Concept of Linewidth Enhancement Factor in Semiconductor Lasers: Its Usefulness and Limitations

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Abstract—**The usefulness and the limitations of the concept of the linewidth enhancement factor α in semiconductor lasers are examined by considering the laser dynamics without the rate-equation approximation. The rate equations with a constant value of α can be used for semiconductor lasers operating continuously or modulated directly such that the carrier density does not change significantly during each modulation cycle. A new set of generalized Bloch equations should be used whenever subpicosecond optical pulses are involved.**

A remarkable feature of semiconductor lasers is that both the optical gain and the refractive index of the active region change with injected carrier density. Simultaneous variations of the mode gain and the mode index with external pumping result in an amplitude-phase coupling that affects many laser characteristics [1]–[3]. Perhaps the most well-known effect of such an amplitude-phase coupling is the enhancement of the laser linewidth by a factor of $1 + \alpha^2$, where the parameter α is referred to as the linewidth enhancement factor (LEF) [1]. In the simple rate-equation model commonly used for semiconductor lasers, α is introduced phenomenologically by assuming that both the optical gain and the refractive index vary linearly with the carrier density [2]. Because of its importance in governing the laser behavior, the LEF has been extensively studied both theoretically and experimentally [3]–[9]. However, in spite of its usefulness, the concept of LEF is viewed with suspicion because of its phenomenological origin. The objective of this paper is to discuss the limitations behind the concept of the LEF and identify the conditions under which such a concept can be used in the theory of semiconductor lasers.

In the density-matrix formulation [10]–[12] the dynamic response of a semiconductor laser is modeled by considering transitions between the individual conduction and valence band states and integrating over the entire range of transition frequencies in a way similar to an inhomogeneously broadened two-level system. The calculation of gain and the refractive index under steady-state conditions requires a knowledge of the band structure [4], [5]. In the transient regime the procedure becomes impractical since the gain and the refractive index cannot even be defined without making the rate-equation approximation. We have recently developed a set of generalized Maxwell–Bloch equations [13] which allows us to study semiconductor-laser dynamics without explicitly performing an integration over the band states. In our model, details of the band structure are included through a single parameter s that can be calculated numerically or used as a fitting parameter. The generalized Bloch equations obtained from the density-matrix equations are [13]

$$\frac{dW}{dt} = \Lambda_p - \gamma_L(W - W_{th}) + \frac{\mu}{i\hbar}(E^*p - Ep^*) \tag{1}$$

$$\frac{dp}{dt} = -\gamma_T(1 + i\Delta)p - s^2\gamma_T S + \frac{\mu}{2i\hbar}EW \tag{2}$$

$$\frac{dS}{dt} = -\gamma_T(1 + i\Delta)S + \gamma_T p + \frac{\mu}{2\hbar}EU \tag{3}$$

$$\frac{dU}{dt} = -\gamma_c(U - \bar{U}) - \gamma_L(U - U_{th}) - \frac{\mu}{\hbar}(E^*S + ES^*) \tag{4}$$

where $W = \langle \rho_{11} - \rho_{22}\rangle$, $p = \langle \tilde{\rho}_{12}\rangle$, $S = \langle \delta\tilde{\rho}_{12}\rangle$, $U = \langle \delta(\rho_{11} - \rho_{22})\rangle/s^2$, and $\tilde{\rho}_{12}$ is the slowly varying part of $\rho_{12} = \tilde{\rho}_{12} \exp(-i\omega_L t)$. Here ρ_{11}, ρ_{22}, and ρ_{12} are the density-matrix elements [10]. The angle brackets denote averaging over the band states, i.e., $\langle x\rangle = \int x(\delta)D(\delta)\,d\delta$, where $\delta = (\omega - \omega_0)/\gamma_T$ is the normalized detuning from a reference frequency ω_0. The parameter Δ in (2) and (3) is defined as $\Delta = (\omega_0 - \omega_L)/\gamma_T$, where ω_L is the laser frequency. We choose $\omega_0 = \omega_L$ since this choice makes $\Delta = 0$ and simplifies the analysis. In (1)–(4), Λ_p is the pumping rate, μ is the dipole moment, γ_T is the dipole relaxation rate inversely related to the intraband relaxation time τ_{in}, γ_c is the intraband relaxation rate of electrons, and γ_L is the interband carrier relaxation rate that is related to the carrier lifetime τ_e as $\tau_e = \gamma_L^{-1}$. Typically γ_T^{-1} and γ_c^{-1} are ~ 0.1 ps whereas $\gamma_L^{-1} \sim 1$ ns. W_{th} and U_{th} are the values of W and U in thermal equilibrium, whereas $\bar{U}$ in (4) denotes the quasi-equilibrium value of U established by the intraband relaxation processes.

