Optical switching in $\lambda/4$-shifted nonlinear periodic structures

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We show that $\lambda/4$-shifted distributed-feedback nonlinear devices can be used as an all-optical switch at relatively low input intensities. The $\lambda/4$ shift opens a narrow transmission window whose peak position within the stop band depends on the input intensity, a feature that can be used for low-power optical switching. The nonlinear coupled-mode approach is used to analyze the stationary operating regime of such a device and determine the transmittivity as a function of the input intensity. A closed-form solution, rather than a numerical one, is found for what we believe is the first time.

Optical properties of periodic nonlinear structures have been studied extensively, both theoretically[1-6] and experimentally,[7,8] in areas such as optical switching,[4] optical bistability,[1] and pulse compression.[5] In view of the ever-increasing importance of integrated photonics devices, the use of these structures in all-optical switching applications deserves particular attention. In this Letter we analyze the switching properties of a $\lambda/4$-shifted distributed-feedback (DFB) nonlinear device operating in the stationary (cw) regime. For the first time, to our knowledge, the closed-form expression for the structure transmittivity is given, as opposed to the different numerical methods used to analyze nonuniform nonlinear structures. Although we emphasize the analysis of the $\lambda/4$-shifted device, the method herein is readily extended to arbitrary phase shifts. Of some interest is the generalization to include multiple phase shifts as well.

Propagation in a linear periodic medium is characterized by the stopgaps within the corresponding photonic band structure in which no traveling-wave solutions are permitted. In the presence of material nonlinearity the band structure changes, shifting the position of stop bands and consequently permitting transmission at previously forbidden frequencies. In the cw regime and for small coupling strengths this type of switching can be analyzed exactly by solution of the corresponding set of nonlinear coupled-mode equations[1] that describe its complicated bistable behavior. The time-dependent case was also studied both numerically[4,9] and analytically,[3] revealing a rich underlying dynamics that, in special cases, can lead to slowly moving soliton-type solutions referred to as Bragg solitons.

To switch from the nontransmitting to the transmitting state, one usually tunes close to the edge of the stop band to minimize the required switching intensity. However, the switching intensity remains relatively high and is the single most important obstacle in the way of practical applications of these devices. Considerable effort has been devoted recently to optimize the design of the grating by tapering[10] and reflectance matching.[11]

The $\lambda/4$-shifted DFB device, first proposed by Haus and Shank,[12] has been used[13] extensively to design and fabricate stable single-mode semiconductor laser sources. In the absence of gain and nonlinearity the photonics band structure of this device is readily calculated[14] and is found to have a narrow transmission peak in the middle of the stop band. It is expected that introduction of the Kerr-type nonlinearity will lead to a frequency shift of this transmission peak as the input intensity is increased. A tunable filter that uses this feature along with an intense control beam was proposed.[15] To produce optical switching, one needs only to shift the transmission peak by a small amount equal to its own width, which we show is possible even for low input intensities.

Consider the phase-shifted structure shown in Fig. 1, having the uniform periodic regions ($-L \le z < 0$ and $0 \le z \le L$) with the linear refractive index

$$n_L(z) = n_0 + \Delta n \cos(2\beta_B z + \Omega),$$
$$\Omega = \begin{cases} \Omega_1, & z < 0 \\ \Omega_2, & z \ge 0 \end{cases}, \qquad (1)$$

where β_B is the Bragg wave number and $\Omega_2 - \Omega_1$ is the phase shift at $z = 0$. The material nonlinearity is modeled by an instantaneous isotropic Kerr-type response such that the refractive index increases by an amount $n_{NL} = n_2|E|^2$. To solve the propagation problem in the cw regime, we proceed by

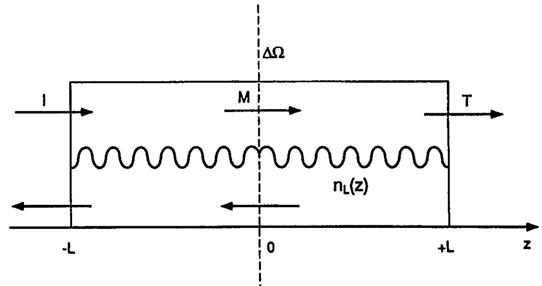

Fig. 1. DFB structure with the phase shift $\Delta\Omega$ at the center located at $z = 0$. The sinusoidal curve represents the periodic linear refractive index $n_L(z)$. Normalized intensities I, M, and T are defined in the text.

