Dynamic and Noise Properties of Tunable Multielectrode Semiconductor Lasers Including Spatial Hole Burning and Nonlinear Gain

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***Abstract*—A general formalism based on the Green's function method is given for multielectrode semiconductor lasers. The effects of both spatial hole burning and nonlinear gain are included in this formalism. An effective nonlinear gain is introduced by taking into account the influence of the laser structure and the associated distribution of the mode intensity along the cavity length. The results obtained for Fabry–Perot and distributed feedback lasers show that the effective nonlinear gain could be considerably enhanced. Affected by the laser structure, the nonlinear gain has a different power dependence than expected from material considerations alone. By including this effective nonlinear gain, the frequency and intensity modulation properties of multielectrode semiconductor lasers are studied. A general linewidth expression is given which includes contributions from spontaneous emission and carrier shot noise. It is found that the effective α-factor affecting the linewidth is in general different from its counterpart affecting modulation and injection locking properties due to spatial hole burning and nonlinear gain. For lasers with uniform intensity distribution, the effective α-factor affecting the linewidth increases or remains constant with increasing output power depending on the model used for the nonlinear gain. For $\lambda/4$ phase-shifted distributed feedback (DFB) lasers, the effective α-factor affecting the linewidth is slightly larger or smaller than that for uniform lasers depending on the value of the normalized grating coupling coefficient. The linewidth due to various contributions is calculated for both uniform intensity distributed lasers and phase-shifted DFB lasers.**

I. Introduction

Multielectrode semiconductor lasers have attracted considerable attention in recent years [1], [2]. This is largely motivated by the worldwide development of advanced communication systems, in which optical sources with narrow spectral linewidth, wavelength tunability, and flat frequency modulation response are required. Although the theory of such lasers is well developed, some phenomena limiting the laser's ultimate performance have not yet been completely understood. For example, linewidth rebroadening occurring at high output powers [1], [2] is contradictory to the standard laser theory, which predicts a monotone linewidth decrease with increasing output power [3]. Various factors, such as side mode influence [4], $1/f$ noise at high output powers [5]–[7], etc., have been involved to explain this phenomenon. For highly single mode semiconductor lasers driven by a noiseless current source, much attention has been paid to longitudinal inhomogeneities and the nonlinear gain.

Longitudinal inhomogeneities include two principal aspects: the structural inhomogeneity and the functional inhomogeneity. The structural inhomogeneity results from the multielectrode nature of the laser structure, in which an optical grating can be introduced in one or more sections and the injection current in each section is separately controlled. The functional inhomogeneity originates from the phenomenon of spatial hole burning [8], [9], occurring even in a homogeneous cavity structure. Its existence is due to the fact that the intensity distribution is not uniform in most laser structures, especially in phase-shifted distributed feedback (DFB) lasers. This nonuniformity introduces a nonuniform carrier density distribution, which, in turn, affects again the field intensity distribution, and so on.

Longitudinal inhomogeneities can be easily included by using the Green's function method initially proposed by Henry [10]. In this method, the solution of wave equation corresponding to a point source (spontaneous emission event in the laser case) is called the Green's function. For distributed excitation sources, the general solution is obtained through a spatial integration of the Green's function weighted by the correspondent excitation source. Using this method, the spontaneous emission rate coupled to the lasing mode is related to the laser structure [10]. This method has been generalized to include complex phase-amplitude coupling effects in DFB and composite-cavity laser structures [11]. More recently, Tromborg *et al.* have included spatial hole burning by considering the Wron-

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skian, appearing in the Green's function formulation, as a functional of carrier density distribution in multielectrode DFB and distributed Bragg reflector (DBR) lasers [12]. This treatment enables them to take into account any nonuniform carrier density distribution due to the presence of multielectrodes and/or due to spatial hole burning. However, the results show that the spatial hole burning alone can not explain linewidth rebroadening.

Another phenomenon that should be considered is the nonlinear gain, which should include the material nonlinear gain (spectral-hole burning, carrier heating, *etc.*) [13]–[16] and the nonlinear gain introduced by nonuniform lateral carrier density distribution [17]. The material nonlinear gain due to the spectral-hole burning has been studied by using a density matrix formalism [13], [14]. By studying its influence on Fabry–Perot type lasers, it was concluded that the material nonlinear gain can lead to linewidth rebroadening [15]. Recent results obtained by Olesen *et al.* give a contrary conclusion for the same type of laser structure [18]. An effort was made by Tromborg *et al.* to include the material nonlinear gain for lasers with a nonuniform intensity distribution using the Green's function method [12]. In their analysis, the Green's function method is directly applied even for a nonlinear dielectric constant. Conceptually, this is not permitted as the method is only valid for a linear dielectric constant. Consequently, the Wronskian, resulting from solutions of wave equation for a linear dielectric constant, can never be an explicit function of photon density distribution.

The purpose of this paper is to give an analysis of lasers by including both spatial hole burning and the nonlinear gain. Different from previous approachs, the material nonlinear gain is taken into account through a perturbation method. This enables us to treat the nonlinear gain within the validity of the Green's function method. Using this perturbation method, an effective nonlinear gain is introduced, which depends also on the mode distribution along the cavity length. The spatial hole burning is included through a power dependent carrier density distribution.

This paper is organized as follows. In Section II, the Green's function method is generalized to multielectrode lasers. In Section III, the effective nonlinear gain is discussed. In Section IV, a small-signal analysis is performed, leading to expressions of modulation transfer functions and spectral linewidth. In Section V, results for lasers with uniform intensity distribution are given. Results for a phase-shifted DFB laser is presented in Section VI. Finally a conclusion is given at the end of the paper.

II. Generalized Rate Equation

The starting point of our analysis is the propagation equation in the frequency domain. We shall concentrate our attention on the longitudinal axis alone, although the same principle of analysis could be used for transverse tuning devices such as tunable-twin-guide DFB lasers [19]. The electrical field in the laser cavity is thus governed by the inhomogeneous scalar Helmholtz equation [10]:

$$\nabla_z^2 E_\omega(z) + k_0^2 \epsilon_\omega(N, S) E_\omega(z) = F_\omega(z) \quad (1)$$

where ∇_z^2 is the Laplacian operator for the longitudinal coordinate z, $k_0 = \omega/c$ is the wavenumber in vacuum, $\epsilon_\omega(N, S)$ is the dielectrical constant depending on the frequency ω, carrier density is N, and photon density is S. Finally, $F_\omega(z)$ is the Langevin force describing the spontaneous emission. Equation (1) is valid for either Fabry–Perot or DFB, DBR, lasers.

