

# INFLUENCE OF REFRACTIVE INDEX NONLINEARITIES ON MODULATION AND NOISE PROPERTIES OF SEMICONDUCTOR LASERS

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The modulation response and the spectral linewidth of singlemode semiconductor lasers are analysed by taking into account the nonlinear gain and the nonlinear refractive index in the rate equations. It is shown that the effect of nonlinear gain and index can be included through an effective linewidth enhancement factor  $\alpha_{eff}$  that is different for frequency modulation and for spectral linewidth. The effect of the nonlinear index is particularly strong in the case of frequency modulation where  $\alpha_{eff}$  can become zero or even negative for lasers operating on the red side of the gain peak. In the case of laser noise,  $\alpha_{eff}$  causes linewidth saturation but no rebroadening at high output powers. Our results indicate that gain and index nonlinearities are not the cause of linewidth rebroadening.

The role of nonlinear gain on the modulation and noise characteristics of semiconductor lasers has attracted much attention in recent years [1-6]. For example, it can affect the frequency-modulation (FM) response as well as the spectral linewidth. In both cases, the effect of nonlinear gain is included by defining an effective linewidth enhancement factor (LEF)  $\alpha_{eff}$  that varies with the optical power but is different in the two cases [6]. The strong-signal analysis carried out by using the density-matrix formalism shows that the nonlinear gain is accompanied by a non-negligible power-dependent change in the refractive index whenever the laser does not operate exactly at the gain peak [1]. A previous analysis that included nonlinear index used an approximation that may not be valid at very high powers [3]. Thus, a complete rate-equation analysis is needed to understand the influence of nonlinear index on the modulation and noise properties of semiconductor lasers. The purpose of this Letter is to show how  $\alpha_{eff}$  changes with the inclusion of nonlinear index.

The strong-signal analysis for a singlemode semiconductor laser gives the following approximate expressions for the gain and the index [1]:

$$\Delta n = -\frac{g_L}{2k_0} \left[ \alpha_0 - \frac{\beta_p}{1 + \sqrt{1+p}} \right] \quad (1)$$

$$g = \frac{g_L}{\sqrt{1+p}} \quad (2)$$

where  $g_L$  is the linear gain,  $k_0 = \omega/c$  is the wavenumber,  $\omega$  the lasing frequency and  $c$  the light velocity in a vacuum,  $\alpha_0$  is the material LEF,  $\beta$  is a parameter related to the intraband relaxation time and to the gain slope at the operating frequency [1],  $p = P/P_s$ ,  $P$  is the photon number and  $P_s$  is the saturation photon number related to the material parameters [3]. By using eqns. 1 and 2, we obtain the modified rate equations for the photon number  $P$ , phase  $\phi$  and the electron number  $N$  [3]:

$$\frac{dP}{dt} = [G_L/\sqrt{1+p} - \gamma]P + R_{sp} + F_p(t) \quad (3)$$

$$\frac{d\phi}{dt} = \frac{\alpha_0}{2}(G_L - \gamma) - \frac{\beta}{2} \frac{G_L P}{1 + \sqrt{1+p}} + F_\phi(t) \quad (4)$$

$$\frac{dN}{dt} = I/q - N/\tau_c - G_L P/\sqrt{1+p} + F_N(t) \quad (5)$$

where  $G_L = \Gamma v_g g_L$ ,  $\Gamma$  is the mode confinement factor,  $v_g$  is the group velocity,  $R_{sp}$  is the rate of spontaneous emission coupled into the lasing mode,  $\gamma$  the cavity decay rate,  $q$  the electron charge,  $\tau_c$  the carrier lifetime, and  $F_p(t)$ ,  $F_\phi(t)$  and  $F_N(t)$  are the random variables representing the spontaneous emission and carrier shot noise.