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using the standard coupled-mode approach.[16] We emphasize that the present analysis is not limited to a particular device realization (thin-film waveguide, nonlinear multilayer stack, or fiber grating device) and can be applied whenever the standard coupled-mode assumption is valid. The electric field is represented as the sum of the forward- and backward-traveling waves:

$$E(z) = E_+(z)\exp(i\beta z) + E_-(z)\exp(-i\beta z). \quad (2)$$

Taking the usual coupled-mode assumptions of small coupling strengths $\Delta n << n_0$ and a slowly varying field envelope,[16] one can easily derive the following set of equations:

$$\frac{dE_+}{dz} = i\kappa E_-\exp[-i(2\Delta\beta z - \Omega)] + i\gamma(|E_+|^2 + 2|E_-|^2)E_+, \quad (3a)$$

$$\frac{dE_-}{dz} = -i\kappa E_+\exp[i(2\Delta\beta z - \Omega)] - i\gamma(2|E_+|^2 + |E_-|^2)E_-, \quad (3b)$$

where $\Delta\beta = \beta - \beta_B$ is the detuning from the Bragg wavelength, $\kappa = \pi\Delta n/\lambda_B$ is the linear coupling coefficient, and $\gamma = \pi n_2/\lambda_B$ is the nonlinear parameter governing the self- and cross-phase modulation.

Each uniform region of the grating can be treated separately, following the approach of Ref. 1, provided that the proper boundary conditions are enforced at $z = 0$. By separating the magnitude and the phase of the field in Eqs. (3) in the form $E_\pm(z) = |E_\pm(z)|\exp[i\phi_\pm(z)]$ one can find the conserved quantities for each region:

$$\Gamma_i = |E_+||E_-|\cos\Psi_i + (2\Delta\beta + 3\gamma|E_-|^2)|E_+|^2/(2\kappa), \quad (4)$$

$$|E_{iT}|^2 = |E_+|^2 - |E_-|^2, \quad (5)$$

where $i = 1, 2$ and $\Psi_i(z) = 2\Delta\beta z + \phi_+(z) - \phi_-(z) - \Omega_i$. It is important to mention that no boundary conditions were introduced in the derivation of Eqs. (4) and (5). By imposing the nonreflective boundary condition at the right-hand end of the structure in Fig. 1 $[E_-(L) = 0]$ the transmitted flux in the second region $(0 \le z \le L)$ is given by $|E_{2T}|^2 = |E_+(L)|^2$. Continuity of the electric field at $z = 0$ implies that the transmitted flux remains constant throughout the structure, i.e.,

$$|E_{1T}|^2 = |E_{2T}|^2 = |E_T|^2 = |E_+(L)|^2. \quad (6)$$

At this point we introduce an auxiliary intensity parameter $I_c = 4n_0\lambda/3\pi n_2 L$ and normalize all intensities to I_c. Specifically, we define $T = |E_T|^2/I_c$ as the transmitted flux at $z = L$, $M = |E_+(0)|^2/I_c$ as the forward flux in the center of the structure at $z = 0$, and $I(z) = E_+(z)|^2/I_c$ as the forward flux in the first region $(-L < z < 0)$. From Eqs. (4)–(6) it can be shown that

$$\cos\Psi_2(0) = -\frac{1}{\kappa L}\left(\frac{M - T}{M}\right)^{1/2}(2M + \Delta\beta L). \quad (7)$$

In the case of a $\lambda/4$-shifted grating the phase shift at $z = 0$ is set to π, which together with Eqs. (4) and (6) leads to an expression for Γ_1 in the following form:

$$\Gamma_1 = I_c[4M(M - T) + (\Delta\beta L)(2M - T)]/\kappa L. \quad (8)$$

Finally, Eqs. (4) and (8) permit us to construct the expression for forward flux in the first region $(-L < z < 0)$:

$$\left(\frac{L}{2}\frac{dI}{dz}\right)^2 = (\kappa L)^2 I(I - T) - [4M(M - T) + (\Delta\beta L)(2M - T) - I(\Delta\beta L) - 2I(I - T)]^2. \quad (9)$$

The integration of Eq. (9) is the standard elliptic problem that can be solved if the integration interval and zeros of the polynomial on the right-hand side are known. These zeros, which depend on the parameters κL, $\Delta\beta L$, and T, are generally complex values. For the sake of brevity we will discuss the general solution of Eq. (9) elsewhere, reporting only the most important results here.

In the case of the zero detuning ($\Delta\beta L = 0$), M is found to be related to T as

$$M = T[1 + nd(u, k)]/2, \quad (10)$$

where $nd(u, k)$ is the elliptic function with argument $u = \sqrt{T^2 + (\kappa L)^2}$ and modulus $k^2 = [1 + (T/\kappa L)^2]^{-1}$. Zeros of the polynomial in Eq. (9) can be explicitly calculated in this case as

$$I_i = \frac{T}{2}\left[1 \pm \frac{k}{k'}\left(x \pm \sqrt{x + k^2}\right)^{1/2}\right], \quad i = 1, 2, 3, 4, \quad (11)$$

where $x = k'^2\{2[nd^2(u,k) - 1] + k^2\}/k^2$ and $k' = \sqrt{1 - k^2}$ is the complement of k. It is seen that all zeros are real $(I_1 > I_2 > I_3 > I_4)$. The input flux $I_0 = I(-L)$ can now be calculated and is given by

$$I_0 = I_3 + \frac{(I_2 - I_3)(I_1 - I_3)}{(I_1 - I_3) - (I_1 - I_2)sn^2[F(\varphi_0, k_0) - 4/g_0, k_0]}, \quad (12)$$

where sn and $F(\varphi_0, k_0)$ are the elliptic functions,[17] $g_0 = 2/[(I_1 - I_3)(I_2 - I_4)]^{1/2}$, $k_0 = g_0[(I_1 - I_2)(I_3 -$

