Importance of Self-Induced Carrier-Density Modulation in Semiconductor Lasers

George R. Gray and Govind P. Agrawal

Abstract—**Semiconductor lasers have a built-in mechanism for modulating the carrier density at multiples of the longitudinal-mode spacing. This mechanism is believed to be relatively unimportant for solitary laser diodes since the mode spacing typically exceeds 50 GHz. We present a general numerical model capable of including self-saturation, cross saturation, and four-wave mixing occurring due to both interband and intraband effects, and show that the self-induced carrier-density modulation plays an important role in solitary laser diodes. In particular, it can severely degrade the gain margin and the mode-suppression ratio in single-mode semiconductor lasers when the operating current is increased. Degradation depends on the linewidth enhancement factor and the laser length and can be especially severe when the cavity length exceeds 1 mm.**

GAIN saturation plays an important role in governing the performance of semiconductor lasers. In the case of multimode operation, one should include not only self- and cross-saturation but also the contribution of four-wave mixing (FWM), a phase-sensitive phenomenon that leads to energy exchange among various modes. The importance of FWM was first recognized by Bogatov *et al.* [1]. In their model, beating of the neighboring modes modulates the carrier density at the mode-spacing frequency Ω and generates an asymmetric contribution to the gain spectrum such that the gain is enhanced for modes lying on the long-wavelength side but reduced for shorter-wavelength modes. Because the total carrier density is modulated, the resulting asymmetry is governed by the linewidth-enhancement factor α. The model explained well the experimental data on external-cavity semiconductor lasers for which the longitudinal mode spacing was comparable to the reciprocal of the carrier lifetime τ_e^{-1}. For solitary lasers, however, the intermode spacing Ω often exceeds 100 GHz. It is generally believed that the carrier density is unable to respond at such high frequencies because of a relatively long carrier lifetime ($\tau_e \sim 1$ ns). For this reason, Ishikawa *et al.* proposed an alternative model [2] in which he replaced τ_e by the intraband relaxation time τ_{in} (< 1 ps) since the electron population can respond to beat frequencies as large as 1 THz by redistributing itself within the conduction band. Such a modified model has been used with success to explain the experimental observations on mode jumping and bistable operation of solitary semiconductor lasers [3], [4]. However, the model suffers from a fundamental inconsistency: Since the model is based on intraband dynamics, gain asymmetry should not depend on the linewidth enhancement factor α which governs the index changes induced by variation in the total carrier density (intraband dynamics leaves the total density unchanged). Instead, a different amplitude-phase coupling coefficient arising from intraband effects should appear in the model. Such a coefficient, however, is much smaller than α and hence is too small to generate the observed gain asymmetry.

Attempts have been made to explain the observed gain asymmetry by considering carrier dynamics through the density-matrix formalism [5]–[9]. There are two distinct mechanisms of FWM in such a model, referred to here as interband and intraband, depending on whether the total carrier density or the distribution of electrons within the conduction band is modulated at the beat frequency of the two neighboring longitudinal modes. Both interband and intraband FWM provide an asymmetric component to the nonlinear gain. However, the magnitude of the intraband FWM, as mentioned above, is too small to explain the experimental data [3], [4]. Yamada has included both interband and intraband mechanisms within the density-matrix formalism [7] and found that gain asymmetry results mainly from the interband contribution that corresponds to modulation of the total carrier density. However, his analysis includes carrier-density modulation at only one frequency. In general, one should consider modulation at multiples of the intermode spacing. The objective of this letter is two-fold. First, we present a numerical model capable of simulating the noise and dynamical aspects of semiconductor lasers under quite general conditions. This model contains self-saturation, cross saturation, and FWM terms arising from both intraband and interband effects. Second, we show that the interband FWM, occurring as a result of carrier-density modulation, contributes significantly to the nonlinear gain, even when $\Omega\tau_e \gg 1$, and must be included for most solitary lasers especially when $\Omega/2\pi$ is less than 100 GHz. As evidence of the importance of self-induced carrier-density modulation, we show numerically that it leads to a degradation of the mode suppression ratio (MSR) in nearly single-mode lasers as the laser power is increased.