We first consider small-signal modulation and solve the rate equations by neglecting the Langevin noise terms. By using the standard linearisation procedure and the Fourier transform, the chirp-to-modulated-power ratio (CPR) is found to be given by [4]

$$CPR = \frac{\Delta\omega(\Omega_m)}{2\pi \Delta P(\Omega_m)} = \frac{\alpha_{eff}^{mod}}{4\pi P} (j\Omega_m + \Omega_c) \quad (6)$$

where  $\Omega_m$  is the modulation frequency and  $\Omega_c$  is the static (DC) chirp. The effective LEF for modulation is found to be related to the material LEF  $\alpha_0$  as

$$\alpha_{eff}^{mod} = \alpha_0 \sqrt{1+p} - \frac{\beta_p}{1 + 1/\sqrt{1+p}} \quad (7)$$

The effective LEF for modulation is plotted as a function of the normalised output power in Fig. 1 for three values of  $\beta$  ( $\beta = -1, 0, 1$ ) by choosing  $\alpha_0 = 2$  and 5. For lasers operating

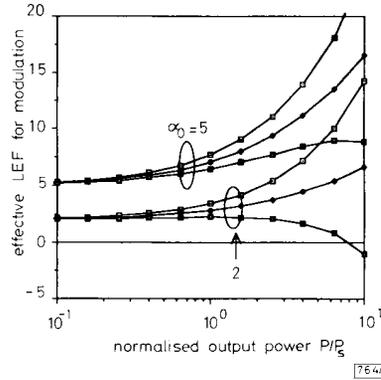


Fig. 1 Effective linewidth enhancement factor for modulation with normalised output power

Material LEF  $\alpha_0 = 2$  and 5  
 $\square$   $\beta = -1$   
 $\diamond$   $\beta = 0$   
 $\circ$   $\beta = 1$

at the gain peak ( $\beta = 0$ ), the contribution of nonlinear index vanishes, and the effective LEF increases with power as  $\sqrt{1+p}$ . For negative wavelength detuning ( $\beta < 0$ ), the effective LEF increases monotonically with  $p$ . By contrast, for positive wavelength detuning ( $\beta > 0$ ), it takes its maximum value at a certain value of  $p$ , and then begins to decrease. It becomes zero when  $p = \alpha_0 \beta / (\alpha_0 \beta + 2)$  and can even become negative for larger values of  $p$ .

Next, we calculate the spectral lineshape by solving eqns. 3-5 at a constant current by keeping the Langevin noise terms. The spectral linewidth induced by spontaneous emission is given by

$$\Delta\nu = \frac{R_{sp}}{4\pi P} [1 + (\alpha_{eff}^{noise})^2] \quad (8)$$

where the effective LEF for noise is given by

$$\alpha_{eff}^{noise} = \alpha_0 \frac{1+p/2}{\sqrt{1+p} + r/2} - \frac{\beta(r+1)p - 2\sqrt{1+p} + r + 2}{2\sqrt{1+p} + r/2} \quad (9)$$

The parameter  $r = (G_N \tau_c P_s)^{-1}$  has a typical value of  $\sim 0.01$ . It should be stressed that the  $\alpha_{eff}$  affecting the linewidth is different from its counterpart affecting the modulation response. This difference is due to the fact that current modulation and spontaneous emission do not affect the laser dynamics in the same manner. For instance, an increase in electron population due to current modulation leads to an increase in the output power. In contrast, an increase in the

output power due to the spontaneous emission results in a decrease of electron population to conserve the laser-oscillation condition.