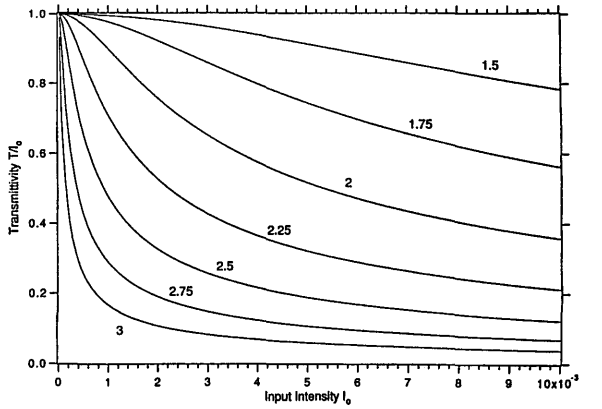

Fig. 2. Transmittivity (T/I_0) versus I_0 for the structure of Fig. 1 when $\Delta\Omega = \pi$ and $\Delta\beta = 0$. Each curve corresponds to a different coupling parameter κL, varied between 1.5 and 3.

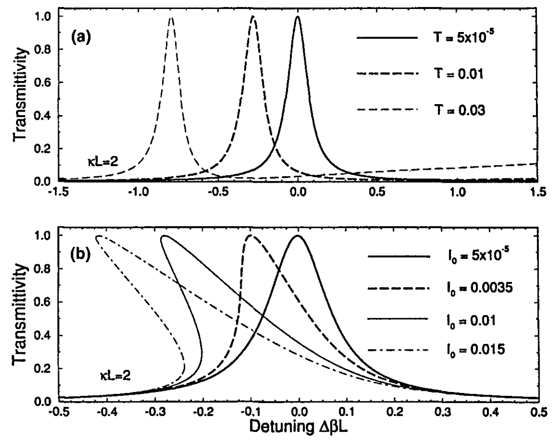

Fig. 3. (a) Variation of T/I_0 with the Bragg detuning $\Delta\beta L$ for three values of the normalized output flux T and for $\kappa L = 2$. (b) Variation of T/I_0 with $\Delta\beta L$ for four values of the normalized input I_0 and $\kappa L = 2$.

$I_4)]^{1/2}/2$, and $\sin \varphi_0 = [(I_1 - I_3)(M - I_2)/(I_1 - I_2)(M - I_3)]^{1/2}$.

To estimate the value of the required switching intensity, we plot in Fig. 2 the device transmittivity T/I_0 versus input intensity I_0 at zero detuning ($\Delta\beta L = 0$). As expected, at very low intensities ($I_0 << 1$) the grating is completely transparent, and transmission $T/I_0 \approx 1$. Transmittivity decreases with increasing input intensity because the nonlinear index shifts the transmission peak toward lower frequencies, which eventually makes the device opaque at λ_B. One can reverse this process by detuning the laser from the Bragg wavelength such that $\Delta\beta L < 0$ at low intensities and then increasing I_0 to achieve the transmitting state. However, the detuning parameter $\Delta\beta L$ has to be chosen carefully, in accordance with the knowledge of the bistable region position. To appreciate the low intensities necessary for this type of switching, we consider the case of $\kappa L = 3$ in Fig. 2. To change the transmission state from 100% to below 20% (or vice versa), we need only to change I_0 by an amount ~0.001. If one considers a 1-cm-long GaAs device ($n_2 = 1.6 \times 10^{-10}$ esu) with $\kappa L = 3$ and $\lambda_0 = 1\ \mu$m, it is clear that such a change corresponds to ~0.1 MW/cm^2. This value should be contrasted with the ~0.2 GW/cm^2 value necessary to switch the equivalent uniform DFB device[1] at $\Delta\beta L = 0$.

We can obtain transmission behavior as a function of the detuning $\Delta\beta L$ by using Eq. (9) and by following a similar procedure. Figure 3(a) shows the variation of T/I_0 with $\Delta\beta L$ for three fixed values of the output intensity T. For small values of T (solid curve) the nonlinear effects are negligible, and the device exhibits a narrow central transmission window located exactly in the center of the stop band. For larger values of T the intradevice intensity is large enough for the nonlinear index to shift the transmission peak from the center of the stop band. The conventional transmittivity spectrum in which the input intensity I_0 (rather than the output intensity T) is a fixed parameter is quite different and is shown in Fig. 3(b). It exhibits bistability for certain values of I_0 ($I_0 > 0.004$) and $\Delta\beta L < 0$, permitting the all-optical switching to be controlled by frequency tuning rather than intensity tuning. Another interesting aspect of this switch is to study its dynamics with the appropriate numerical methods.[4,18]

In conclusion, we have shown that a $\lambda/4$ DFB nonlinear structure operated close to the Bragg frequency can be used for optical switching at low intensities, thus greatly relaxing the requirement for high-power sources.

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