Effect of two-photon absorption on the amplification of ultrashort optical pulses

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The effect of two-photon absorption on the amplification of ultrashort optical pulses is studied theoretically by solving a generalized nonlinear Schrödinger equation. An input pulse can be simultaneously amplified and compressed, although the compression factor is smaller in the presence of two-photon absorption. The amplified pulse evolves toward a chirped soliton that is the solitary-wave solution of the underlying propagation equation. It can split into several chirped solitons whose number, width, and peak power depend on the amplifier parameters.

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The amplification of ultrashort optical pulses in fiber amplifiers has attracted considerable attention[1–9] in view of its potential applications in fields such as optical communications and photonic switching. In a recent paper [8] the amplification process was modeled through a Ginzburg-Landau equation that included both gain dispersion and gain saturation. However, the effect of two-photon absorption (TPA) on the amplification process was neglected because its effects are expected to be small for the case of commonly used erbium-doped silica fibers. TPA is known to play an important role in all-optical switching [10–12]. It is also known to affect the soliton dynamics and can lead to the breakup of higher-order solitons in undoped optical fibers [13]. The TPA effects are likely to become important for new types of fibers made with semiconductor-doped glasses or lead-silicate glasses [14,15] exhibiting relatively high nonlinearities. It is therefore important to consider how TPA affects the performance of doped fiber amplifiers. In this Brief Report we extend the results of Ref. [8] to include the effect of TPA on the amplification of ultrashort optical pulses.

If the coherent effects [7] are neglected by assuming that the pulse width is much larger than the dipole relaxation time T_2 of the dopants, pulse amplification is governed by a generalized nonlinear Schrödinger equation (or a Ginzburg-Landau equation) [8]

$$\frac{\partial A}{\partial z}+\frac{1}{v_g}\frac{\partial A}{\partial t}+\frac{i}{2}(\beta_2+ig_0T_2^2)\frac{\partial^2 A}{\partial t^2}-i\gamma|A|^2A = \frac{g_0}{2}A-\frac{\alpha_2}{2}|A|^2A \ , \quad (1)$$

where β_2 is the group-velocity-dispersion (GVD) coefficient [16], $\gamma=2\pi n_2/\lambda$ is related to the nonlinear-index parameter n_2, and g_0 is the small-signal gain at the operating wavelength λ. This equation is similar to the Ginzburg-Landau equation used in Ref. [8], except that the effect of TPA has been included through the last term in Eq. (1) where α_2 is the TPA coefficient. Gain saturation during amplification of a single pulse can be neglected in most fiber amplifiers since the saturation energy is typically much larger than the pulse energy. It is useful to write Eq. (1) in a normalized form,

$$i\frac{\partial u}{\partial \xi}-\tfrac{1}{2}(s+id)\frac{\partial^2 u}{\partial \tau^2}+|u|^2u=\frac{i}{2}(\mu-\mu_2|u|^2)u \ , \quad (2)$$

where ξ, τ, and u are normalized variables defined as

$$\xi=\frac{z}{L_D}, \quad \tau=\frac{t-z/v_g}{T_0} \quad u=\sqrt{\gamma L_D}\,A \quad (3)$$

with

$$L_D=\frac{T_0^2}{|\beta_2|}, \quad d=\frac{gT_2^2}{|\beta_2|}, \quad \mu=g_0L_D, \quad \mu_2=\frac{\alpha_2}{\gamma} \ . \quad (4)$$

Here T_0 is the input pulse width, L_D is the corresponding dispersion length, and $s=\mathrm{sgn}(\beta_2)=+1$ or -1 depending on whether β_2 is positive or negative. TPA is governed by the parameter μ_2 that can be written as $\mu_2=\lambda\alpha_2/2\pi n_2$. For silica fibers $\mu_2\ll 1$, although it can become ~ 1 for semiconductor-doped glasses or other high-n_2 materials [12]. As an example, μ_2 was estimated to be about 0.01 in lead-silicate fibers [14] and may approach 0.1 for As_2S_3 glasses [15].

To study pulse amplification, Eq. (2) is solved numerically by using a split-step algorithm [14]. The input amplitude $u(0,\tau)=\mathrm{sech}\tau$ corresponds to that of a fundamental soliton. Figure 1 shows the energy gain and the full width at half maximum (FWHM) τ_{FWHM} of the central peak as a function of the propagation distance ξ for an amplifier with 10-dB gain per dispersion length ($\mu=2.3$) and $T_2/T_0=0.2(d=0.092)$ for μ_2 in the range 0–0.2. As one would expect the energy gain is reduced considerably in the presence of TPA. The pulse width τ_{FWHM} decreases as the pulse is amplified, a feature that can be used to simultaneously amplify and compress picosecond optical pulses. The compression factor is reduced in the presence of TPA. Figure 2 compares the pulse shapes at $\xi=1$ after the optical pulse has propagated over one dispersion length. Even for $\mu_2=0.1$ the peak power is reduced by a factor of 3 and the compression is reduced by a factor of 2 because of TPA.