The complex dielectrical constant is written as

$$\epsilon_\omega(N, S) = [n + j(g - \alpha_L)/(2k_0)]^2 \quad (2)$$

where n is the refractive index, g is the gain, and α_L the internal loss. The dielectrical constant can be split into linear part ϵ_L and nonlinear part ϵ_{NL}:

$$\epsilon_\omega(N, S) = \epsilon_L(\omega, N) + \epsilon_{\rm NL}(\omega, N, S). \quad (3)$$

The nonlinear part is intensity dependent. The inclusion of the nonlinear dielectric constant makes the theoretical analysis somewhat complicated, as the Green's function method is only valid for the linear case [20]. To overcome this difficulty, the nonlinear part of the dielectric constant is moved to the right-hand side so that (1) takes the form:

$$\nabla_z^2 E_\omega + k_0^2 \epsilon_L E_\omega = F_\omega - k_0^2 \epsilon_{\rm NL} E_\omega. \quad (4)$$

It should be stressed that the nonlinear part is being treated nonperturbatively here. The general solution of the scalar equation is obtained by using the Green's function formalism and is given by [20]:

$$E_\omega(z) = -\int_{(L)} G_\omega(z, z') k_0^2 \epsilon_{NL} E_\omega \, dz' + \int_{(L)} G_\omega(z, z') F_\omega(z') \, dz' \quad (5)$$

where the integration is performed over the total cavity length. $G_\omega(z, z')$ is the Green's function given by [20]:

$$G_\omega(z, z') = \frac{Z_+(z>) Z_-(z<)}{W(\omega, N(z))} \quad (6)$$

where $z> = \max(z, z')$ and $z< = \min(z, z')$, $Z_+(z)$, $Z_-(z)$ are two independent solutions of the homogeneous equation, satisfying the boundary conditions for the left or right facet and $W(\omega, N(z))$ is the Wronskian of these solutions. The Wronskian is a functional of the frequency ω and the carrier density distribution $N(z)$. It is not explicitly dependent on the coordinate z. Our approach is different from that of Tromborg *et al.*, in which the Wronskian is considered also as a functional of photon density distribution [12]. This is not permitted within the validity of the Green's function method. In our approach, the photon density distribution does not appear explicitly in the Wronskian, although it affects the carrier density distribution through the spatial hole burning effect [8], [9].

For a laser system, it is important to consider the laser oscillation condition. In the linear case, this condition is expressed by setting the Wronskian of functions $Z_+(z)$ and $Z_-(z)$ equal to zero. These two functions $Z_+(z)$ and $Z_-(z)$ are then identical to one solution $Z_0(z)$, satisfying the boundary conditions at both facets and representing the longitudinal distribution of the electrical field in the laser cavity. In the present case where nonlinear dielectric constant is considered, the above conclusions are only a first order approximation. Within the validity of this approximation, we obtain from (5) and (6):

$$\frac{WE_\omega(z)}{Z_0(z)} = -k_0^2 \int_{(L)} Z_0(z')\epsilon_{NL} E_\omega \, dz' + \int_{(L)} Z_0(z') F_\omega(z') \, dz'. \quad (7)$$

As the RHS of the above equation is independent of the coordinate z, the only solution of the electrical field in the frequency domain $E_\omega(z)$ is of the type:

$$E_\omega(z) = \beta_\omega Z_0(z). \quad (8)$$

For small perturbations due to the nonlinear gain and the driving noise sources (spontaneous emission, carrier shot noise, etc.), the Wronskian is expanded around the linear operating point:

$$W(\omega, N(z)) = \frac{\partial W}{\partial \omega}(\omega - \omega_0) + \int_{(L)} \frac{\partial W}{\partial N} \Delta N(z) \, dz \quad (9)$$

where ω_0 is the emission frequency at this point and ΔN is the deviation of the carrier density distribution from this point. By using the properties of Fourier transform the following rate equation for $\beta_0(t)$ is obtained [10], [11]:

$$\frac{d\beta_0}{dt} = \left\{ j(\overline{\omega} - \omega_0) - j \int_{(L)} W_N \Delta N \, dz + \frac{1}{2} G_{NL}(N, S) \right\} \cdot \beta_0(t) + F_{\beta 0}(t) \quad (10)$$

where $W_N = (\partial W/\partial N)/(\partial W/\partial \omega)$, $\overline{\omega}$ is the actual lasing frequency, different from ω_0 due to the nonlinear gain, $\beta_0(t)$ is the slowly varying complex amplitude given by:

$$\beta_0(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \beta_\omega \exp j(\omega - \overline{\omega})t \, d\omega. \quad (11)$$

$F_{\beta 0}$ is the Langevin force associated with the complex amplitude in the time domain. The spontaneous emission rate R_{sp} is given by [10]:

$$R_{sp} = \frac{4\omega_0^2}{c^3} \frac{\int_{(L)} Z^* g_0 n_0 n_{sp} Z \, dz \int_{(L)} Z^* n_0 n_{g0} Z \, dz}{|\partial W/\partial \omega|^2}. \quad (12)$$

The integration in (12) is restricted to laser sections with gain only since passive sections with different bandgap do not contribute directly to the spontaneous emission rate.

The term G_{NL} is the effective nonlinear gain given by:

$$G_{NL}(N, S) = -\frac{j 2k_0^2}{\partial W/\partial \omega} \int_{(L)} Z_0^2(z) \epsilon_{NL}(N, S) \, dz. \quad (13)$$

This newly defined nonlinear gain takes into account both the material and the structural dependences and will be discussed in detail in Section III.

Some assumptions have been made in deriving the above rate equation. First, the nonlinear gain term is, strictly speaking, a convolution between the effective nonlinear gain and the field complex amplitude. This is replaced by a simple multiplication. This assumption is valid as long as the field complex amplitude variations are much slower than the optical frequency. Second, the influences of nonlinear gain on field distribution have been neglected. Our theory can be considered as a first-order perturbation theory. A more complete theory would take into account these higher-order effects.

III. Effective Nonlinear Gain

For Fabry–Perot, DFB and DBR lasers with a refractive index and gain varying smoothly in the cavity, it can be shown that [12]:

$$\frac{\partial W}{\partial N} = k_0 n g_d(-\alpha_H + j) Z_0^2(z) \quad (14)$$

$$\frac{\partial W}{\partial \omega} = \frac{2k_0}{c} \int_{(L)} n n_g Z_0^2(z) \, dz \quad (15)$$

where g_d is the differential gain and α_H is the linear material linewidth enhancement factor [3]. Inserting (15) into (13) together with $\epsilon_{NL} = jng_{NL}/k_0$, the effective nonlinear gain can be finally written as [21]:

$$G_{NL}(N, S) = v_g \frac{\int_{(L)} Z_0^2(z) g_{NL}(N, S) \, dz}{\int_{(L)} Z_0^2(z) \, dz} \quad (16)$$

where g_{NL} is assumed to be complex. Its real part represents the nonlinear gain and the imaginary part the nonlinear refractive index. Thus the effective nonlinear gain is a spatial average of the material local nonlinear gain weighted by the squared field distribution rather than by the intensity. This newly defined nonlinear gain takes into account both material and structural dependences. It simplifies to the material gain for uniform intensity distributions. The effective nonlinear gain has the same origin as the longitudinal spontaneous emission enhancement factor K_z [10], [11]: the different longitudinal mode distributions $\{Z_m(z), m = 0, 1, \cdots\}$, forming a complete set, are not orthogonal in the Hermitian product sense, due to the presence of optical gain in the cavity [22].