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The standard multimode rate equations, generalized to include terms accounting for nonlinear gain and FWM arising from intraband effects ($\zeta_j^{\rm int}$) and interband effects ($\delta_j^{\rm cdm}$), can be written as

$$\frac{dE_j}{dt} = -\frac{1}{2}(1-i\alpha)(G_j-\gamma_j)E_j + \zeta_j^{\rm int} + \delta_j^{\rm cdm} + F_j(t) \quad (1)$$

where

$$\zeta_j^{\rm int} = -\frac{1}{2}\left(\beta_j P_j + \sum_k \theta_{jk}P_k\right)E_j + \sum_k \kappa_{jk}E^*_{2k-j}E_k^2 \quad (2)$$

and

$$\delta_j^{\rm cdm} = C\frac{1}{2}(1-i\alpha)A\left[\sum_{m=1}^{M-1}(\Delta N_m E_{j-m} + \Delta N_m^* E_{j+m})\right] \quad (3)$$

with

$$\Delta N_m \equiv -(N-N_0)\frac{\sum_j E_j E^*_{j-m}/P_s}{1-im\Omega\tau_e + P_t/P_s}. \quad (4)$$

In (1), E_j is the complex, slowly varying field component at the frequency ω_j, α is the linewidth-enhancement factor, γ_j is the total cavity loss (decay rate) for each mode, and F_j is a complex Langevin source term. $G_j = v_g g_j(N)\Gamma$, where v_g is the group velocity, Γ is the confinement factor, and $g_j(N) = a(N-N_0)/V - \delta g_j$ is the usual optical gain, where a is the gain coefficient, N is the electron number, N_0 is the number required for transparency, V is the active region volume, and δg_j is the mode-dependent reduction from the peak value due to gain rolloff. In (2) β_j, θ_{jk}, and κ_{jk} are the respective self-saturation, cross-saturation and FWM coefficients arising from *intraband* effects. (These coefficients are in general complex and their explicit dependence on material parameters can be found in [5] for the case of spectral hole-burning. The contribution of other sources of nonlinear gain such as carrier heating can be included by suitably modifying β_j, θ_{jk}, and κ_{jk}.) In (3) and (4), C is a geometrical overlap factor [10], whose magnitude is estimated to be 0.5–1.0, $A \equiv \Gamma v_g a/V$, τ_e is the carrier lifetime, P_t is the total photon number, and $P_s = 1/(A\tau_e)$. We have ignored group-velocity dispersion, since its main effect is to shift the mode frequencies slightly (< 100 MHz), an amount much smaller than the shifts introduced by the nonlinear effects considered here. It can be included in a straightforward manner by making the parameter A mode dependent.

Equation (3) represents the contribution to the field rate equation from self-induced carrier-density modulation. In the derivation of (3), the carrier density is assumed to be modulated at multiples of the longitudinal-mode spacing Ω. Similar to the case of amplifiers [10], this allows the carrier rate equation to be solved approximately, in which the rapidly oscillating part of the carrier number, ΔN, couples together the field components at different frequencies, as shown in (4). This high-frequency oscillation leads to an additional term in the field rate equation (1) in terms of only slowly varying quantities. The substitution of (4) into (3) yields both cross-saturation terms as well as four-wave mixing terms.

Equation (1), together with the standard rate equation for the slowly-varying part of the carrier number, are the generalized multimode rate equations which include self-saturation, cross saturation, and FWM arising from both intraband and interband sources. The equations are solved numerically. In the absence of FWM, the photon number rate equations are decoupled from the mode phases. With the inclusion of FWM effects, this is no longer the case, since the FWM terms depend explicitly on the individual mode phases. The interband contribution, as expressed by (3), depends on several laser parameters, including the linewidth-enhancement factor α, the gain derivative a, and the laser length L through the mode spacing Ω. Since there is also a dependence on the total photon number P_t, one may expect the effect of the self-induced carrier-density modulation to change with operating current.

