Effective Nonlinear Gain in Semiconductor Lasers

Guang-Hua Duan, Philippe Gallion, and Govind P. Agrawal

Abstract—**An effective nonlinear gain is introduced for semiconductor lasers by taking into account the effect of laser structure and the associated distribution of the mode intensity along the cavity length. It should be used in the analysis of laser dynamics and noise in place of the material nonlinear gain parameter. A general expression for the effective nonlinear gain is given by using the Green's function method. The results obtained for Fabry–Perot and distributed feedback lasers show that the effective nonlinear gain could be considerably enhanced. The exact value of the enhancement factor depends on cavity parameters. Affected by the laser structure, the nonlinear gain has a different power dependence than expected from material considerations alone.**

Introduction

The nonlinear gain is known to have an important influence on the dynamics and noise properties of semiconductor lasers [1], [2]. The previous studies have used a form of the nonlinear gain derived from the consideration of the material properties alone such as spectral hole burning or carrier heating [3], [4]. Based on the density matrix formalism, an exact form of nonlinear gain due to spectral hole burning has been obtained [3]. In every model, the nonlinear gain is a function of the local photon density. However, in most semiconductor lasers, the photon density is not uniform along the cavity due to the output coupling. A natural question is what is the effective nonlinear gain that should appear in the rate equations governing the laser dynamics and noise properties. The purpose of this letter is to clarify the influence of photon density distribution on the nonlinear gain.

Analysis

The starting point of our analysis is the wave equation in the frequency domain. Assuming perfect transverse and lateral index guiding, we concentrate our attention to the longitudinal axis. The one-dimensional propagation equation is solved by using the Green's function method, which has been used to describe laser dynamics and noise [5], [6]. However, the Green's function method is only valid for a linear dielectric constant. To include the material nonlinear gain, the initial Green's function method was modified. The details will be presented in a forthcoming paper. The final result for the effective nonlinear gain is given by

$$g_{\rm NL}^{\rm EFF} = \frac{2k_0^2}{v_g \partial W/\partial\omega} \int_0^L Z_0^2(z)\, n g_{\rm NL}(N, P)\, dz \qquad (1)$$

where $Z_0(z)$ is the field distribution inside the cavity, $N(z)$ the carrier density, $P(z)$ the photon density, $k_0 = \omega/c$, with ω the optical frequency and c the light velocity in vacuum, v_g is the group velocity in the laser medium, n is the linear refractive index, $g_{\rm NL}$ is the nonlinear gain, and W is the Wronskian of the linear laser system [5], [6]. The integration is performed over the cavity length L. It has been shown that the refractive index changes also with an increase in the output power [3]. This nonlinear refractive index can be formally included by a complex material nonlinear gain [2]. When the refractive index and the optical gain vary smoothly in the cavity, as in the case of Fabry–Perot, DFB, and DBR lasers, the effective nonlinear gain can be written as

$$g_{\rm NL}^{\rm EFF} = \frac{\int_0^L Z_0^2(z)\, g_{\rm NL}(N, P)\, dz}{\int_0^L Z_0^2(z)\, dz}. \qquad (2)$$

Thus, the effective nonlinear gain is a spatial averaging of the material local nonlinear gain weighted by the squared field distribution rather than by the intensity. This newly defined nonlinear gain takes into account both material and structural dependences. It reduces to the material gain for an uniform intensity distribution. The effective nonlinear gain has the same origin as the longitudinal spontaneous emission enhancement factor K_z [5], [6]: the different longitudinal mode distributions $\{Z_m(z),\ m = 0, 1, \dots\}$, forming a complete set, are not orthogonal in the Hermitian sense, due to the presence of optical gain in the cavity [7].

Several consequences of the above considerations can be predicted from (2):

i) Different laser structures can give rise to different values and forms of the effective nonlinear gain for the same material. This is due to the field distribution $Z_0(z)$ dependence on the cavity structure.

ii) As the field distribution generally includes a spatially dependent phase, the material nonlinear gain can result in an effective nonlinear index and vice versa.

iii) As the intensity distribution is not uniform and changes with the output power due to spatial hole burning [8], the effective nonlinear gain will have, in general, a different power dependence than the material nonlinear gain.

