Transient Multimode Dynamics in Nearly Single-Mode Lasers

Antonio Mecozzi, A. Sapia, Paolo Spano, and Govind P. Agrawal, *Senior Member, IEEE*

Abstract—**The transient dynamics of nearly single-mode semiconductor lasers is studied analytically and numerically for lasers biased below threshold. The side-mode excitation probability is evaluated by solving a Fokker–Planck equation approximately. The validity of the approximate solution is verified through Monte Carlo simulations of the corresponding Langevin equations. The results show the relevance of the carrier-density overshoot during laser turn-on in determining the side-mode excitation probability. They also indicate the dependence of this probability on various device parameters such as the gain margin between the main and side modes. The experiments performed by using distributed feedback (DFB) semiconductor lasers show qualitative agreement with theory. The experiments also suggest that the inhomogeneous nature of the carrier-density distribution may be important for understanding the transient multimode dynamics in DFB lasers.**

Introduction

Fluctuations of physical quantities usually represent only small corrections to a well-defined deterministic evolution. There are few exceptions to this general behavior of physical systems: one of them is the decay from an unstable equilibrium point [1]. A material point on the top of a hill, for example, leaves its initial unstable position due to the effect of ever-present fluctuations driving the system towards a new stable equilibrium state. The effect of these fluctuations is twofold. First, the time the system needs to attain the new equilibrium state is a random quantity. The second effect of the fluctuations is to randomize the direction of departure from the unstable state. The system, in fact, will leave the initial state preferably along a direction very close to that of maximum slope but, due to initial fluctuations, sometimes the direction can be very far from this one. The statistical properties of the first passage time [2], that is, the time the system needs to reach for the first time a given distance from the initial unstable point, have been extensively studied in the literature [3]–[5], and are used to explain many physical phenomena. In spite of its importance, there are, to the best of our knowledge, no studies on the effects of the fluctuations in determining the initial direction of leaving from an unstable state.

It has been generally recognized that the Q switching and the gain switching of lasers can be usefully studied in the general framework of the decay from an unstable state [6]–[10]. In these systems, in fact, the effect of spontaneous-emission noise is to drive the system from an initial state in which the cavity is almost empty of photons towards a final state in which coherent radiation is established inside the cavity. Here the effect of the spontaneous emission is to randomize the delay between the buildup of the laser radiation and the instant of the switching of the cavity in Q-switched lasers, or the attainment of threshold in gain-switched lasers. The second effect of the spontaneous-emission noise arises in lasers in which more than one mode is present. Even though the laser usually starts to oscillate in the main mode, i.e., the mode having the minimum cavity loss, occasionally the spontaneous-emission noise can trigger the laser to oscillate on a side mode as well.

In laser systems, the equations for the fields contain an initial unstable state corresponding to a mean value of the field equal to zero in each mode. This instability is not clearly shown in the equations for the photon densities, due to the presence of the mean spontaneous-emission rate. Finding the probability that the lasing action starts on the main mode is equivalent to finding, in the multidimensional phase space of the system, the direction of departure from this unstable point.

The evaluation of the probability that a side mode is excited transiently in a nearly single-mode laser is also an important task from the point of view of the applications. In fact, mode-partition noise greatly affects the bit error rate (BER) and hence the performance of semiconductor lasers when they are used in optical transmission systems, particularly when dispersion shifted fibers are not employed. It has been recently shown [11] how random excitations of a side mode during transients can have a detrimental effect on the BER even in distributed feedback (DFB) lasers having a side-mode suppression ratio larger than 30 dB in stationary conditions. The problem of the excitation of a side mode during the transient operation has been initially studied in many papers using the rate equation approach by considering only mean evolution of the spectrum during the transient operation without considering fluctuations [12]–[14]. From the above considerations, one can understand that this is a rough approximation of the process because the stochastic nature of the emission can produce effects not accounted for in that approach. The importance of the spontaneous-emission noise in this context has been already pointed out by using numerical simulations by Jensen *et al.* [15], Liu and Choy [16], [17], and Miller [18], [19].

In this paper we develop an analytical theory of the dynamic evolution of the spectrum of semiconductor lasers having high side-mode suppression in stationary conditions, namely DFB and distributed Bragg reflector (DBR) lasers. This theory, valid only for bias levels below threshold due to the involved approximations, shows the relevance of the overshoot of the carrier density in determining the transient excitation of a secondary mode.

