Modulation Instability in Erbium-Doped Fiber Amplifiers

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***Abstract*—The onset of modulation instability in erbium-doped fiber amplifiers (EDFA's) is studied through a stability analysis of the underlying nonlinear Schrödinger equation. The existence of gain in EDFA's lowers the threshold for modulation instability considerably compared with the case of undoped fibers. Modulation instability generates multiple pulses when a single pulse is amplified. It can also create multiple subpulses in mode-locked fiber lasers, a feature observed experimentally. Numerical simulations show that EDFA's can convert a continuous-wave optical signal into a train of high-repetition rate femtosecond pulses.**

ERBIUM-DOPED fiber amplifiers (EDFA's) have attracted considerable attention recently because of their potential applications in fiber-optic communication systems [1]. Since the group-velocity dispersion (GVD) is anomalous at the operating wavelength of EDFA's, such amplifiers are susceptible to modulation instabilities occurring in optical fibers even in the absence of dopants [2]. In undoped optical fibers, modulation instabilities are generally not of much concern because of their relatively high threshold; typically input peak powers should exceed 1 W for the onset of modulation instability in passive fibers [2]. It is likely that the internal gain of active fibers would lead to the onset of modulation instability at much reduced input power levels. This letter investigates modulation instabilities in EDFA's by solving the underlying nonlinear Schrödinger equation and discusses its implication for mode-locked fiber lasers.

The starting point is the nonlinear Schrödinger equation obeyed by the slowly varying amplitude $A(z, t)$ of the signal propagating inside the EDFA. It can be written as [3]

$$\frac{\partial A}{\partial z} + \frac{i}{2}(\beta_2 + igT_2^2)\frac{\partial^2 A}{\partial t^2} - i\gamma |A|^2 A = \frac{1}{2}gA \quad (1)$$

where β_2 is the GVD coefficient, $\gamma = 2\pi n_2/(\lambda a_{\text{eff}})$ is the nonlinear parameter, λ is the optical wavelength, a_{eff} is the effective core area, and g is the small-signal gain of the EDFA. The finite gain bandwidth of the EDFA is included through the parameter T_2 in (1) which represents the dipole relaxation time of the erbium dopants. It was chosen to be 100 fs.

To study the onset of modulation instability, we follow a standard approach [2] by considering the stability of the steady-state solution when a CW beam of input power P_0 is launched into the EDFA. The results are expected to remain valid even for the case of optical pulses as long as the pulse duration is much longer than the time scale of modulation instability (~ 1 ps). The steady-state solution of (1), obtained by neglecting the second-derivative term, is given by

$$A(z) = \sqrt{P_0}\exp\left(i\gamma\int_0^z P_0 \exp(gz)\,dz + \frac{gz}{2}\right). \quad (2)$$

It shows that the EDFA imposes a nonlinear phase shift on the amplified CW signal. By considering a small perturbation from the steady-state in the form

$$A(z, t) = (\sqrt{P_0} + u + iv)\exp\left(i\gamma\int_0^z P_0\exp(gz)\,dz + \frac{gz}{2}\right) \quad (3)$$

and linearizing (1) in u and v, we obtain the following set of two coupled equations:

$$\frac{\partial u}{\partial z} = \frac{1}{2}gT_2^2\frac{\partial^2 u}{\partial t^2} + \frac{1}{2}\beta_2\frac{\partial^2 v}{\partial t^2} \quad (4)$$

$$\frac{\partial v}{\partial z} = \frac{1}{2}gT_2^2\frac{\partial^2 v}{\partial t^2} - \frac{1}{2}\beta_2\frac{\partial^2 u}{\partial t^2} + 2\gamma P_0\exp(gz)u. \quad (5)$$

These equations can be solved approximately by assuming a solution of the form

$$u(z) = u_0(z)\exp\left[i\int K(z)\,dz - i\Omega t\right] \quad (6)$$

$$v(z) = v_0(z)\exp\left[i\int K(z)\,dz - i\Omega t\right] \quad (7)$$

where Ω is the frequency of perturbation. The wavenumber K is z dependent because of the gain provided by the amplifier. By substituting (6) and (7) in (4) and (5) and assuming that u_0 and v_0 vary slowly with z, the wavenumber K is found to satisfy the following dispersion relation:

$$K(\Omega, z) = \frac{i}{2}gT_2^2\Omega^2 \pm \frac{1}{2}|\beta_2|\Omega\left[\Omega^2 + (4\gamma P_0/\beta_2)\exp(gz)\right]^{1/2}. \quad (8)$$

In the absence of the amplifier gain ($g = 0$), K becomes z independent and reduces to that obtained for passive fibers [2]. Modulation instability occurs whenever K has a negative imaginary part since the perturbation then grows exponentially along the fiber length. It is useful to define the total

Manuscript received November 25, 1991; revised March 13, 1992. The work was supported in part by the National Science Foundation (Grant No. ECS-9010599) and the Army Research Office.