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Results for Fabry–Perot and DFB Lasers

Neglecting the nonlinear refractive index, the nonlinear gain can be approximated in the low output power regime by $g_{NL} = -g_L \epsilon P(z)$ where ϵ is referred to as the gain compression factor and g_L as the linear gain. By relating the photon density with the mode distribution by $P(z) = P_0 |Z_0(z)|^2$, with P_0 a spatially independent parameter proportional to the output power, the g_{NL}^{EFF} can be written as

$$g_{NL}^{EFF} = -\epsilon P_0 g_L \frac{\int_0^L Z_0^2(z) |Z_0(z)|^2 \, dz}{\int_0^L Z_0^2(z) \, dz}. \quad (3)$$

It is assumed for the moment that the spatial variation of the linear gain due to spatial hole burning in the laser cavity could be neglected [8]. By comparing this effective nonlinear gain with the nonlinear material gain g_{NL}^{MAT} corresponding to the average photon density in the cavity:

$$g_{NL}^{MAT} = -\epsilon P_0 g_L \int_0^L |Z_0(z)|^2 \, dz / L \quad (4)$$

a correction factor of effective nonlinear gain can be introduced, which relates the effective gain compression factor with the material gain compression factor. By using (3) and (4), the correction factor C is given by

$$C = \frac{\int_0^L Z_0^2(z) |Z_0(z)|^2 \, dz}{\left(\int_0^L Z_0^2(z) \, dz\right)\left(\int_0^L |Z_0|^2(z) \, dz / L\right)}. \quad (5)$$

Equations (2) and (5) are the main results of this letter. The correction factor C given in (5) shows how the laser structure can change the value of the material parameter ϵ. This factor is generally complex. The real part represents the change in the material nonlinear gain, the imaginary part represents the contribution of the material nonlinear gain to the effective refractive index. The latter directly affects the phase variation of the electrical field in the laser cavity and thus contributes to the frequency chirp and the linewidth.

We have calculated the correction factor for different types of laser structure. Fig. 1 shows the result for a Fabry–Perot laser with one facet reflectivity of 30% ($R_2 = 0.3$) and a varying reflectivity R_1 of the other. The real part of C is slightly larger than unity for small values of reflectivity ($R_1 < 10^{-4}$). Beyond this value, the correction factor keeps at unity. The imaginary part of C decreases with the increasing facet reflectivity R_1. The value of the imaginary part is quite small compared to that of the real part.

The correction factor C is plotted as a function of the normalized coupling coefficient in Fig. 2 for a conventional DFB lasers. Both facets of the laser are assumed AR-coated ($R_1 = R_2 = 0.$). The real part of the correction factor increases from 0.87 for $\kappa L = 1.0$ to 1.3 for $\kappa L = 5.0$. The imaginary part of the correction factor changes from negative values to positive values with the increasing normalized coupling coefficient. It is not surprising that the effective nonlinear gain in Fabry–Perot and conventional DFB lasers is not very different from the material nonlinear gain corresponding to the average photon density, as the intensity distribution in these lasers is rather uniform.

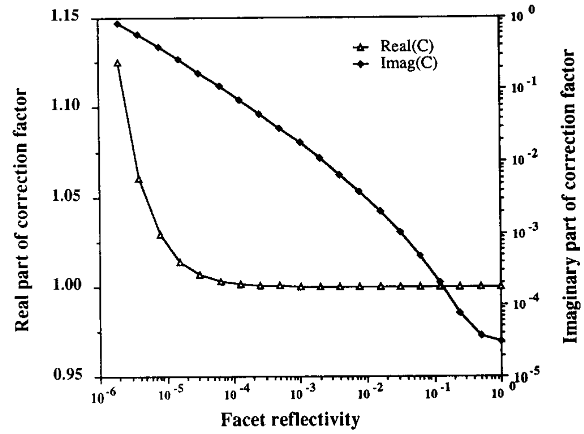

Fig. 1. The real and imaginary parts of the correction factor C as a function of facet reflectivity R_1 for a Fabry–Perot laser. The reflectivity of the other facet R_2 is assumed to be 30%.

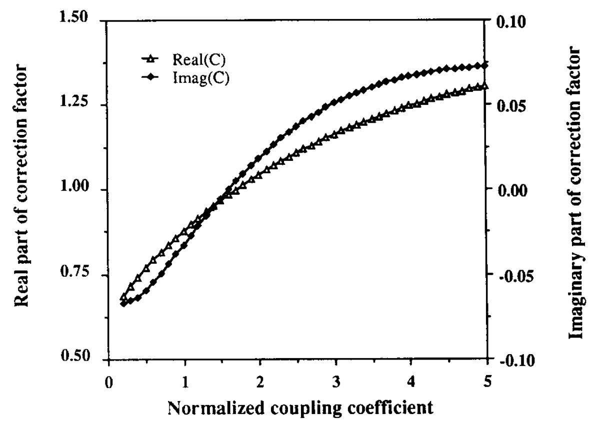

Fig. 2. The real and imaginary parts of the correction factor C as a function of the normalized coupling coefficient κL for a conventional DFB laser with AR-coated facets.