As one application of our computer model, we consider the question of mode stability in a nearly single-mode laser such as a DFB laser, which oscillates predominantly in a single-longitudinal mode. For the numerical simulations, we choose five longitudinal modes (mode 3 is located at the gain peak) and assign mode 4 (located on the high-frequency side of the gain peak) a lower loss than the other modes; i.e., $\gamma_4 < \gamma_3 \equiv \gamma$. We use a built-in loss margin, $(\gamma - \gamma_4)/\gamma = \Delta\gamma/\gamma = 5\%$, which provides a very high mode-suppression ratio (MSR), defined as the ratio of main mode average power to the average power in the strongest side mode. The gain bandwidth is taken as 10 THz (FWHM), the carrier lifetime is $\tau_e = 2$ ns, the overlap factor $C = 1$, and the gain derivative $a = 3.2 \times 10^{-16}$ cm^2. Additional relevant parameter values can be found in [5]. The carrier-density modulation generates both a gain and an index grating [9]. The index grating, which depends on the linewidth enhancement factor α, provides a mechanism by which the dominant mode saturates the gain of the high-frequency side modes while actually enhancing the gain for the low-frequency side modes. Thus, as output power is increased, we expect the longer wavelength modes to grow in power relative to the shorter wavelength modes. At high operating power, this gain enhancement can actually become comparable to the built-in loss margin, causing a degradation in MSR. To demonstrate this, we plot in Fig. 1(a) the effective gain margin between the main mode (mode 4) and the adjacent mode on the long wavelength side (mode 3 located at the gain peak) as a function of operating current for four different values of α. The laser current is increased up to twice the threshold value. In order to treat the effects of linear gain, nonlinear gain and FWM in a convenient manner, we define an effective gain according to $P_j \propto (\gamma_j - G_{j,\rm eff})^{-1}$. The effective gain margin between main mode

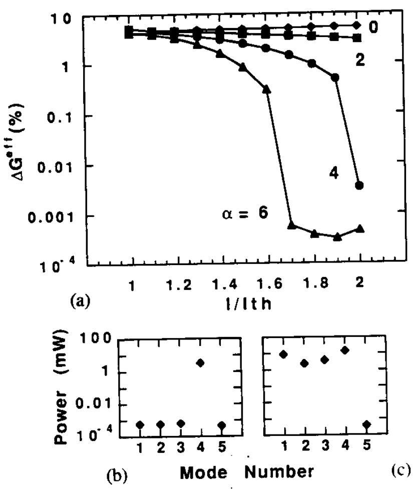

Fig. 1. (a) Effective gain margin between modes 3 and 4 plotted as a function of I/I_{th} for four different values of linewidth-enhancement factor α. (b) Mode spectrum at $I = 1.1 \times I_{th}$ for $\alpha = 4$. (c) Mode spectrum at $I = 2 \times I_{th}$ for $\alpha = 4$. In all cases the mode spacing is 50 GHz.

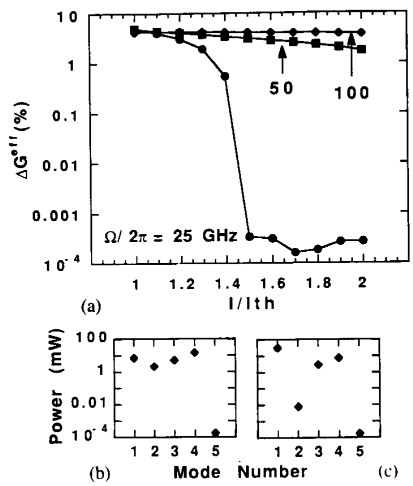

Fig. 2. (a) Effective gain margin between modes 3 and 4 plotted as a function of I/I_{th} for three different longitudinal mode spacings Ω. (b) Mode spectrum at $I = 1.7 \times I_{th}$ and (c) at $I = 2 \times I_{th}$ for the case of $\Omega/2\pi = 25$ GHz. In all cases $\alpha = 3$.

($j = 4$) and side mode ($j = 3$) is then defined as $\Delta G^{\text{eff}} = [\Delta\gamma - (G_{4,\text{eff}} - G_{3,\text{eff}})]/\gamma$. We choose $\Omega/2\pi = 50$ GHz ($L \approx 750$ μm).

The most striking feature of Fig. 1 is the strong dependence of the effective gain margin on the linewidth enhancement factor α. When $\alpha = 0$ there is little gain asymmetry, since the index grating arising from carrier-density modulation disappears. As seen in Fig. 1, the gain grating is not strong enough to cause any decrease in the gain margin; in fact, the effective gain margin, as well as MSR, actually improve with power at all operating currents when $\alpha = 0$ [6]. For nonzero α, however, the ΔG^{eff} decreases with increasing current, which eventually causes the MSR to degrade. Fig. 1(b) and (c) show the time-averaged mode spectra for the case $\alpha = 4$ at $I = 1.1 \times I_{th}$ and $I = 2 \times I_{th}$, respectively. It is easily shown that the MSR is related to the effective gain margin according to $\text{MSR} = 1 + \Delta G^{\text{eff}} P_m / n_{sp}$ where n_{sp} is the population-inversion factor; for our simulations we use $n_{sp} = 1.8$. At a critical value of the current, ΔG^{eff} falls below 1% in a phase-transition-like manner, and the MSR degrades abruptly from its initial value of >30 dB to as low as 10 dB. For the particular case of $\alpha = 4$, as the built-in loss margin of 5% decreases to 3×10^{-3}% at $I = 2 \times I_{th}$, the MSR decreases suddenly from >37 dB at $I = 1.1 \times I_{th}$ to merely 2 dB. A mode jump can occur at higher values of current, as the effective gain margin decreases further.