In fact, when the mode distribution forming a complete orthogonal set is used, as is assumed in the classical laser theory, the effective nonlinear gain is given by [23]:

$$G_{NL}(N, S) = v_g \frac{\int_{(L)} |Z_0(z)|^2 g_{NL}(N, S) \, dz}{\int_{(L)} |Z_0(z)|^2 \, dz}. \quad (17)$$

Several consequences of the effective nonlinear gain can be predicted by comparing (16) and (17):

i) Different laser structures can give rise to different values and forms of the effective nonlinear gain for the same material. This is due to the dependence of the field distribution $Z_0(z)$ on the cavity structure.

ii) As the field distribution generally includes a spatially dependent phase, the material nonlinear gain can result in an effective nonlinear index and *vice versa*.

iii) As the intensity distribution is not uniform and changes with the output power due to spatial hole burning, the effective nonlinear gain will have, in general, a different power dependence than the material nonlinear gain.

In order to understand the implications of (16), we have to consider a specific functional form of the nonlinear gain g_{NL}. However, the functional form depends on the mechanism responsible for the nonlinear gain (spectral-hole burning, carrier heating, etc.). In many cases of practical interest, one can assume that g_{NL} decreases linearly with the photon density ($S(z)$) as:

$$g_{NL} = -g_L S(z)/P_s \tag{18}$$

where P_s is referred to as the saturation photon density and g_L as the linear gain. This expression is obviously valid at low powers such that $S(z) << P_s$. P_s is a material parameter that depends on details of carrier relaxation within the conduction band. An expression of P_s can be found in [15] for the case in which spectral-hole burning is the origin of nonlinear gain. Its typical value is in the range of 3–6 $\times$ 10^{16} cm^{-3}. Equation (18) assumes that g_{NL} is real such that the refractive index is power independent. In the case of lasers operating away from the gain peak, the nonlinear gain is accompanied by index changes [15] that can be included through a complex g_{NL}. In this paper, such changes are neglected by treating g_{NL} as purely real.

By relating the photon density to the mode distribution through $S(z) = P_0 |Z_0(z)|^2$, with P_0 a spatially independent parameter proportional to the output power, the G_{NL} can be written as:

$$G_{NL} = -\frac{P_0}{P_s} v_g g_L \frac{\int_{(L)} Z_0^2(z) |Z_0(z)|^2 \, dz}{\int_{(L)} Z_0^2(z) \, dz}. \tag{19}$$

It is assumed for the moment that spatial variation of the linear gain due to spatial hole burning in the laser cavity can be neglected. By comparing this effective nonlinear gain with the nonlinear material gain G_{NL}^{MAT} corresponding to the average photon density in the cavity:

$$G_{NL}^{MAT} = -\frac{P_0}{P_s} v_g g_L \int_{(L)} |Z_0(z)|^2 \, dz/L \tag{20}$$

a correction factor for the gain can be introduced, which relates the effective nonlinear gain to the material nonlinear gain. By using (19) and (20), the correction factor C is given by:

$$C = \frac{G_{NL}}{G_{NL}^{MAT}} = \frac{\int_{(L)} Z_0^2(z) |Z_0(z)|^2 \, dz}{\left(\int_{(L)} Z_0^2(z) \, dz\right) \left(\int_{(L)} |Z_0(z)|^2 \, dz/L\right)}. \tag{21}$$

The correction factor C given by (21) shows how the laser structure can affect the value of the material parameter P_s. Note that C is generally complex. The real part represents the change in the material nonlinear gain, the imaginary part represents the contribution of the material nonlinear gain to the effective refractive index. The latter directly affects phase variation of the optical field in the laser cavity and thus contributes to the frequency chirp and the spectral linewidth.

We have calculated the correction factor for different types of laser structures. Fig. 1 shows the result for a Fabry–Perot laser with one fixed facet reflectivity of 30% ($R_2 = 0.3$) and a varying reflectivity R_1 of the other. The real part of C is slightly larger than unity for small values of reflectivity ($R_1 < 10^{-4}$). Beyond this value, the correction factor keeps at unity. The imaginary part of C decreases with increasing facet reflectivity R_1. The value of the imaginary part is quite small in comparison to that of the real part.

The correction factor C is plotted as a function of the normalized coupling coefficient in Fig. 2 for a conventional DFB laser. Both facets of the laser are assumed to be AR-coated ($R_1 = R_2 = 0$). The real part of the correction factor increases from 0.87 for $\kappa L = 1.0$ to 1.3 for $\kappa L = 5.0$. The imaginary part of the correction factor changes from negative values to positive values with increasing normalized coupling coefficient. It is not surprising that the effective nonlinear gain in Fabry–Perot and conventional DFB lasers is not very different from the material nonlinear gain (corresponding to the average photon density), as the intensity distribution in these lasers is rather uniform.

The result for an AR-coated $\lambda/4$ phase-shifted DFB laser is shown in Fig. 3. The real part of the correction factor becomes larger than unity for $\kappa L > 1.25$ and reaches 2 for $\kappa L = 4.0$. The imaginary part of the correction factor changes sign at $\kappa L = 1.25$ and becomes negligible for larger values of κL. This is due to the fact that for $\kappa L = 1.25$, the field intensity distribution is nearly uniform inside the cavity [8]. For κL larger than 1.25, the field intensity is more concentrated at the center of the cavity. Otherwise, the field intensity is concentrated near the two facets [8].

To evaluate its output power dependence, the effective nonlinear gain is calculated by using (16) for an AR-coated $\lambda/4$ phase-shifted DFB laser with $\kappa L = 3.0$. In our calculations, the field distribution and the linear gain

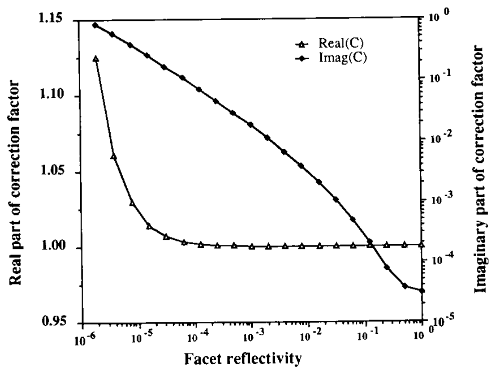

Fig. 1. The real and imaginary parts of the correction factor C as a function of facet reflectivity R_1 for a Fabry–Perot laser. The reflectivity of the other facet R_2 is assumed to be 30%.

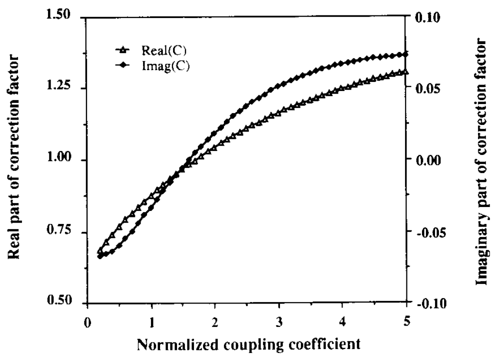

Fig. 2. The real and imaginary parts of the correction factor C as a function of the normalized coupling coefficient κL for a conventional DFB laser with AR-coated facets.