The result for a AR-coated $\lambda/4$ phase-shifted DFB laser is shown in Fig. 3. The real part of the correction factor becomes larger than unity for $\kappa L > 1.25$ and attains 2 for $\kappa L = 4.0$. The imaginary part of the correction factor changes sign at $\kappa L = 1.25$ and becomes negligible for larger values of κL. This is due to the fact that for $\kappa L = 1.25$, the field-intensity distribution is nearly uniform inside the cavity [8]. For κL larger than 1.25, the field intensity is more concentrated in the center of the cavity. Otherwise, the field intensity is concentrated near the two facets [8].

To evaluate its output power dependence, the effective nonlinear gain is calculated by using (2) for a AR-coated $\lambda/4$ phase-shifted DFB laser with $\kappa L = 3.0$. In our calculations, the field distribution and the linear gain are calculated by including the spatial hole burning [8]. The material nonlinear gain is assumed to have the form $g_{NL} =$

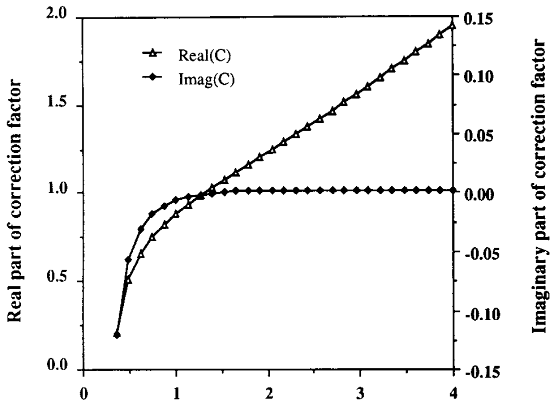

Fig. 3. The real and imaginary parts of the correction factor C as a function of the normalized coupling coefficient κL for a $\lambda/4$ phase-shifted DFB laser with AR-coated facets.

$g_L/\sqrt{1+2\epsilon P(z)} - g_L$ [3]. The real and imaginary parts of the effective nonlinear gain are plotted as a function of the output power in Fig. 4(a) and (b), respectively. The material nonlinear gain corresponding to the average photon density is also plotted in Fig. 4(a) for comparison. It can be seen that at low output powers (< 1 mW) the effective nonlinear gain and the material nonlinear gain give a similar power dependence. The ratio of their values is close to 1.5, the same as the correction factor C given in Fig. 3 for $\kappa L = 3.0$. When the output power increases, the intensity distribution becomes more uniform due to the spatial hole burning [8]. Consequently, the real part of the effective nonlinear gain approaches the material nonlinear gain at an output power of about 10 mW and continues to decrease for higher output powers. At the same time, the imaginary part of the nonlinear gain tends to change sign.

Conclusion

The concept of an effective nonlinear gain is introduced, which takes into account both the material and structural dependences. The effective nonlinear gain can be twice the magnitude of the material contributions for some device parameters in the case of DFB lasers. The effective nonlinear gain has generally a different power dependence than the material nonlinear gain. The consequences of the effective nonlinear gain on laser dynamics and noise are under study.

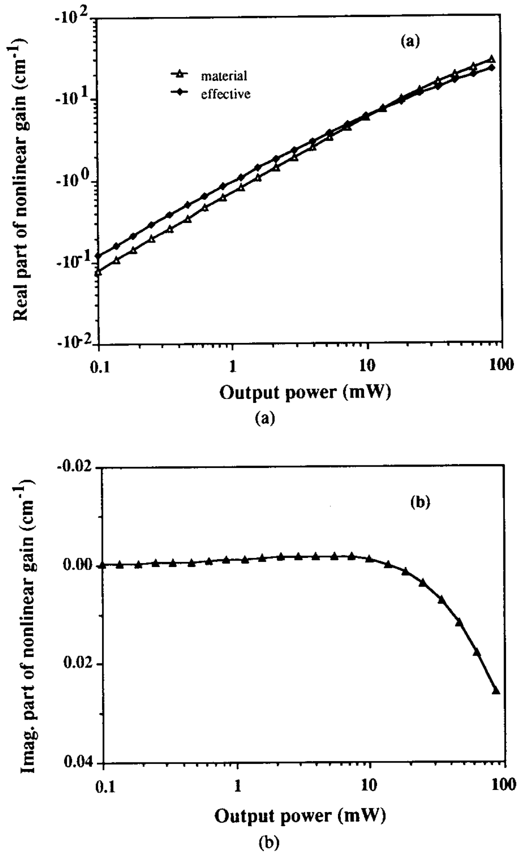

Fig. 4. (a) The real part and (b) the imaginary part of the effective nonlinear gain g_{NL}^{EFF} as a function of output power for a phase-shifted DFB laser with $\kappa L = 3.0$.

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