Manuscript received March 16, 1990; revised November 2, 1990. The work performed at the Fondazione Ugo Bordoni was carried out in the framework of the agreement between the Fondazione Ugo Bordoni and the Italian Post and Communication Administration. The work performed at the University of Rochester was supported in part by the U.S. Army Research Office.

A. Mecozzi, A. Sapia, and P. Spano are with the Fondazione Ugo Bordoni, 00142 Rome, Italy.

G. P. Agrawal is with the Institute of Optics, University of Rochester, Rochester, NY 14627.

IEEE Log Number 9142989.

0018-9197/91/0300-0332$01.00

General Theory

The dynamics of a semiconductor laser can be theoretically modeled by using the well-known rate equations for the density of minority carriers and for the density of photons in the cavity modes [20]:

$$\frac{dn}{dt} = C - \frac{n}{\tau_{sp}} - \frac{c}{\eta n_g}\sum_\nu g_\nu S_\nu + \left(\frac{2n}{\eta\tau_{sp}}\right)^{1/2} F_n(t) - \frac{1}{\eta}\sum_\nu\left(\frac{2\gamma}{\tau_{sp}}D_\nu S_\nu n\right)^{1/2} F_\nu(t) \quad (1)$$

$$\frac{dS_\nu}{dt} = \frac{\gamma}{\tau_{sp}}D_\nu n + \frac{c}{n_g}(g_\nu - \alpha_\nu)S_\nu + \left(\frac{2\gamma}{\tau_{sp}}D_\nu S_\nu n\right)^{1/2} F_\nu(t) \quad (2)$$

with

$$g_\nu = \eta\frac{n_g}{c}A(D_\nu n - n_0)\Big/\left(1 + \sum_\nu S_\nu/\chi\right). \quad (3)$$

The meaning of the symbols, the same used in [15] and [16], is summarized in Table I. $F_\nu(t)$ are uncorrelated Gaussian white noise terms introduced in order to take into account fluctuations [21]–[23]. They have the following correlation functions:

$$\langle F_\nu(t)\rangle = 0, \quad \langle F_\nu(t)\,F_\mu(t')\rangle = \delta_{\nu\mu}\delta(t - t'). \quad (4)$$

Equations (1) and (2) are a set of stochastic differential equations in the Ito sense (see [2]).

We limit our analysis to the case of a two-mode laser (ν = 1, 2). This is justified for the nearly single-mode DFB laser, where only one side mode can have an appreciable probability to be excited during the transient. The aim of this paper is to analyze the evolution of the field inside the cavity, and consequently the evolution of the output power, of a semiconductor laser in which the injected current is suddenly switched from a value C_0 below to a value C above the threshold current. The evolution of photon densities during the switching can be characterized by the probability density function (PDF) $P(S_1, S_2, n; t)$, defined as the probability per unit volume to find the system at time t around the state defined by S_1, S_2, n. The time dependence of $P(S_1, S_2, n; t)$ can be obtained by solving a Fokker–Planck equation (FPE) obtained from the Langevin equations (1) and (2) by using the well-known techniques [2].

Let us suppose that at $t = 0$ the injected current of the laser is suddenly switched from C_0 to a value C above the threshold current C_{th}. As a consequence, $P(S_1, S_2, n; t)$ evolves from a given initial distribution according to the following FPE [2]:

$$\frac{\partial P(S_1, S_2, n; t)}{\partial t} = -\tilde{\nabla} J(S_1, S_2, n; t) \quad (5)$$

where the tilde indicates the transpose, $J(S_1, S_2, n; t)$ is the probability current defined as

$$\tilde{J}(S_1, S_2, n; t) = [\tilde{A}(S_1, S_2, n) - \tfrac{1}{2}\tilde{\nabla}B(S_1, S_2, n)]P(S_1, S_2, n; t) \quad (6)$$

and

$$\nabla = \begin{bmatrix} \dfrac{\partial}{\partial S_1} \\ \dfrac{\partial}{\partial S_2} \\ \dfrac{\partial}{\partial n} \end{bmatrix} \quad (7)$$