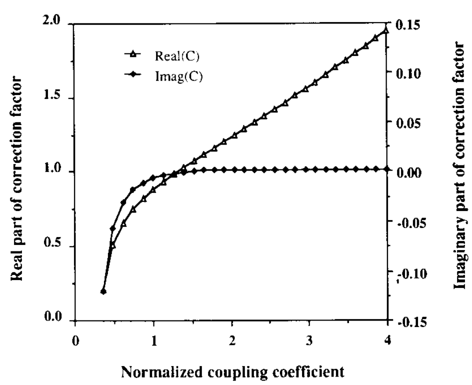

Fig. 3. The real and imaginary parts of the correction factor C as a function of the normalized coupling coefficient κL for a $\lambda/4$ phase-shifted DFB laser with AR-coated facets.

are calculated by including the spatial hole burning. The material nonlinear gain is assumed to have the form [15]:

$$g_{\mathrm{NL}} = g_{\mathrm{L}}/\sqrt{1 + 2S(z)/P_s} - g_{\mathrm{L}}. \tag{22}$$

Compared with the expression given in [15], a factor 2 is added in (22) to give the same small signal approximation result as (18). The real and imaginary parts of the effective nonlinear gain divided by $v_g(G_{\mathrm{NL}}/v_g)$ are plotted as a function of the output power in Fig. 4(a) and (b), respectively. The material nonlinear gain corresponding to the average photon density is also given in Fig. 4(a) for comparison. It can be seen that at low output powers (< 1 mW), the effective nonlinear gain and the material nonlinear gain exhibit nearly the same power dependence. The ratio of their values is close to 1.5, the value of the correction factor C given in Fig. 3 for $\kappa L = 3.0$. When the output power increases, the intensity distribution becomes more uniform due to spatial hole burning [8], [9]. Consequently, the real part of the effective nonlinear gain approaches the material nonlinear gain at an output power of about 10 mW and continues to decrease for higher output powers. At the same time, the imaginary part of the nonlinear gain tends to change sign.

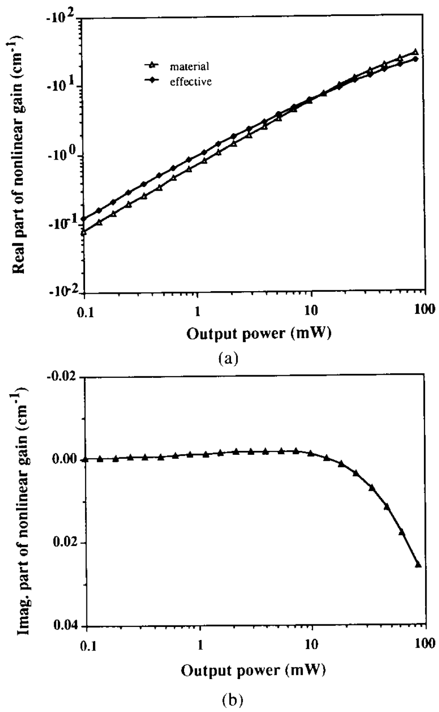

Fig. 4. (a) The real part and (b) the imaginary part of the effective nonlinear gain G_{eff}/v_g as a function of output power for a phase-shifted DFB laser with $\kappa L = 3.0$.

IV. Small-Signal Analysis

To study the dynamic and noise properties, one has to use the rate equation for the carrier density in the laser cavity:

$$\frac{dN(z, t)}{dt} = \boldsymbol{J}(z, t) - R(N(z, t)) - v_g g(z, t)S(z, t) + F_N(z, t) \tag{23}$$

where $\boldsymbol{J}(z, t) = J(z, t)/(ed)$, $J(z, t)$ is the current density, e is the electron charge, d is the thickness of the active layer, $S(z, t)$ is the photon density, $R(N(z, t))$ is the non-stimulated carrier recombination rate, and $F_N(z, t)$ is the Langevin force representing the carrier shot noise. The photon density is related to the field intensity distribution by:

$$S(z, t) = C_0 |Z_0(z)|^2 P(t), \quad C_0 = \left(\int_{(L)} |Z_0(z)|^2 \, dz/L \right)^{-1} \tag{24}$$

where C_0 is a normalization constant, and $P(t)$ is the average photon density in the cavity. The assumption that the spatial and temporal dependences of the photon density can be separated is a direct consequence of the Green's function method [see (8)]. This is a good approximation for modulation frequencies much lower than the mode separation frequency of the cavity.

The above analysis defines the basis of our formalism. In the following, it will be used to discuss the dynamic and noise properties in multisection lasers. It is more practical to convert the field complex amplitude rate equation into photon density P and phase ϕ rate equations by using $\beta_0(t) = \sqrt{P(t)} \exp(j\phi(t))$:

$$\frac{dP}{dt} = 2 \int_{(L)} W_{Ni} \Delta N(z) \, dz \, P(t) + G_{NLr}(N, S) P(t) + F_P(t) \tag{25a}$$

$$\frac{d\phi}{dt} = \omega_0 - \overline{\omega} - \int_{(L)} W_{Nr} \Delta N(z) \, dz + \frac{1}{2} G_{NLi}(N, S) + F_\phi(t). \tag{25b}$$

The subscripts r and i represent the real and the imaginary parts, respectively. The stationary solution is obtained by setting $d/dt = 0$ and eliminating the noise terms. The obtained $\overline{\omega}$, $\overline{P}$, $\overline{N}(z)$, $Z_0(z)$, etc. are power dependent stationary values and distributions because of the effects of spatial hole burning and nonlinear gain. Small deviations from the stationary point are obtained by linearizing the rate equations. If $\Delta P(\Omega)$, $\Delta N(z, \Omega)$, $\Delta\omega(\Omega)$ represent the Fourier transform of deviations, the small signal solution is given by:

$$\Delta N(z, \Omega) = \frac{\Delta \boldsymbol{J}(z, \Omega) + F_N - A(z)|Z_0(z)|^2 \Delta P(\Omega)}{j\Omega + 1/\tau_R} \tag{26a}$$

$$\Delta P(\Omega) = \frac{F_P}{j\Omega - G_{NLPr}\overline{P} + 2\overline{P} \int_{(L)} U_2 A(z) |Z_0(z)|^2 \, dz} + \int_{(L)} H_P(z, \Omega)(\Delta \boldsymbol{J} + F_N) \, dz \tag{26b}$$

$$\Delta\omega(\Omega) = \frac{\int_{(L)} U_1 A(z) |Z_0(z)|^2 \, dz + G_{NLPi}/2}{j\Omega - G_{NLPi}\overline{P} + 2\overline{P} \int_{(L)} U_2 A(z) |Z_0(z)|^2 \, dz} \cdot F_P + F_\phi + \int_{(L)} H_f(z, \Omega)(\Delta \boldsymbol{J} + F_N) \, dz \tag{26c}$$

where $A(z)$, τ_R, U_1, U_2 are given by:

$$A(z) = C_0 v_g \left(\frac{\partial g}{\partial P} \overline{P} + \overline{g} \right) \tag{27a}$$

$$\frac{1}{\tau_R} = \left. \frac{\partial R(N(z))}{\partial N(z)} \right|_{N(z) = \overline{N}(z)} + v_g \frac{\partial g}{\partial N} \overline{S}(z) \tag{27b}$$

$$U_1 = \frac{W_{Nr} - 1/2 \, \partial G_{NLi}/\partial N}{j\Omega + 1/\tau_R}$$

$$U_2 = \frac{W_{Ni} + 1/2 \, \partial G_{NLr}/\partial N}{j\Omega + 1/\tau_R} \tag{27c}$$

$$\frac{\partial G_{NL}}{\partial N} = \frac{v_g \partial g_{NL}(N, S)/\partial N Z_0^2(z)}{\int_{(L)} Z_0^2(z) \, dz}. \tag{27d}$$