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The parameter s^2 appearing in (2) is defined as

$$s^2 = \langle \delta^2(\rho_{11}^{ss} - \rho_{22}^{ss})\rangle / \langle \rho_{11}^{ss} - \rho_{22}^{ss} \rangle \tag{5}$$

where the superscript *ss* stands for steady-state values. All band-structure details appear in our model through a single parameter s, defined to be dimensionless. Physically, s is a measure of the spread of the electron and hole populations.

The generalized Bloch equations (1)–(4) govern the dynamic response of semiconductor lasers and are valid even in the femtosecond time domain. However, the concept of LEF is not useful in the femtosecond regime since neither gain nor refractive index can be defined as long as the induced polarization p is not proportional to the optical field E. One can define a time-dependent ratio Re (p)/Im (p) that reduces to α under steady-state conditions. This ratio plays the role of LEF in the femtosecond regime.

For time scales longer than a few picoseconds one can consider the rate-equation limit of the generalized Bloch equations by assuming that both γ_T and γ_c are $\gg \gamma_L$ and setting dp/dt, dS/dt, and $dU/dt = 0$ in (2)–(4). Both p and S are then proportional to E, and one can introduce the susceptibility χ through the relation

$$P = 2\mu N_t p = \epsilon_0 \chi E \tag{6}$$

where P is the induced polarization, N_t is the total dipole density, and ϵ_0 is the vacuum permittivity. The susceptibility χ is found to be given by

$$\chi = -\frac{\mu^2 N_t}{\epsilon_0 \hbar \gamma_T (1 + s^2)} (s^2 U + iW) \tag{7}$$

where

$$U = \frac{\bar{U}}{1 + |E|^2/\tilde{I}_s} \tag{8}$$

and the intraband saturation intensity $\tilde{I}_s$ is given by

$$\tilde{I}_s = (\hbar^2/\mu^2)\gamma_c \gamma_T (1 + s^2). \tag{9}$$

Equation (7) provides an analytic expression for the complex susceptibility of a semiconductor laser under external pumping. The real and imaginary parts of χ are related to the refractive index and the optical gain, respectively. The LEF, defined as $\alpha = \mathrm{Re}\,\chi/\mathrm{Im}\,\chi$, is given by a remarkably simple expression

$$\alpha = \frac{s^2 U}{W} = \frac{\langle \delta(\rho_{11} - \rho_{22})\rangle}{\langle(\rho_{11} - \rho_{22})\rangle}. \tag{10}$$

This expression for α makes it evident that the origin of α lies in the asymmetric nature of electron and hole distributions in semiconductors. If we define $w = \rho_{11} - \rho_{22}$ as the local value of the inversion, $\alpha = \langle \delta w \rangle / \langle w \rangle$ can be interpreted as the average detuning weighted with the local inversion. The main point to note is that, in contrast with the commonly made assumption, α changes with both the carrier density N (proportioned to W) and the intensity $|E|^2$. It is also time dependent whenever N and $|E|^2$ change with time.

One may ask under what conditions α can be treated as constant. In the case of CW operation of a semiconductor laser, both U and W in (10) are replaced by their steady-state values. The LEF is then obviously time independent. The inversion W remains approximately clamped to its threshold value and does not change significantly with laser power. At law operating powers (< 10 mW), U can be replaced by $\bar{U}$ since intraband saturation is negligible. The LEF factor can then be treated as a constant parameter as long as the laser power remains below the intraband saturation power (typically > 100 mW). For directly modulated semiconductor lasers, α is, in principle, time dependent since W (proportional to the carrier density N) changes with time. However, variations in N or W are quite small as long as the laser is not biased well below threshold, and α can be taken approximately constant. The situation is different for semiconductor laser amplifiers in which variations in N or W are quite large during pulse amplification. The use of a constant α is less justified for optical amplifiers than for semiconductor lasers. For both lasers and amplifiers, the concept of α becomes questionable when ultrashort (subpicosecond) optical pulses are involved. The full set of generalized Bloch equations should then be used since the rate-equation approximation no longer remains valid in that case.