Figure 1 shows that τ_{FWHM} begins to increase dramati-

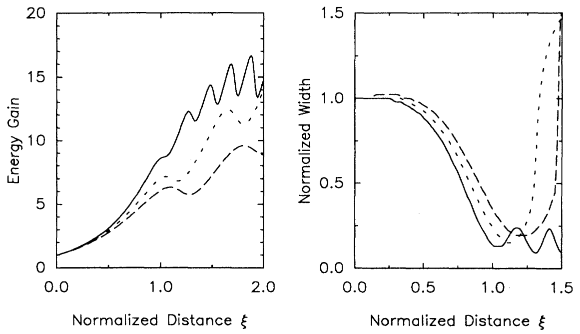

FIG. 1. Energy gain and the FWHM of the input pulse as a function of the amplifier length when a fundamental soliton is amplified in a fiber with 10-dB gain per dispersion length. Solid, dotted, and dashed curves correspond to $\mu_2=0$, 0.1, and 0.2, respectively, where μ_2 is related to the TPA coefficient.

cally after $\xi>1.2$, a feature that is absent when $\mu_2=0$. This behavior can be understood from Fig. 3 where pulse evolution over the range $\xi=0-2$ is shown for $\mu_2=0.1$. The amplified pulse splits into two pulses beyond $\xi>1.2$ such that a minimum occurs at $\tau=0$. Both subpulses are amplified until they reach a maximum amplitude near $\xi=1.8$. Further propagation leads to generation of additional subpulses. Pulse splitting is also observed for $\mu_2=0$ [6,8], but the presence of TPA absorption makes pulse splitting occur at shorter distances. Another difference induced by TPA is that whereas the central peak remains intact in the absence of TPA [8], it can disappear in the presence of TPA as seen in Fig. 3 for $\xi=1.8$.

To understand the effects of TPA in a more analytic manner, one should consider whether Eq. (2) permits solitary-wave solutions. In the absence of TPA ($\mu_2=0$), Eq. (2) is known to have a solitary-wave solution of the form [2]

$$u(\xi,\tau)=N\,\mathrm{sech}(p\tau)\exp\{iq\ln[\cosh(p\tau)]+i\Gamma\xi\}\;, \tag{5}$$

where the parameters N, p, q, and Γ depend on μ and d.

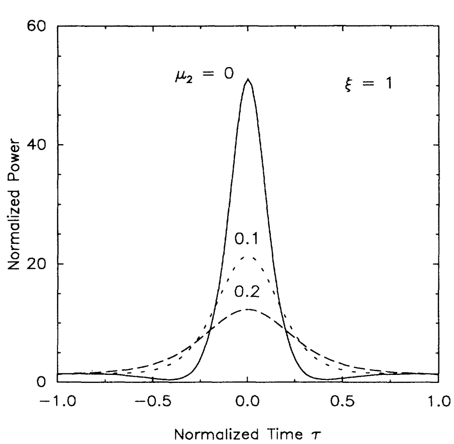

FIG. 2. Comparison of pulse shapers at $\xi=1$ for $\mu_2=0$, 0.1, and 0.2. Other parameters are the same as in Fig. 1.

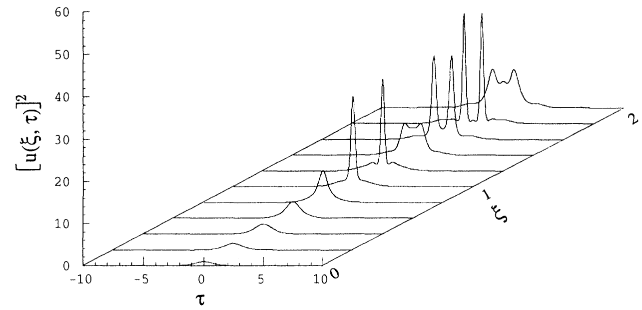

FIG. 3. Evolution of amplified pulse over two dispersion lengths when $\mu_2=0.1$. Other parameters are the same as in Fig. 1.