The local modulation transfer functions denoted by $H_p(z, \Omega)$ and $H_f(z, \Omega)$ are given by:

$$H_P(z, \Omega) = \frac{2U_2}{j\Omega/\overline{P} + 2 \int_{(L)} U_2 A(z) |Z_0(z)|^2 \, dz - G_{NLPr}} \tag{28a}$$

$$H_f(z, \Omega) = \frac{-U_1(j\Omega/\overline{P} - G_{NLPr}) + G_{NLPi} U_2}{j\Omega/\overline{P} + 2 \int_{(L)} U_2 A(z) |Z_0(z)|^2 \, dz - G_{NLPr}} \tag{28b}$$

$$G_{NLP} = \frac{\partial G_{NL}}{\partial P} = C_0 v_g \frac{\int_{(L)} Z_0^2(z) |Z_0(z)|^2 \partial g_{NL}(N, S)/\partial S \, dz}{\int_{(L)} Z_0^2(z) \, dz}. \tag{28c}$$

In obtaining the above solutions, it has been assumed that the field distribution $Z_0(z)$ remains unchanged even in the presence of perturbation due to the noise and/or modulation.

In Section II (Fourier transform from ω to t), it has been assumed that the carrier density distribution $\Delta N(z)$ is a constant in the time domain. This is a reasonable approximation, as the temporal variation of $\Delta N(z)$ is much slower than the optical frequency ω_0 ($\sim 10^{15}$ s^{-1}). By contrast, the same $\Delta N(z)$ is considered time dependent in this Section (Fourier transform from t to Ω). This is not inconsistent with the first part as long as $\Omega << \omega_0$ (typically $\Omega < 10^{11}$ s^{-1}).

A. Dynamic Response

The dynamic responses can be obtained from (26)–(28) by eliminating all noise terms. For the case of uniformly injected single electrode laser, we have:

$$\Delta P(\Omega) = \int_{(L)} H_P(z, \Omega)\, dz \frac{\Delta I(\Omega)}{eV} \qquad (29a)$$

$$\Delta \omega(\Omega) = \int_{(L)} H_f(z, \Omega)\, dz \frac{\Delta I(\Omega)}{eV} \qquad (29b)$$

In obtaining (29), we have used $\Delta J = \Delta I/(eV)$, with V the volume of active layer. A convenient, parasite-free measure of FM is the chirp-to-modulated power ratio (CPR) [17]. From (28) and (29), the CPR is found to be:

$$\text{CPR} = \frac{\Delta\omega}{2\pi \Delta P} = \frac{\alpha_{\text{eff}}^{\text{mod}}}{4\pi \overline{P}} (j\Omega - G_{\text{NL}Pr}\overline{P} + G_{\text{NL}Pi}\overline{P}/\alpha_{\text{eff}}^{\text{mod}}) \qquad (30a)$$

where an effective α-factor affecting the modulation properties is defined:

$$\alpha_{\text{eff}}^{\text{mod}} = -\frac{\int_{(L)} U_1(z, \Omega)\, dz}{\int_{(L)} U_2(z, \Omega)\, dz}. \qquad (30b)$$

The CPR expression (30a) has the same form as the classical one [17], but with the effective parameters. It is interesting to note that the newly defined α-factor (30b) is modulation-frequency dependent for lasers with nonuniform intensity distributions.

In the case of a two section laser, the excitation source is the modulation current density $\Delta J(z) = \Delta I_1/s_1$, for values of z lying within Section I and $\Delta J(z) = \Delta I_2/s_2$, for z within Section II, where s_1 and s_2 are the cross section of areas. The carrier density, the photon density and the frequency deviation can be obtained from (26):

$$\Delta P(\Omega) = A_{P1}\Delta I_1 + A_{P2}\Delta I_2 \qquad (31a)$$

$$\Delta \omega(\Omega) = A_{\phi 1}\Delta I_1 + A_{\phi 2}\Delta I_2 \qquad (31b)$$

where the transfer functions A_{P1}, A_{P2}, $\cdots$ can be obtained directly from (26)–(28). Using (31a) and (31b), conditions for pure frequency and pure amplitude modulation are given by:

$$\frac{\Delta I_1}{\Delta I_2} = -\frac{A_{P2}}{A_{P1}} \quad \text{for pure FM} \qquad (32a)$$

$$\frac{\Delta I_1}{\Delta I_2} = -\frac{A_{\phi 2}}{A_{\phi 1}} \quad \text{for pure AM.} \qquad (32b)$$

The transfer functions A_{P1}, A_{P2}, $\cdots$ usually depend on modulation frequency Ω. Pure frequency modulation or pure intensity modulation have been obtained using this method for multielectrode distributed-Bragg-reflector (DBR) and DFB lasers [24], [25].

B. Phase Noise and Spectral Linewidth

The Langevin forces representing the spontaneous emission $F_p(\Omega)$, $F_\phi(\Omega)$ are delta-correlated [10]:

$$\langle F_X(\Omega) F_{X'}^*(\Omega')\rangle = 2D_{XX'}\delta(\Omega - \Omega'), \qquad X, X' = P, \phi. \qquad (33a)$$

The nonzero diffusion coefficients are given by [10]:

$$2D_{\phi\phi} = R_{sp}/(2I); \qquad 2D_{pp} = 2\overline{P}R_{sp}/V \qquad (33b)$$

where $I\ (= \overline{P}V)$ is the total photon number in the laser cavity. The carrier shot noise is assumed to be delta-correlated in time and in space. The correlation relations are found to be [12]:

$$\langle F_N(z, \Omega) F_N^*(z, \Omega')\rangle = 2D_{NN}(z, \Omega)\delta(z - z')\delta(\Omega - \Omega') \qquad (34a)$$

$$2D_{NN}(z, \Omega) = 2[v_g g(z) n_{sp} \overline{S}(z) + R(N(z))]/V \qquad (34b)$$

$$\langle F_P(\Omega) F_N(z, \Omega')\rangle = 2D_{PN}(z, \Omega)\delta(\Omega - \Omega') \qquad (35a)$$

$$2D_{PN}(z, \Omega) = -2v_g g(z) n_{sp}\overline{S}(z)/V \qquad (35b)$$

The frequency variation driven by noise at zero frequency ($\Omega = 0$) is obtained from (26c):

$$\Delta\omega = F_\phi - \alpha_{\text{eff}}^{\text{noise}} F_P/(2\overline{P}) + \int_{(L)} H_f(z, 0) F_N(z, 0)\, dz. \qquad (36)$$

