Intensity and phase noise in microcavity surface-emitting semiconductor lasers

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The noise characteristics of vertical-cavity surface-emitting (VCSE) lasers are studied by using the Langevin rate equations modified suitably to include the enhanced spontaneous emission occurring in such microcavity lasers. The intensity and frequency noise spectra show the effects induced by suppression of relaxation oscillations. However, such a suppression depends on the output power as well as on transverse dimensions of the VCSE laser. The laser linewidth increases considerably as a result of enhanced spontaneous emission.

Vertical-cavity surface-emitting (VCSE) semiconductor lasers have attracted considerable attention recently[1-7] because of their potential applications in technologically important areas such as optical computing and optical signal processing. An interesting aspect of such lasers from a theoretical standpoint is that the cavity length is extremely small ($L \sim 1\ \mu$m). The most dramatic effect of the microcavity is that the rate of spontaneous emission into the lasing mode is enhanced by orders of magnitude compared with the conventional edge-emitting semiconductor lasers. The rate-equation analysis of VCSE lasers predicts novel features such as suppression of relaxation oscillations and thresholdless operation. One would expect that enhanced spontaneous emission into the lasing mode would affect the noise characteristics of such lasers considerably. The objective of this letter is to study the intensity and phase noise in VCSE lasers.

Conventional rate equations for semiconductor lasers incorporate the effect of spontaneous emission through a parameter R_{sp} that represents the rate of spontaneous emission into the laser mode. If N represents the number of electrons inside the laser cavity, R_{sp} is related to N as[8]

$$R_{sp} = \beta_{sp} B N^2 / V, \quad (1)$$

where V is the cavity volume, B is the bimolecular recombination coefficient ($B \sim 10^{-10}\ \mathrm{cm}^3/\mathrm{s}$), and β_{sp} is the spontaneous emission factor. For conventional semiconductor lasers β_{sp} is quite small (typically $\sim 10^{-5}$), because the laser cavity supports many cavity modes. However, the number of cavity modes supported by VCSE lasers is generally much smaller than that of conventional lasers. As a result, even though the total rate of spontaneous emission is about the same as in conventional lasers, the fraction into one cavity mode becomes enhanced ($\beta_{sp} \sim 10^{-3}$). The maximum value ($\beta_{sp} = 0.5$) can be approached in properly designed lasers.[6]

For simplicity of discussion, we focus on the case in which the density of photons and charge carriers is nearly uniform inside the active region. The conventional rate equations can then be used to model VCSE lasers. If P and N represent, respectively, the number of photons and electrons inside the laser cavity, the rate equations are given by[8]

$$\dot{P} = (G - \gamma) P + R_{sp} + F_P(t), \quad (2)$$

$$\dot{N} = I/q - \gamma_e N - GP + F_N(t), \quad (3)$$

$$\dot{\phi} = \tfrac{1}{2}\alpha (G - \gamma) + F_\phi(t), \quad (4)$$

where ϕ is the optical phase, I is the injection current, α is the linewidth enhancement factor, $\tau_e = \gamma_e^{-1}$ is the carrier lifetime, and $\tau_P = \gamma^{-1}$ is the photon lifetime. The gain G and the cavity loss rate γ are defined by the usual relations

$$G = \Gamma v_g (a/V)(N - N_0), \quad \gamma = v_g(\alpha_{mir} + \alpha_{int}), \quad (5)$$

where Γ is the confinement factor, v_g is the group velocity, a is the gain coefficient, N_0 is the transparency value of N, α_{int} is the internal loss, and the mirror loss $\alpha_{mir} = -\ln(R_1 R_2)/2L$. The mirror reflectivities R_1 and R_2 are typically 98% or more for VCSE lasers whereas the cavity length $L \sim 1\ \mu$m, resulting in $\alpha_{mir} \sim 100\ \mathrm{cm}^{-1}$. The cavity length L is defined such that it includes the effective penetration depth of optical field into distributed Bragg reflectors. The Langevin noise sources $F_P(t)$, $F_N(t)$, and $F_\phi(t)$ in Eqs. (2)–(4) are random processes with zero mean and a delta-correlated correlation function

$$\langle F_i(t) F_j(t') \rangle = 2 D_{ij} \delta(t - t'), \quad (6)$$

where i and j equal P, N, or ϕ. The diffusion coefficients depend on the rate of spontaneous emission[8] R_{sp} and are considerably enhanced in VCSE lasers.