TABLE I
Meaning of the Symbols Used in the Paper

S_ν	density of photons in the νth mode
n	carrier density
η	mode confinement factor
d	thickness of the active layer
W	width of the active layer
D_ν	line shape factor
τ_{sp}	spontaneous lifetime
n_g	group index
γ	fraction of spontaneous emission coupled into the mode
A	differential gain
n_0	carrier density at transparency
α_1	loss of the main mode
α_2	loss of the side mode
J	injection carrier density
e	electron charge
c	speed of light in vacuum
$C = J/ed$	injection current of the laser in electron/s.
χ	saturation photon density
$\Delta g = c(\alpha_2 - \alpha_1)/n_g$	gain difference between the modes in s^{-1}
g_ν	net gain of the νth mode

$$A = \begin{bmatrix} \dfrac{\gamma}{\tau_{sp}}D_1 n + \dfrac{c}{n_g}(g_1 - \alpha_1)S_1 \\ \dfrac{\gamma}{\tau_{sp}}D_2 n + \dfrac{c}{n_g}(g_2 - \alpha_2)S_2 \\ C - \dfrac{n}{\tau_{sp}} - \dfrac{c}{\eta n_g}\sum_\nu g_\nu S_\nu \end{bmatrix} \quad (8)$$

$$B = \begin{bmatrix} \dfrac{2\gamma}{\tau_{sp}}D_1 n S_1 & 0 & -\dfrac{2\gamma}{\tau_{sp}}D_1 n S_1 \\ 0 & \dfrac{2\gamma}{\tau_{sp}}D_2 n S_2 & -\dfrac{2\gamma}{\tau_{sp}}D_2 n S_2 \\ -\dfrac{2\gamma}{\tau_{sp}}D_1 n S_1 & -\dfrac{2\gamma}{\tau_{sp}}D_2 n S_2 & \dfrac{n}{\tau_{sp}} + \sum_\nu \dfrac{2\gamma}{\tau_{sp}}D_\nu n S_\nu \end{bmatrix}. \quad (9)$$

In order to characterize the statistics of the power partition between the two modes during the switching of the laser radiation, let us define the PDF $\Phi(S_1)$ $(0 \le S_1 \le S_T)$ in the following way. $\Phi(S_1)\,dS_1$ is the probability that, when the total photon density is for the first time $S_1 + S_2 = S_T$, the main-mode photon density is between S_1 and $S_1 + dS_1$. $\Phi(S_1)$ does not depend on the evolution of the system when the total photon density becomes larger than S_T, i.e., outside the domain $D = \{0 \le S_1 \le S_T, 0 \le S_2 \le S_T - S_1, 0 \le n < \infty\}$. Moreover, since we are interested in the first exit from D, we have to be careful to avoid the system crossing again the boundary of D once it has already exited. From the point of view of the stochastic process, this is equivalent to removing the points representing the process when it has reached for the first time any point of the surface $\Sigma = \{S_2 = S_T - S_1, 0 \le n < \infty\}$, i.e., when the total photon density of the laser $S_1 + S_2$ has reached for the first time the value S_T. The same constraint can be given to the FPE by imposing the so called absorbing boundary condition on D, corresponding to imposing $P(S_1, S_T - S_1, n; t) = 0$ for any t.

Let us suppose now that $P(S_1, S_2, n; t)$ is the solution of (5)–(9), corresponding to the initial distribution $P(S_1, S_2, n; 0)$, obtained by imposing the absorbing boundary conditions on the domain D; let $J(S_1, S_2, n; t)$ be the probability current obtained

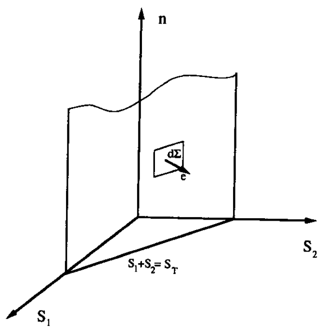


Fig. 1. Surface $d\Sigma$ on the boundary of the domain D (see text).

with this PDF through (6). The probability that the system exits for the first time through an element $d\Sigma$ of the surface Σ is (see Fig. 1)

$$\Pr(d\Sigma) = \int_0^\infty dt\, \bar{J}(S_1, S_T - S_1, n; t)e\, d\Sigma \tag{10}$$

where

$$e = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} \tag{11}$$

is the unit vector perpendicular to the surface Σ. We are interested in the probability of the first exit through Σ around a given value S_1 of the intensity of the main mode, whatever the value of n is. This probability can be obtained by writing $d\Sigma = dn\, d\lambda$ and integrating over all the possible values of n, and is given by

$$\Pr(d\lambda) = \int_0^\infty dt \int_0^\infty dn\, \bar{J}(S_1, S_T - S_1, n; t)e\, d\lambda$$
$$= \Phi(S_1)\, dS_1. \tag{12}$$