The effective phase-amplitude coupling factor for the linewidth is obtained from (26c) and written as:

$$\alpha_{\text{eff}}^{\text{noise}} = -\frac{\int_{(L)} \left[W_{Nr} - \frac{1}{2}\frac{\partial G_{\text{NL}i}}{\partial N}\right] \tau_R A(z)|Z_0|^2\, dz + \frac{1}{2} G_{\text{NL}Pi}}{\int_{(L)} \left[W_{Ni} + \frac{1}{2}\frac{\partial G_{\text{NL}r}}{\partial N}\right] \tau_R A(z)|Z_0|^2\, dz + \frac{1}{2} G_{\text{NL}Pr}}. \qquad (37)$$

Using (14)–(16) and neglecting the nonlinear refractive index, the effective α-factor for the linewidth can also be written as:

$$\alpha_{\text{eff}}^{\text{noise}} = -\text{Real}(\chi)/\text{Imag}(\chi) \qquad (38a)$$

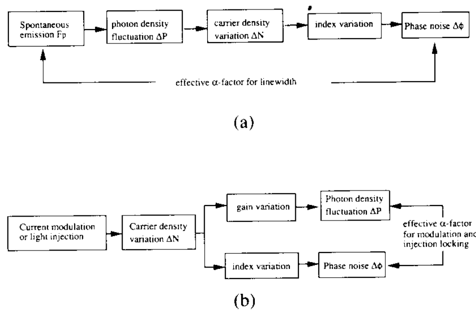

Fig. 5. The interaction scheme in a semiconductor laser due to (a) photon density fluctuation caused by spontaneous emission, (b) current modulation or light injection.

$$\chi = \frac{\int_{(L)} [(-\alpha_H + jg/g_L)g_d\tau_R A(z) - jC_0 \partial g/\partial S] \,|Z_0|^2 Z_0^2(z)\, dz}{\int_{(L)} Z_0^2(z)\, dz}. \tag{38b}$$

By comparing (30b) and (37), we found that the effective α-factor for the linewidth is different from its counterpart for modulation at zero-modulation frequency. This is explained in Fig. 5. In the modulation case, current modulation creates carrier density variations, which give rise to a simultaneous change of photon density and phase through the change of gain and the index. The $\alpha_{\text{eff}}^{\text{mod}}$ represents the ratio between variations of photon density and phase. In the linewidth case, the Langevin force F_p due to spontaneous emission gives rise to a change of the photon density, which leads to a carrier density variation. The latter introduces an additional phase noise through the change of the refractive index. The α-factor for the linewidth relates this additional phase change to the Langevin force F_p. Thus these two effective α-factors have different definitions. More importantly, current modulation and spontaneous emission affect laser dynamics differently. For instance, an increase in electron population due to current modulation leads to an increase in the output power. In contrast, an increase in the output power due to the spontaneous emission results in a decrease of electron population to conserve the laser-oscillation condition.

The linewidth is given by the value of the power spectrum density of frequency noise at zero frequency divided by 2π. By using (33)–(36), the final expression is given by:

$$\Delta\nu = \frac{R_{\text{sp}}}{4\pi I}(1 + (\alpha_{\text{eff}}^{\text{noise}})^2) + \frac{1}{2\pi} \cdot \int_{(L)} |H_f(z, 0)|^2 2D_{\text{NN}}(z, 0)\, dz - \frac{\alpha_{\text{eff}}^{\text{noise}}}{2\pi\bar{P}} \int_{(L)} 2\text{Real}(H_f(z, 0)) 2D_{\text{PN}}(z, 0)\, dz. \tag{39}$$

The first term represents the contribution of phase fluctuations due to spontaneous emission and that of photon density fluctuations via the well-known phase-amplitude coupling. The second term is the contribution due to the carrier shot noise. The last term represents the contribution of cross correlation between the photon density and carrier density. The carrier shot noise induced linewidth has also been pointed out by using other methods [26], [27].

The above discussion completes our formalism on laser dynamics and noise. To get some insight into the influence of nonlinear gain, the theory is applied at first to lasers with uniform intensity distribution. Results for phase-shifted DFB lasers are given thereafter.

V. Application to Lasers with Uniform Intensity Distribution

For Fabry–Perot lasers with high facet reflectivities ($R_1 = R_2 = 0.3$) and specially designed DFB lasers [28], the field intensity distribution is nearly uniform inside the cavity. In this case, the spatial hole burning effect can be neglected. The effective nonlinear gain becomes simply the material nonlinear gain. By using $W_N = v_g g_d(-\alpha_H + j)/2$ [11], the rate equations in this case are written as:

$$\frac{dP}{dt} = v_g g_d \Delta N P(t) + G_{\text{NL}r}(N, P)P(t) + F_P(t) \tag{40a}$$

$$\frac{d\phi}{dt} = \bar{\omega} - \omega_0 + \frac{1}{2}\alpha_H v_g g_d \Delta N(z) + \frac{1}{2} G_{\text{NL}i}(N, P) + F_\phi(t). \tag{40b}$$

The modulation and noise properties have been widely studied using similar or the same equations previously. We are essentially interested in the linewidth and the phase-amplitude coupling factor.

A. Linewidth Calculation

Using the same method as the previous section, the linewidth is written as:

$$\Delta\nu = \frac{R_{sp}}{4\pi I}(1 + (\alpha_{\text{eff}}^{\text{noise}})^2) + \frac{1}{2\pi}|H_f(0)|^2 2D_{\text{NN}} - \frac{\alpha_{\text{eff}}^{\text{noise}}}{2\pi\bar{P}} 2\text{Real}(H_f(0))2D_{\text{PN}} \tag{41}$$

The effective phase-amplitude coupling factor is given by:

$$\alpha_{\text{eff}}^{\text{noise}} = \frac{\alpha_H + \dfrac{1}{gd}\dfrac{\partial g_{\text{NL}i}}{\partial N} - \dfrac{1}{g_d v_g \tau_R}\dfrac{\partial g_{\text{NL}i}/\partial P}{g + \bar{P}\partial g/\partial P}}{1 + \dfrac{1}{gd}\dfrac{\partial g_{\text{NL}r}}{\partial N} - \dfrac{1}{g_d v_g \tau_R}\dfrac{\partial g_{\text{NL}r}/\partial P}{g + \bar{P}\partial g/\partial P}} \tag{42}$$

where $g_{\text{NL}} = G_{\text{NL}}/v_g$ and:

$$\frac{1}{\tau_R} = \frac{1}{\tau_e} + v_g\left(g_d + \frac{\partial g_{\text{NL}}}{\partial N}\right)\bar{P} \tag{43}$$

where τ_e is the carrier lifetime corresponding to the non-stimulated carrier recombination. It is assumed that the index nonlinearities are neglected in what follows. Using the nonlinear gain expression in the low output regime (18) and (42), (43), the effective α-factor for the linewidth for $P \to 0$ is:

$$\alpha_{\text{eff}\,0}^{\text{noise}} = \frac{\alpha_H}{1 + r}; \qquad r = P_c/P_s, \qquad P_c = \frac{1}{v_g g_d \tau_e} \tag{44}$$

where P_c denotes the interband saturation photon density. Using the values given in Table I, P_c is estimated to be 3×10^{14} cm^{-3}. The typical intraband saturation photon density is $P_s \approx 3\text{–}6 \times 10^{16}$ cm^{-3}. Thus the ratio P_c/P_s is typically of the order 0.01, leading to a correction of the α-factor of the order of 1%.