The average values of P and N under cw operation are obtained by neglecting the Langevin noise sources and setting the time derivatives to zero in Eqs. (2)–(4). The main effect of enhanced spontaneous emission is that the carrier number N does not clamp but keeps increasing with an increase in P. This behavior is related to the soft turn-on of VCSE lasers, because the transition from spontaneous to stimulated emission is less abrupt when R_{sp} increases.[7] From the standpoint of laser noise, R_{sp} cannot be treated as constant, an assumption commonly made in the case of semiconductor lasers.[8] The diffusion constants in Eq. (6) are also affected by an increase in R_{sp} with increasing laser power.

The noise characteristics of VCSE lasers are obtained by linearizing Eqs. (2)–(4) in terms of small fluctuations around their average values and solving the resulting equations in the frequency domain. Since the entire procedure is well known,[8] we write the results directly. The intensity and phase noise are characterized by the relative-intensity-noise (RIN) spectrum $S_p(\omega)$ and the frequency-noise spectrum $S_f(\omega)$, respectively. Their general expressions are

$$S_P(\omega)=\frac{2R_{sp}}{P}\frac{\Gamma_N^2+\omega^2+(G_NP+2R_{sp}/N)[(G_NP+2R_{sp}/N)(1+\gamma_eN/R_{sp}P)-2\Gamma_N]}{[(\Omega_R-\omega)^2+\Gamma_R^2][(\Omega_R+\omega)^2+\Gamma_R^2]}, \quad (7)$$

$$S_f(\omega)=\frac{2R_{sp}}{P}\left(1+(\alpha G_NP)^2\frac{G'^2+2\Gamma_PG'+(\Gamma_P^2+\omega^2)(1+\gamma_eN/R_{sp}P)}{[(\Omega_R-\omega)^2+\Gamma_R^2][(\Omega_R+\omega)^2+\Gamma_R^2]}\right), \quad (8)$$

where $G_N = \Gamma v_g a/V$, $G' = G(1 - 2\epsilon_{NL}P)$ and Ω_R, Γ_R, Γ_P, and Γ_N are given by

$$\Omega_R=[G'(G_NP+2R_{sp}/N)-\tfrac{1}{4}(\Gamma_N-\Gamma_P)^2]^{1/2}, \quad (9)$$

$$\Gamma_R=(\Gamma_N+\Gamma_P)/2, \quad \Gamma_P=R_{sp}/P+\epsilon_{NL}GP, \quad (10)$$

$$\Gamma_N=\gamma_e+N(\partial\gamma_e/\partial N)+G_NP(1-\epsilon_{NL}P). \quad (11)$$

The parameters Ω_R and Γ_R are the frequency and the damping rate of relaxation oscillations. For conventional lasers $\Omega_R/2\pi \sim$ 2–3 GHz and $\Gamma_R \sim 2$ GHz at a power level of a few milliwatts. The output power P_{out} is related to the photon number P by the relation $P_{out} = h\nu v_g\alpha_{mir}P/2$; typically $P\approx 10^4$ at 1 mW. The effect of nonlinear changes in gain is included by replacing G by $G(1 - \epsilon_{NL}P)$, where the nonlinear gain parameter is chosen $\epsilon_{NL} = 1 \times 10^{-7}$. The nonlinear-gain effects are negligible on the results presented here. To study the noise characteristics we consider a VCSE laser with cylindrical cavity of 1 μm length and 6 μm radius. The other parameters are $R_1=R_2=0.99$, $a=2.5\times 10^{-16}$ cm^2, $\alpha_{int} = 50$ cm^{-1}, $\Gamma=0.4$, $B=1\times 10^{-10}$ cm^3/s, and $\alpha=5$.