The use of perturbation theory shows that Eq. (2) has an approximate solution given by [15] (valid only for $\beta_2<0$ or $s=-1$)

$$u(\xi,\tau)=2\eta\,\mathrm{sech}(2\eta\tau)\exp[4i\eta^2\xi-\tfrac{2}{3}i\eta(2d+\mu_2)\tau] \tag{6}$$

with $\eta=\frac{3}{4}\mu(d+2\mu_2)^{-1}$ even in the presence of TPA provided μ, d, and μ_2 are $\ll 1$.

In the case of fiber amplifiers μ can be larger than 1 ($\mu=2.3$ in Figs. 1–3), and the perturbative solution (6) is not valid. It turns out that a solitary-wave solution of Eq. (2) exists for arbitrary values of μ, d, and μ_2 and for both positive and negative values of β_2. In fact, the solution (5) is a solution of Eq. (2) when N, p, and Γ are given by

$$N^2=\tfrac{1}{2}p^2[s(q^2-2)-3qd]\;, \tag{7a}$$

$$p^2=-\mu[d(1-q^2)-2sq]^{-1}\;, \tag{7b}$$

$$\Gamma=-\tfrac{1}{2}p^2[s(1-q^2)+2qd]\;, \tag{7c}$$

and q is a solution of the quadratic equation

$$(d-\mu_2 s/2)q^2+3(s+\mu_2 d/2)q+\mu_2 s-2d=0\;. \tag{7d}$$

The solitary-wave solution (5) represents a chirped soliton. It exists for both normal and anomalous dispersions (by choosing $s=1$ and -1, respectively) as long as N and p are real and positive. In the case of anomalous dispersion ($s=-1$) the solution (5) reduces to the exact solution of Ref. [17]. In the absence of TPA ($\mu_2=0$) it reduces to the solution given in Ref. [2]. It also reduces to the perturbative solution (6) if μ, μ_2, and d are assumed to be small. Physically, N represents the soliton amplitude, p^{-1} is related to the soliton width, q provides the extent of frequency chirping, and Γ is the propagation constant.

Figure 4 shows dependence of the soliton amplitude N and the soliton width p^{-1} on the TPA parameter μ_2 for the anomalous GVD case by choosing $s=-1$ in Eqs. (7). The parameter $d=\mu$ if we use $T_0=T_2$ for the normalization. The gain parameter μ is related to the amplifier gain G through $G=\exp(\mu)$. The effect of TPA is to decrease the soliton amplitude and increase the soliton width. The same qualitative behavior persists even in the case of normal GVD. Since the soliton widths obtained

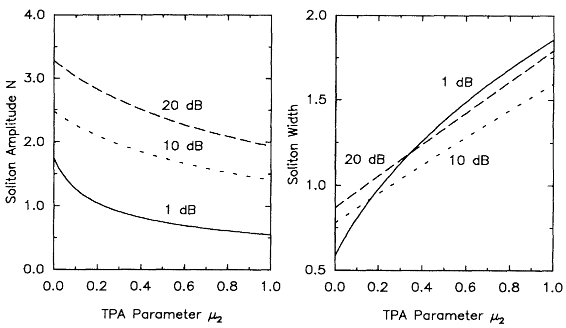

FIG. 4. Amplitude and width of chirped solitons as a function of the TPA parameter μ_2 for amplifiers with 1-, 10-, and 20-dB gain per dispersion length. Soliton width is normalized to the dipole relaxation time T_2.

in Fig. 4 are comparable to T_2, one should reexamine the question of validity of the soliton given by Eq. (5). Strictly speaking, Eq. (1) does not remain valid for $T_0 \sim T_2$ since the coherent effects are expected to become important [7]. The important point to note is that TPA is not detrimental to the existence of chirped solitons in fiber amplifiers. One can understand the amplification process seen in Fig. 3 in terms of these chirped solitons as follows. The input pulse experiences simultaneous amplification and compression during the early stages of the amplification process and evolves toward the chirped soliton given by Eq. (5). The numerical results verify the existence of frequency chirp. Since the soliton amplitude is fixed by the parameter N for a given set of amplifier parameters, further amplification leads to the generation of multiple solitons whose number keeps on increasing with the propagation distance. Malomed has shown [18] that the frequency chirp provides an attractive force that helps to bind multiple chirped solitons.

In conclusion, this paper has studied the effects of TPA on the pulse amplification in fiber amplifiers. An input pulse can be simultaneously amplified and compressed although the compression factor is smaller in the presence of TPA. The amplified pulse evolves toward a chirped soliton that is the solitary-wave solution of the generalized nonlinear Schrödinger equation. The pulse can split into several chirped solitons whose number depends on the gain and the length of the fiber amplifier.

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