This result shows that the effective α-factor for linewidth differs from the linear material parameter α_H even for zero output power. Mathematically, this is due to the fact that the first order derivative $\partial g_{\text{NL}}/\partial P$ is not zero. This result seems surprising since the nonlinear material gain becomes negligible in the low output regime. However, the effective α-factor depends not only on the magnitude of the nonlinear gain, but also on its first order derivative. The latter does not vanish even in the low output regime.

As a consequence, the effective α-factor at high output powers should be compared with α_{eff0} rather than with the linear parameter α_H. First, it is assumed that the nonlinear gain has the same form as in a two-level system:

$$g = \frac{g_d(N - N_0)}{1 + p}; \qquad p = P/Ps. \tag{45}$$

In this case, the α_{eff} is exactly the same as α_{eff0}. Second, a more accurate nonlinear gain expression given by (22) is used. The ratio between α_{eff} and α_{eff0} is found to be:

$$\frac{\alpha_{\text{eff}}^{\text{noise}}}{\alpha_{\text{eff0}}^{\text{noise}}} = \frac{(1 + p)(1 + r)}{\sqrt{1 + 2p + r}}. \tag{46}$$

TABLE I
LASER PARAMETERS

Parameters	Symbols	Values
Cavity length	L	400 μm
Thickness of active layer	—	0.15 μm
Width of active layer	—	2.0 μm
Effective refractive index	n	3.50
Group index	n_g	3.56
Internal loss	α_{int}	40 cm^{-1}
Linear linewidth enhancement factor	α_H	4.0
Bragg wavelength	λ_{B0}	1.50 μm
Carrier lifetime	τ_e	2 ns
Differential gain	g_d	$1.5 \cdot 10^{-16}$ cm^2
Transparent carrier density	N_0	$1.0 \cdot 10^{18}$ cm^{-3}
Derivative of loss	$d\alpha/dN$	$-2.5 \cdot 10^{-17}$ cm^2
Derivative of refractive index	dn/dN	$-1.0 \cdot 10^{-20}$ cm^3
Spontaneous emission factor	n_{sp}	2.0
Saturation photon density	P_s	$5.0 \cdot 10^{16}$ cm^{-3}

The ratio increases with increasing photon density, as is shown in Fig. 6 for various values of r. The material α-factor [see (48b)] normalized by α_{eff0} is also shown in the same figure for comparison. It can be seen that the increase of the material α-factor is more rapid than that of the effective α-factor for the linewidth.

The frequency tuning efficiency is obtained by integrating the local tuning efficiency over the cavity length:

$$H_f(0) = \int_{(L)} H_f(z, 0)\, dz = \frac{1}{2} v_g g_d \alpha_H \tau_R \cdot \frac{\partial g_{\text{NL}r}/\partial P}{-v_g \tau_R \partial g/\partial N \partial(gP)/\partial P + \partial g_{\text{NL}r}/\partial P}. \tag{47}$$

By using (41), (46), and (47), the three contributions to the linewidth are plotted in Fig. 7 as a function of the inverse of the normalized photon density. It can be seen that the contribution from spontaneous emission is dominant. This contribution decreases with increasing photon density. The contributions from the carrier shot noise and the cross correlation increase with output power because of the increase of the noise diffusion coefficients D_{NN} and D_{NP}. However, their values remain many orders smaller than that due to spontaneous emission.

It is important to note that the thermal effects are not included in our analysis. In practical cases, thermal effects are dominant mechanism for the frequency tuning at low frequency. Another important point is that the drive current noise has also been neglected in our model. However, the drive current noise is converted by nonlinear gain, especially by thermal effects, into phase noise [29]. The recent result on tunable lasers showed a very impor-

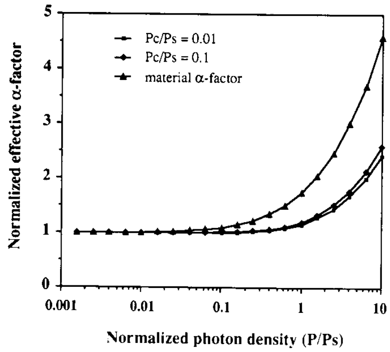

Fig. 6. The normalized effective α-factor for the linewidth as a function of the normalized photon density for a uniform intensity distribution laser. The parameter is the ratio of the interband saturation power to the intraband saturation power. The normalized material α-factor is also shown for comparison.

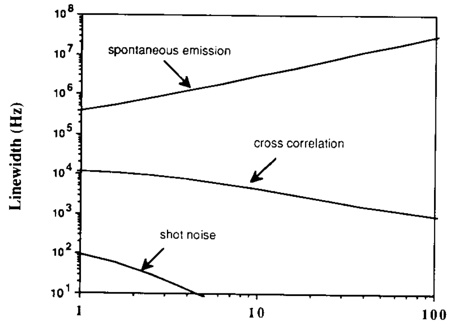

Fig. 7. Calculated linewidth due to spontaneous emission, carrier shot noise, and their cross correlation as a function of the inverse of the normalized photon density for a laser with uniform intensity distribution.

tant linewidth increase and lineshape change due to the drive current noise [30], [31].

B. Measurement on the Power Dependence of the Effective α-Factor

Various methods such as chirp-to-modulated-power-ratio (CPR) measurement [32] and the injection locking technique [33] are used to measure the effective α-factor and its dependence on the output power. It was usually recognized that the α-factor measured from these methods is the same as that for the linewidth. Using (30), the CPR for lasers with uniform intensity distribution is given by [17], [32]:

$$\text{CPR} = \frac{\Delta\omega}{2\pi\Delta P} = \frac{\alpha_{\text{eff}}^{\text{mod}}}{4\pi\overline{P}}\left(j\Omega - v_g \frac{\partial g}{\partial P}\overline{P}\right) \tag{48a}$$

where $\alpha_{\text{eff}}^{\text{mod}}$ is the effective α-factor for modulation given by:

$$\alpha_{\text{eff}}^{\text{mod}} = \frac{\alpha_H}{1 + \dfrac{1}{g_d}\dfrac{\partial g_{\text{NL}}}{\partial N}} \tag{48b}$$

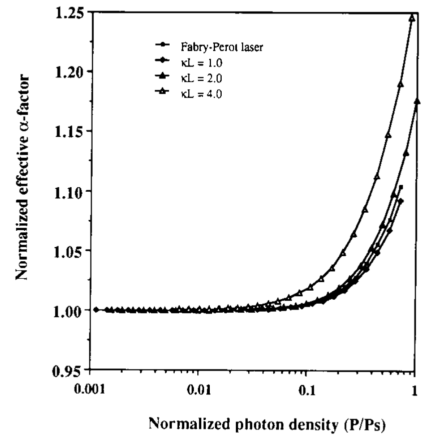

Fig. 8. The normalized effective α-factor for the linewidth as a function of the normalized photon density for a phase-shifted DFB laser. The parameter is the normalized coupling coefficient κL. The normalized α-factor for the linewidth in a Fabry–Perot laser is also shown for comparison.

