has the strongest impact. It enhances the maximum compression over a large range of N's, with the effect becoming more dramatic as the modulation amplitude is increased. For a relatively long pulse [$s = 0.1$ in Fig. 4(c)] the modulation has the opposite effect on the maximum compression. Conversely, the point of maximum compression is affected by the phase modulation in an intuitive manner even for $s = 0.1$. As Figs. 4(b) and 4(d) clearly indicate, focusing modulation always shortens the distance to the point of maximum compression and defocusing modulation always increases this distance. The control of pulse compression through spatial phase modulation can be analytically studied by use of the moment method of Ref. 10.

From the results presented here we may draw the following conclusions. First, spatiotemporal coupling can lead to pulse compression even in normally dispersive, self-focusing Kerr media. Second, the degree of compression increases and the distance to the point of maximum compression decreases as either the pulse width increases or the strength of the nonlinearity increases. Finally, this compression phenomenon can be controlled with spatial phase modulation. Several other points should also be made. First, in none of the numerical simulations was a reduction in the FWHM by more than a factor of 2 achieved, nor was there ever any reduction in the temporal variance.[10] Also, in every case in which compression was achieved, it was immediately followed by a splitting of the pulse.[10–13] The inclusion of both spatial transverse dimensions may affect the pulse compression characteristics, although pulse splitting occurs even in that case. As others have shown, splitting is critically dependent on the presence of normal dispersion.[13]

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