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IEEE Log Number 9200599.

1041-1135/92$03.00

integrated gain at the frequency Ω as

$$h(\Omega) = -2\int_0^L \mathrm{Im}\left[K(\Omega, z)\right] dz \tag{9}$$

where the factor of 2 converts $h(\Omega)$ to the power gain.

Equation (9) provides the growth of a perturbation at the frequency Ω occurring as a result of modulation instability [perturbation grows by a factor of $\exp(h)$]. In the normal dispersion region, $h(\Omega) = -gL(\Omega T_2)^2$. Since h is negative, the perturbation is damped at all frequencies. This result is different from the passive fibers for which $h(\Omega) = 0$. The amplifier-induced damping is a result of gain dispersion. The amplifier provides less gain in the spectral wings. As a result, perturbations with large Ω amplify less than the signal. In essence, fiber amplifiers with positive GVD are inherently stable against perturbations.

The situation is quite different for EDFA's for which the GVD is anomalous ($\beta_2 < 0$). Similar to the case of undoped fibers ($g = 0$), the integrated gain $h(\Omega)$ becomes positive for certain frequencies, and perturbations at those frequencies grow exponentially along the fiber. A new feature is that the frequency range over which the instability gain exists itself expands exponentially because of the signal amplification. This feature can be seen from (8) by noting that the gain exists for frequencies $\Omega < \Omega_0 \exp(gz/2)$ where

$$\Omega_0 = \left(4\gamma P_0 / |\beta_2|\right)^{1/2} \tag{10}$$

is determined by the input power. Thus, even though frequency components for which $\Omega_0 < \Omega < \Omega_0 \exp(gL/2)$ are stable initially, they become destabilized at a latter stage. Equation (9) can include this feature if the lower limit is replaced by the distance at which the gain first becomes positive. The integration can be performed analytically with the result

$$h(\Omega) = \frac{8fL}{\ln(G) L_{NL}}\Big[\sqrt{G - f^2} + f\sin^{-1}(f/\sqrt{G}) - \sqrt{G' - f^2} - f\sin^{-1}(f/\sqrt{G'})\Big] - f^2\Omega_0^2 T_2^2 \ln(G) \tag{11}$$

where $f = \Omega/\Omega_0$ is the normalized frequency, $L_{\mathrm{NL}} = (\gamma P_0)^{-1}$ is the nonlinear length, $G = \exp(gL)$ is the amplifier gain, and $G' = 1$ for $f \le 1$, but becomes f^2 for $f > 1$.

Fig. 1 shows the gain spectrum of modulation instability by plotting $h(\Omega)$ for several values of the input power P_0 for an EDFA of 100-m length and 30-dB single-pass gain by taking $\beta_2 = -20$ ps^2/km and $\gamma = 10$ W^{-1}/km. The gain exists over a wide range of frequencies extending up to a few THz. When the modulation instability is initiated from noise, the frequency Ω for which the integrated gain $h(\Omega)$ is maximum becomes the modulation frequency since the perturbation at that frequency grows most. Since the perturbation grows as $\exp(h)$, the initial noise is amplified by a factor of 10^8 when $h = 18.4$. Such values of h are large enough to initiate modulation instability from noise. As seen in Fig. 1, modulation instability can readily occur for $P_0 \sim 100$ mW. A CW beam would be converted into a pulse train with a repetition rate ~ 1 THz when the input power is large enough to initiate the modulation instability.

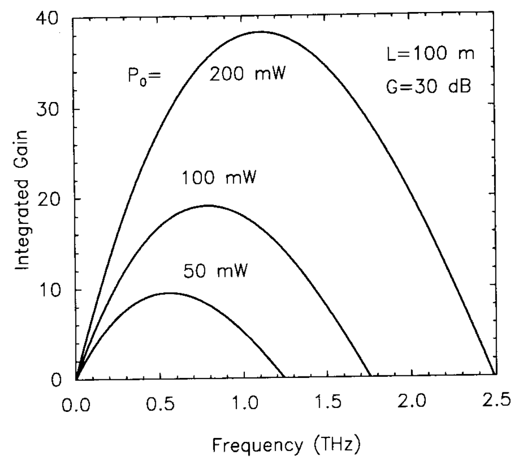

Fig. 1. Gain spectra of modulation instability at several input power levels for a 100-m-long EDFA with 30 dB gain.