Consider first the RIN spectrum. Figure 1 shows the RIN spectra at a power level of 3 mW in the frequency range 0.1–50 GHz for three values of the spontaneous emission factor β_{sp}. The curve for $\beta_{sp} = 5 \times 10^{-5}$ corresponds to the case in which spontaneous emission is about the same as in conventional edge-emitting lasers. For a larger value $\beta_{sp} = 5 \times 10^{-3}$, the RIN is enhanced by about 20 dB and the relaxation-oscillation peak is broadened because of an enhancement of Γ_P as evident from Eq. (10). Finally, for the largest value $\beta_{sp} = 0.5$ relaxation oscillations are so heavily damped that the peak disappears altogether. Indeed, Γ_P becomes so large for this value of β_{sp} that Ω_R^2 becomes negative in Eq. (9); the approach to steady state is nonoscillatory under these conditions.

Figure 2 shows the frequency-noise spectra under conditions identical to those of Fig. 1. The curve for $\beta_{sp} = 5 \times 10^{-5}$ is similar to the case of edge-emitting lasers. However, the low-frequency noise is considerably enhanced compared with edge-emitting lasers even for $\beta_{sp} = 5 \times 10^{-5}$. The enhancement is solely due to a smaller cavity length of VCSE lasers. The low-frequency value is a measure of the linewidth of the single longitudinal mode[8] and suggests that the linewidth of VCSE lasers is considerably enhanced even for small values of β_{sp}. The spectral density of frequency noise does not remain constant in the frequency range 0.1–1 GHz for such large values of β_{sp}. This feature is again related to suppression of relaxation oscillations.

The results shown in Figs. 1 and 2 are obtained by assuming the same cavity volume for the three values of β_{sp}. In practice, the transverse dimensions of the cylindrical cavity play an important role[6] in controlling β_{sp}. In particular $\beta_{sp} = 0.5$ can be realized by reducing the cylinder

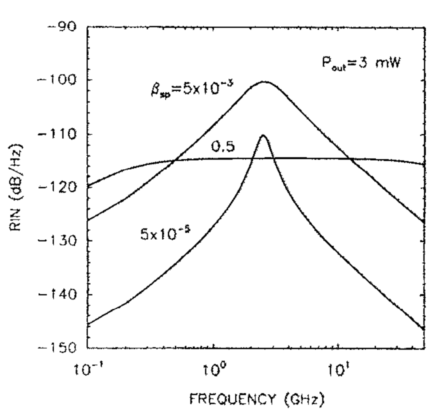

FIG. 1. Intensity-noise spectra of VCSE lasers at 3 mW output power for several values of the spontaneous-emission factor β_{sp}.

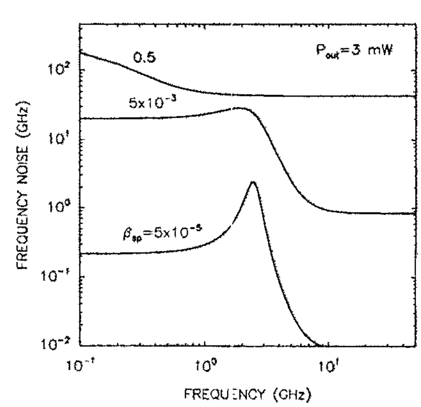

FIG. 2. Frequency-noise spectra of VCSE lasers under the same conditions as of Fig. 1.

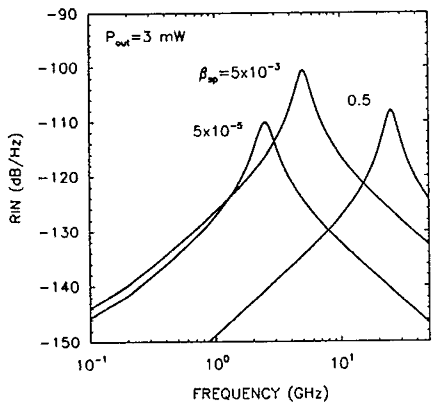

FIG. 3. Intensity-noise spectra under the same conditions as of Fig. 1 except for the radius r of the cylindrical cavity associated with VCSE lasers. Specifically, $r = 6$ μm for $\beta_{sp} = 5 \times 10^{-5}$, $r = 3$ μm for $\beta_{sp} = 5 \times 10^{-3}$, and $r = 0.6$ μm for $\beta_{sp} = 0.5$.