This is exactly the material linewidth enhancement factor in nonlinear gain cases. The same parameter $\alpha_{\text{eff}}^{\text{mod}}$ appears in the technique of injection locking. For every model of nonlinear gain, $\alpha_{\text{eff}}^{\text{mod}}$ is thus always larger than the linear α-factor α_H. The measurement results recently made by Nakajima *et al.* on a strained quantum-well DFB laser confirmed the increase of this parameter with increasing output power [34].

It is thus not appropriate to measure the power dependence of α_{eff} for the linewidth by using the CPR or injection locking method, since such experiments measure the power dependence of $\alpha_{\text{eff}}^{\text{mod}}$ rather than that of $\alpha_{\text{eff}}^{\text{noise}}$. Direct measurement of this dependence from linewidth is also difficult, as many other factors, in addition to the spontaneous emission, contribute at the same time to the linewidth [see (39)].

VI. Application to a Phase-Shifted DFB Laser

For $\lambda/4$ phase-shifted DFB lasers, the intensity distribution is highly nonuniform due to the presence of the phase shift, except for the normalized coupling coefficient κL in the neighborhood of 1.25 [8], [9]. The field distribution is obtained by solving the coupled wave equations and the carrier conservation equation using a matrix method. The self-consistent field distribution is then used to calculate the effective α-factor and the linewidth.

Fig. 8 gives the results of the α-factor for the linewidth normalized by α_{eff0} in (44) as a function of the normalized photon density for $\kappa L = 1.0$, 2.0, and 4.0, respectively. The normalized α-factor for the linewidth in a Fabry–Perot laser is also shown for comparison. The nonlinear gain model described by (22) is used. It can be seen that for all values of the normalized coupling coefficient the effective α-factor increases with increasing photon density. This factor is slightly larger than the α-factor of a uniform laser for $\kappa L > 1.25$ and slightly smaller for $\kappa l <$

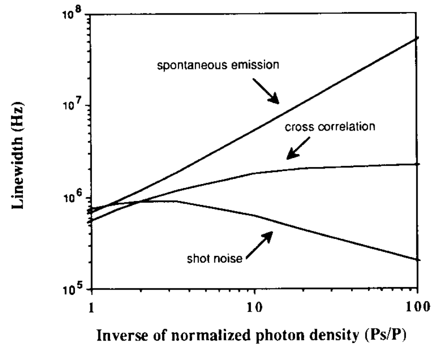

Fig. 9. Calculated linewidth due to spontaneous emission, carrier shot noise, and their cross correlation as a function of the inverse of the normalized photon density for a phase-shifted DFB laser with $\kappa L = 2.0$.

1.25. If nonlinear gain is not taken into account, no difference between the effective and the linear α-factor is observed.

Fig. 9 shows the three contributions to the linewidth as a function of the inverse of the normalized photon density for a phase-shifted DFB laser with $\kappa L = 2.0$. The contribution due to spontaneous emission has the same order of magnitude as in a uniform laser. In contrast, contributions due to carrier shot noise and the cross correlation are much more important than in a uniform laser. This is due to the fact that a more important frequency tuning efficiency is obtained by spatial hole burning. However, the spontaneous emission remains the dominant contribution to the linewidth in the usual output power regime ($P/P_s < 1$).

VII. Conclusion

In this paper the Green's function method has been extended to include spatial hole burning and the nonlinear gain in multielectrode semiconductor lasers. Different from previous approachs, a complex effective nonlinear gain has been introduced replacing the material nonlinear gain. Based on this method, a generalized rate equation has been obtained. Amplitude and frequency modulation transfer functions have been given for single and multielectrode lasers. A linewidth expression has been obtained, which includes contributions from spontaneous emission and carrier shot noise. The spontaneous emission coupled to the lasing mode and the effective α-factor for the linewidth have been found to be dependent on the stationary carrier density distribution. The carrier shot noise leads to a spectral linewidth proportional to the square of frequency tuning efficiency. The contribution of the correlation between photon density and carrier density depends on the tuning efficiency and the effective α-factor for the linewidth. It is found that the effective α-factor affecting the linewidth is in general different from its counterpart affecting modulation and injection locking properties due to spatial hole burning and nonlinear gain.

For lasers with uniform intensity distribution, it is found that the effective α-factor for the linewidth increases or remains constant with increasing output power depending on the model used for the nonlinear gain. For $\lambda/4$ phase-shifted distributed feedback lasers, the effective α-factor for linewidth is slightly larger or smaller than that for uniform lasers depending on the value of the normalized coupling coefficient. The linewidth due to various contributions is calculated for both uniform intensity distributed lasers and phase-shifted DFB lasers.

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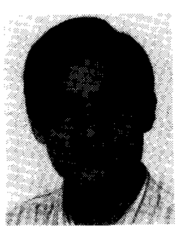

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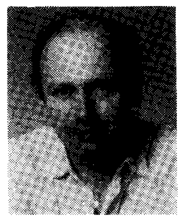
From 1983 to 1985, he was engaged in research on optical fiber measurements at the Guilin Institute of Optical Communications, Guilin, China. After one year of post-Doctoral research, he is now an Assistant Professor at TELECOM PARIS. His research concerns semiconductor lasers and amplifiers and their applications in optical transmission and switching.



Philippe Gallion (M'82) was born on March 30, 1950, in Saint-Dizier, France. He received the Maitrise and the Doctorat de Troisième Cycle from the University of Reims in 1972 and 1975, respectively, and the Doctorat d'Etat from the University of Montpellier in 1986.

As a Senior Lecturer at the University of Reims, his research involved Optical Signal Processing in Electron Microscopy. He joined the Ecole Nationale des Télecommunications (Télécom Paris) in 1978 where he is presently Professor and Head of the Communication Department. He is lecturing in the fields of electromagnetism, optics, quantum electronics, and optical communications and is in charge of the Mastére program in the field of optoelectronics and microwave devices. He is also lecturing at the University Pierre et Marie Curie, Paris. His present research topics include noise, modulation and tunability of semiconductor lasers, optical injection-locking and advanced optical communication, and switching techniques. He is author or co-author of more than 90 technical papers and communications.

Dr. Gallion is a member of the Optical Society of America.

Govind P. Agrawal (M'83-SM'86) was born in Kashipur, India. He received the M.S. and Ph.D. degrees from the Indian Institute of Technology, New Delhi, in 1971 and 1974, respectively.

After spending several years at the Ecole Polytechnique, Palaiseau, France, and the City University of New York, New York, he became a member of the technical staff at AT&T Bell Laboratories. He joined the faculty of the Institute of Optics at the University of Rochester, Rochester, NY, in January 1989. His research interests focus on quantum electronics, nonlinear optics, and laser physics. In particular, he has contributed significantly to the fields of semiconductor lasers, nonlinear fiber optics, and optical communications. He is an author or coauthor of more than 150 research papers, several review articles, and two books entitled *Long-Wavelength Semiconductor Lasers* (Van Nostrand Reinhold, 1986) and *Nonlinear Fiber Optics* (Academic, 1989).

Dr. Agrawal is a fellow of the Optical Society of America and a member of the American Physical Society.