To illustrate the limitation of the concept of the LEF for subpicosecond pulses, consider amplification of a Gaussian pulse in a semiconductor laser amplifier. Equations (1)–(4) are solved numerically with the amplitude

$$E = E_0 \exp(-t^2/2\tau_0^2) \tag{11}$$

by choosing $\tau_{\mathrm{in}} = \gamma_T^{-1} = 0.1$ ps, $\tau_c = \gamma_c^{-1} = 0.2$ ps, $\tau_e = \gamma_L^{-1} = 0.3$ ns, $\Delta = 0$, $s = 2$, and $\bar{U} = 0.5$. The peak amplitude E_0 is chosen such that $E_0^2 = \hbar \gamma_L \gamma_T / \mu^2$ corresponds to the interband saturation intensity. The pump rate is such that the amplifier can provide 30-dB single-pass gain in the absence of gain saturation. Fig. 1 shows variation of α, defined as the ratio Re (p)/Im (p), with time t for three values of the pulse width τ_0. For $\tau_0 \gg 1$ ps, α is nearly constant over the entire pulse, indicating that it can be treated as time independent for picosecond pulses. However, for subpicosecond pulses, α varies considerably over the pulse duration and cannot be assumed to be a constant. For $\tau_0 = 0.5$ ps, α can even change sign. These results clearly indicate that the use of a constant value of α becomes less and less justified as pulses become shorter than a few picoseconds. The generalized Bloch equations presented here are valid as long as the concept of intraband relaxation remains applicable.

In conclusion, the limitations of the concept of LEF in semiconductor lasers were examined by considering the laser dynamics beyond the rate-equation approximation. A constant LEF can be used for semiconductor lasers operating continuously above threshold and for directly

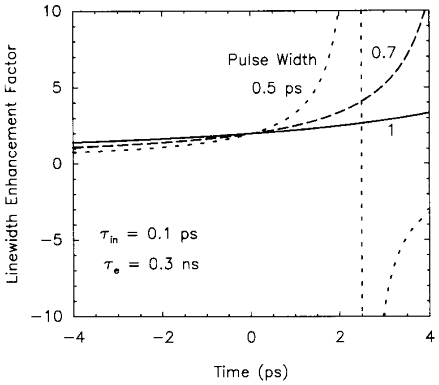

Fig. 1. Variation of linewidth enhancement factor α with time for several values of the pulse width τ_0 when a Gaussian pulse is amplified in a semiconductor laser amplifier with 30-dB gain. The carrier lifetime, the intraband (dipole) relaxation time, and the electron scattering time are $\tau_e = 0.3$ ns, $\tau_{in} = 0.1$ ps, and $\tau_c = 0.2$ ps, respectively. Other parameter values used in the numerical calculations are given in the text.

modulated semiconductor lasers as long as the laser is not biased too far below threshold so that the carrier density remains nearly constant. A new set of generalized Bloch equations [(1)–(4)] should be used whenever subpicosecond or femtosecond optical pulses are involved. These equations would be useful for mode-locked semiconductor lasers and for amplification of femtosecond pulses in semiconductor laser amplifiers.

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Reduction of Damping in High-Speed Semiconductor Lasers

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***Abstract*—We derive an analytical expression for the intrinsic gain suppression factor based on carrier heating. The theory shows good agreement with the published experimental value of $\epsilon = +1.5 \times 10^{-17}$ cm^3 for in-plane lasers. For the first time, we predict and experimentally observe a *negative* gain suppression factor for particular laser designs. A negative gain suppression factor can lead to the elimination of damping in semiconductor lasers. Using vertical-cavity surface-emitting lasers, we observe a negative gain suppression factor of -2.2×10^{-17} cm^3.**

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UNDERSTANDING the difficulties that limit the modulation bandwidth are crucial to the development of high-speed semiconductor lasers. Structure effects such as carrier transport can severely degrade the modulation bandwidth of semiconductor lasers [1]. For properly designed semiconductor lasers without any extrinsic transport limitation, the modulation response is still damped by intrinsic effect such as gain suppression [2], [3].

Experimental measurements of damping in the modulation response of high-speed lasers have been modeled fairly well by assuming the gain is reduced at high photon densities by a factor in the form of $1/(1 + \epsilon S_o)$ [1], [2], $(1 - \epsilon S_o)$ [3], or $1/\sqrt{1 + \epsilon S_o}$ [4], where ϵ is a phe-

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