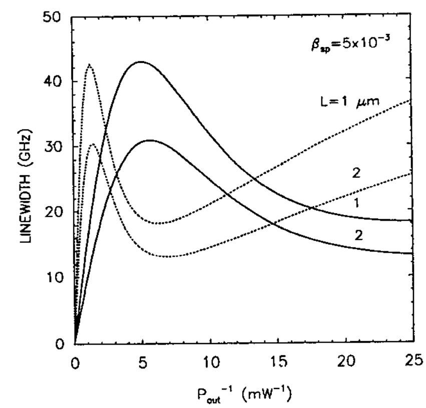

FIG. 4. Power dependence of the laser linewidth for $\beta_{sp} = 5 \times 10^{-3}$ and for cavity lengths 1 and 2 μm. Solid and dashed curves correspond to VCSE lasers of different transverse dimensions $r = 3$ and 6 μm, respectively.

radius r to a value of about 0.6 μm.[6] It is thus important to study how the noise characteristics change if r is decreases as β_{sp} increases. Figure 3 shows the RIN spectra under conditions identical to those of Fig. 1 but with different transverse dimensions. Specifically, $r = 6$, 3, and 0.6 μm for $\beta_{sp} = 5 \times 10^{-5}$, 5×10^{-3}, and 0.5, respectively.

In contrast with Fig. 1, relaxation oscillations are not suppressed even for the largest value of β_{sp}. In fact, the qualitative features of the RIN spectrum are identical for all values of β_{sp} except for a shift of the relaxation-oscillation frequency which exceeds 20 GHz for $\beta_{sp} = 0.5$. The frequency-noise spectra of Fig. 2 also change in the same way when β_{sp} and r are changed simultaneously. The physical reason for the qualitative changes seen in Figs. 1 and 3 is related to the fact that the RIN spectra are being compared at the same output power for two VCSE lasers of different cavity volumes. The output power of 3 mW would be realized at a much higher bias level for the laser of a smaller volume. Since the relaxation-oscillation frequency increases with an increase in the bias level, one would expect changes seen in Figs. 1 and 3 when the comparison is made at the same output power. Results similar to Fig. 1 are obtained at lower output powers.

For conventional edge-emitting semiconductor lasers the laser linewidth $\Delta\nu$ decreases with an increase in the output power as P_{out}^{-1}. Because R_{sp} changes with an increase in the output power for VCSE lasers, such a behavior would not generally hold. Since the linewidth is related to the zero-frequency value of the frequency noise by $\Delta\nu = S_f(0)/2\pi$, we can use Eq. (8) to obtain an analytic expression.

Figure 4 shows the variation of $\Delta\nu$ with the output power for VCSE lasers of different cavity lengths ($L = 1$ and 2 μm) and different transverse dimensions ($r = 3$ and 6 μm). The spontaneous-emission factor $\beta_{sp} = 5 \times 10^{-3}$ in all cases. As the output increases, $\Delta\nu$ first decreases, goes through a minimum, increases to reach a peak value, and then falls again. One can understand this behavior from Eq. (8). At very low output powers ($P_{out} < 0.1$ mW), the damping rate $\Gamma_R \approx R_{sp}/2P$ is dominated by spontaneous emission. The laser is effectively below threshold, and the α^2 term in Eq. (8) does not contribute to the linewidth. As the output power increases, the α^2 term begins to contribute, and the linewidth increases. Finally, the α^2 term saturates (for $P_{out} \sim 1$ mW), and the linewidth decreases following the usual P^{-1} dependence. The minima and maxima seen in Fig. 4 occur at relatively low power levels ($P_{out} < 1$ mW) and reflect the effect of enhanced spontaneous emission on the linewidth of VCSE lasers. These features have not yet been observed experimentally. Note that the linewidth of VCSE lasers (typically $\sim$10 GHz) is larger by more than two orders of magnitude compared with edge-emitting lasers because of enhanced spontaneous emission.

In conclusion, we have studied the noise characteristics of VCSE lasers by using the modified Langevin rate equations. The effect of enhanced spontaneous emission is included by varying the spontaneous-emission factor β_{sp} from 5×10^{-5} to 0.5. Particular attention is paid to the laser linewidth. It is enhanced considerably as a result of enhanced spontaneous emission. Furthermore, the power dependence of the linewidth exhibits new features related to the soft turn-on of VCSE lasers. These features have not yet been observed experimentally.

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