The effectiveness of the self-induced carrier-density modulation is of course strongly dependent upon the laser length, since the fundamental beat frequency scales inversely with length. In Fig. 2 we show this dependence by plotting ΔG^{eff} versus current for three different laser lengths. For lasers with a mode spacing of 100 GHz or more, the interband effect is relatively weak, and there is very little degradation of the built-in loss margin over the range of current plotted. Such a mode spacing corresponds to a laser length of 375 μm assuming a group refractive index of 4. Therefore for lasers of this length or shorter, self-induced carrier-density modulation would lead to degradation of the MSR only when the laser operates far above threshold. However, for a mode spacing of 50 GHz (laser length = 750 μm), the decrease in the gain suppression with increasing current is quite noticeable even twice above threshold. Since lasers with lengths exceeding 500 μm are becoming increasingly more common, the self-induced carrier-density modulation provides a fundamental limit on the high-power mode stability of these lasers. If the asymmetry in the interband gain saturation becomes very pronounced, then the lasing mode will actually jump to a longer wavelength mode with increasing current, similar to a Fabry–Perot laser. This case is shown in Fig. 2(a) by letting $\Omega/2\pi = 25$ GHz. By the time the injection current reaches 70% above threshold, the average mode spectrum is essentially multimode [see Fig. 2(b)], although mode 4 remains the strongest mode. For $I > 1.7 \times I_{th}$ mode 1 is the strongest mode, indicating a mode jump over three longitudinal modes. At $I = 2 \times I_{th}$ the time-average spectrum, as shown in Fig. 2(c), shows mode 1 to be the dominant mode, although the MSR is quite poor.

In conclusion, semiconductor lasers have a built-in mechanism for modulating the carrier density at multiples of the longitudinal-mode spacing. We have shown that this self-induced carrier-density modulation plays an important role in solitary laser diodes, even though the carrier lifetime may be much larger than the laser round-trip time. In particular, the gain margin and the mode-suppression ratio in single-mode semiconductor lasers may be severely degraded when the operating current is increased. Degradation depends on the linewidth enhancement factor and the laser length and can be especially severe when the cavity length exceeds 1 mm. Because quantum-well lasers, and particularly strained quantum-well lasers, have lower values of the linewidth enhancement factor, they should demonstrate single-mode operation over a wider range of output power than conventional semiconductor lasers.

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Analysis of Tunable Semiconductor Lasers with Co-Directional Grating-Assisted Coupler Filter

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Abstract—**A general theoretical model is established for the tunable semiconductor lasers with built-in co-directional grating-assisted vertical coupler filter. Possible internal reflection at the gain-coupler junction is considered and simple expressions for the lasing wavelengths/threshold gain derived. Mechanisms affecting the longitudinal mode selectivity are examined.**

I. INTRODUCTION

TUNABLE semiconductor lasers are key devices in advanced photonic systems [1]. The conventional tunable lasers such as the DBR and twin-guide DFB lasers are limited in their tunable range. A broadly tunable InGaAsP/InP laser based on a vertical coupler filter has been demonstrated by Alferness and co-workers [2]. A similar device was also reported by Illek *et al.* [3]. Despite the impressive performances these devices have demonstrated experimentally, little theoretical work has been published, except for a simplified coupled-mode analysis by Willems and coworkers [4]. A general theoretical model considering the critical factors that affect the lasing characteristics is still lacking. In practice, such a model and a rigorous analysis based on the model would be essential for thorough understanding, further exploitation, and final optimization of the devices. For instance, single longitudinal mode operation has been observed in the reported device [2]. However, the mechanism responsible for the single-mode operation with large side-mode suppression ratio has not yet been fully explained. In this letter, a general model for this device is proposed in which both the filter characteristics and the internal reflection are taken into account and a more rigorous coupled-mode formulation is employed.

A schematic diagram of the co-directional grating-assisted tunable laser is depicted in Fig. 1. The lasing condition of this device can be derived by a transfer

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