The effective LEF for noise is plotted as a function of the normalised output power in Fig. 2 by choosing  $r = 0.01$ . The

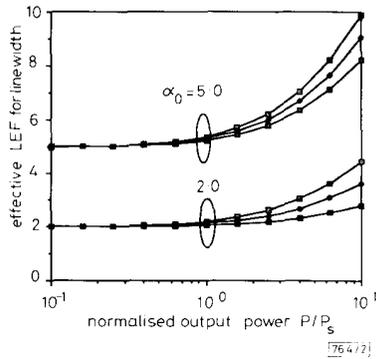


Fig. 2 Effective linewidth enhancement factor for spectral linewidth with normalised output power

Parameter  $r = 0.01$ ; other parameters and symbols have same meaning as in Fig. 1

effect of nonlinear index is similar to that shown in Fig. 1 except that the index-induced change is much smaller in the case of noise compared with the case of modulation. The major qualitative change is that  $\alpha_{eff}$  for noise never becomes smaller than the material LEF  $\alpha_0$ . It is practical to define the normalised linewidth as a ratio of  $\Delta\nu$  with the linear linewidth  $\Delta\nu_s$  corresponding to the intraband saturation output power, where

$$\Delta\nu_s = \frac{R_{sp}}{4\pi P_s} (1 + \alpha_0^2) \quad (10)$$

The normalised linewidth is plotted in Fig. 3 as a function of the normalised output power. For any value of detuning parameter  $\beta$ , we observe linewidth saturation but no rebroadening at high output powers. For negative wavelength detuning, the linewidth saturates faster, whereas for positive wavelength detuning, the linewidth saturation becomes less important.

In conclusion, we show that the effective LEF is different for modulation response and for spectral linewidth by including the nonlinear gain and index. In the modulation case, the effective LEF becomes zero and may even become negative at high output powers when the laser operates on the red side of the gain peak. The effects of index nonlinearities are easier to observe in lasers with a small value of  $\alpha_0$ . Strained-layer quantum-well lasers are well suited for this purpose because not only  $\alpha_0$  is smaller in such lasers but also  $P_s$  is smaller, allowing observation of the effects of gain and index saturation

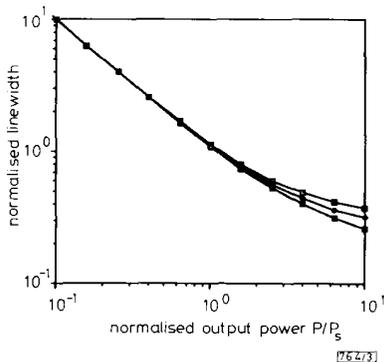


Fig. 3 Normalised linewidth plotted as function of normalised output power  $P/P_s$  for  $\alpha_0 = 5$  and  $r = 0.01$

tion at relatively low power levels (in the range 50–100 mW). In the case of laser linewidth, the effective LEF causes linewidth saturation without rebroadening at high output powers. Thus our results show that the nonlinear gain and index changes cannot explain the linewidth rebroadening observed experimentally.

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## RECOMBINATION CONSTANTS AND $\alpha$ FACTOR IN 1.5 $\mu\text{m}$ MQW OPTICAL AMPLIFIERS TAKING CARRIER OVERFLOW INTO ACCOUNT

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Recombination constants, carrier lifetime, linewidth enhancement factor and related parameters are examined experimentally and theoretically for MQW semiconductor optical amplifiers. The theoretical model which takes carrier overflow into account predicts a band-to-band recombination constant of  $B = 10^{-10} \text{ m}^6/\text{s}$  and an Auger recombination constant of  $C = 15 \times 10^{-29} \text{ m}^6/\text{s}$  which is higher than previously reported for MQW devices. Furthermore linewidth enhancement factors up to 30 are measured.

Introduction: To design MQW laser diodes and optical amplifiers, it is essential to know important parameters such as the linewidth enhancement factor ( $\alpha$  factor) and its related parameters (differential gain  $dg/dN$  and differential refractive index  $dn/dN$ ) and the recombination constants and thereby the carrier lifetime. Measurements of these parameters, that govern important properties such as linewidth, frequency response and resonance frequency, are presented here for four-well MQW devices. The parameters are influenced by the carrier overflow [1], which is due to the small energy discontinuity of the conduction band for InGaAs/InGaAsP MQW structures where the Fermi level can easily exceed the barrier energy even at low bias conditions [2, 3]. This implies that a larger number of electrons in the conduction band are present