The linear-stability analysis can provide only the initial growth rate of modulation instability. When the perturbation amplitude becomes comparable to the CW signal itself, the linear analysis becomes invalid, and (1) should be solved numerically. Fig. 2 shows the result of a numerical simulation when modulation instability is induced by injecting an input signal into the EDFA of the form:

$$A(0, t) = \sqrt{P_0}\left[1 + a_m \sin(2\pi f_m t)\right] \tag{12}$$

where a_m is the amplitude and f_m is the frequency of a weak sinusoidal modulation imposed on the cw beam. In the case of Fig. 2, $a_m = 1\%$, $f_m = 1$ THz, and other parameters are the same as for Fig. 1. The nearly CW input (dashed curve) has been transformed into a pulse train through modulation instability. The pulse width is comparable to the dipole relaxation time T_2 of the erbium ions, assumed to be 100 fs. Numerical results indicate the possibility of period-doubling and chaos under certain operating conditions.

The results of the above analysis are applicable to the quasi-CW situation as long as the pulse width is much longer than the period of modulation instability (~ 1 ps). The input power P_0 in (10) then corresponds to the peak power of the input pulse. Consider, for example, a 50-ps pulse train at 1-Gb/s repetition rate emitted by a mode-locked laser. The peak power of each pulse exceeds 100 mW when the average power is above 5 mW. According to the results of Fig. 1, modulation instability can occur when such a pulse train is amplified by an EDFA. Each pulse would split into many subpulses, spaced apart by a picosecond or so. The width of each individual subpulse is determined by the EDFA bandwidth and is expected to be ~ 100 fs. Such a splitting has been observed, both numerically [3] and experimentally [4], when picosecond optical pulses are amplified by EDFA's. Modulation instability is also expected to affect the mode-locking process in erbium-doped fiber lasers. Indeed, in some recent experiments [5], [6] each pulse was found to be a

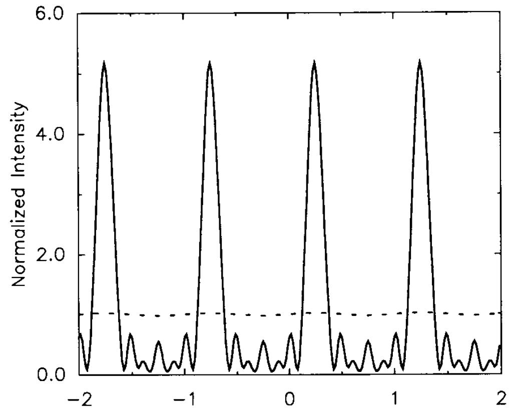

Fig. 2. Numerically simulated pulse train at the amplifier output. Dashed curve shows the input CW signal, sinusoidally modulated at 1 THz with 1% modulation depth to induce modulation instability.

bunch of equispaced femtosecond subpulses whose origin was not clear. In one experiment [5] the pulse spectrum showed side-lobes, suggesting that modulation instability played a role. According to the results of this analysis, modulation instability is most likely responsible for the generation of multiple subpulses in mode-locked fiber lasers.

In conclusion, the existence of gain in EDFA's lowers the threshold for modulation instability considerably compared with the case of undoped fibers. Modulation instability leads to generation of multiple pulses when a single pulse is amplified. It can also create multiple subpulses when EDFA's are used as a gain medium for mode-locked fiber lasers. Such amplifiers can generate a high-repetition rate femtosecond pulse train when a CW signal is launched at the input provided care is taken to suppress the competing nonlinear processes such as stimulated Brillouin scattering.

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Coherent Lightwave Amplification and Stimulated Brillouin Scattering in an Erbium-Doped Fiber Amplifier

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Abstract—**The stimulated Brillouin scattering (SBS) of coherent light pulses launched into an erbium-doped fiber (EDF) is reported. In a 50 m EDF, the SBS peak power reaches about 40 dBm for input powers higher than the threshold power of about 6 dBm. SBS and its amplification in an erbium-doped fiber amplifier (EDFA) result in the deformation of forward-amplified pulse due to the Brillouin power conversion and deterioration of the EDFA population inversion. It is also shown that a very short EDF is effective in suppressing the SBS.**

Manuscript received March 9, 1992.
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IEEE Log Number 9200601.

I. Introduction

EDFA'S have been introduced into many areas of optical fiber transmission systems. These include long span coherent lightwave transmission systems [1] and frequency division multiplexing transmission systems [2]. EDFA's have also been applied to optical time domain reflectometry

1041-1135/92$03.00