Taking into account that $d\lambda = \sqrt{2}\, dS_1$, we finally obtain

$$\Phi(S_1) = \sqrt{2} \int_0^\infty dt \int_0^\infty dn\, \bar{J}(S_1, S_T - S_1, n; t)e. \tag{13}$$

The exact expression for $\Phi(S_1)$ can be found only resorting to a numerical solution of the FPE. Nevertheless, an analytical expression can be found once some reasonable approximations are made. The first approximation is to neglect the Langevin noise terms $F_n(t)$ and $F_\nu(t)$ in the rate equation for the carriers. This is reasonable because the Langevin noise term on the carriers gives rise only to small fluctuations in the gain. These fluctuations have small effects during the transient compared with the fluctuations induced by the Langevin noise term on the photons, as they are amplified by the stimulated-emission process. The second approximation consists of neglecting the stimulated-emission term in (1). This is reasonable, because there is practically no lasing emission until the carrier density rises to a level well beyond its threshold value, and because we are interested in the probability of excitation of the side mode before stimulated emission depletes the population inversion. This approximation, however, limits our analysis to the bias levels below threshold. Its advantage is that it decouples (1) from the set of (2), so that they become

$$\frac{dn}{dt} = C - \frac{n}{\tau_{sp}} \tag{14}$$

$$\frac{dS_\nu}{dt} = rn + g_\nu(n)S_\nu + \sqrt{2rnS_\nu}F_\nu(t) \tag{15}$$

where

$$g_\nu(n) = a(n - n_{th}) - \Delta g_\nu \tag{16}$$

$$\Delta g_1 = 0 \qquad \Delta g_2 = \frac{c}{n_g}(\alpha_2 - \alpha_1), \qquad \alpha_2 > \alpha_1 \tag{17}$$

$$r = \frac{1}{2}\frac{\gamma}{\tau_{sp}} \qquad a = \frac{1}{2}\eta A \tag{18}$$

$$n_{th} = 2\,\frac{\eta A n_g n_0 + c\alpha_1}{\eta A n_g} \tag{19}$$

and we have supposed $D_1 = D_2 = 1/2$, a valid assumption when the frequency difference between the two modes is much less than the spontaneous-emission linewidth. This simplified expression for (1)–(2) is valid below threshold and during the first stage of the exponential growing of the photon density. Initially, the laser is below threshold, and its initial distribution can be written as

$$P(S_1, S_2, n; t = 0) = p_1(S_1; 0)\, p_2(S_2; 0)\, \delta[n - n(0)] \tag{20}$$

where $p_\nu(S_\nu; 0)$ is a negative exponential distribution

$$p_\nu(S_\nu; 0) = \frac{1}{\langle S_\nu(0)\rangle} \exp\left[-\frac{S_\nu}{\langle S_\nu(0)\rangle}\right]. \tag{21}$$

Furthermore, $n(0)$ and $\langle S_\nu(0)\rangle$ are the initial values of the carrier density and of the mean photon density, which read

$$n(0) = C_0\tau_{sp} \tag{22}$$

$$\langle S_\nu(0)\rangle = \frac{rC_0}{a}\left(C_{th} - C_0 + \frac{\Delta g_\nu}{a\tau_{sp}}\right)^{-1} \tag{23}$$

where C_{th} is the threshold current of the laser and is given by

$$C_{th} = \frac{n_{th}}{\tau_{sp}}. \tag{24}$$

The mean photon density $\langle S_2(0)\rangle$ for the side mode is smaller than the main mode because the parameter Δg_2, known as the gain margin, is not zero for lasers such as DFB and DBR lasers. This initial distribution corresponds to two modes which fluctuate independently from each other and are at equilibrium with a given value of the carrier density, determined by the initial value C_0 of the injected current.

Due to the initial δ-like carrier density distribution given by (20), the solution of (14) can be obtained as

$$n(t) = C\tau_{sp}\left[1 - \exp\left(-\frac{t}{\tau_{sp}}\right)\right] + C_0\tau_{sp}\exp\left(-\frac{t}{\tau_{sp}}\right) \tag{25}$$

and the result inserted into (15). As a consequence of our approximation, the stochastic differential equations for the two modes are decoupled, and the PDF of the system still reads

$$P(S_1, S_2, n; t) = p_1(S_1; t)\, p_2(S_2; t)\, \delta[n - n(t)] \tag{26}$$

where $p_\nu(S_\nu; t)$ is the solution of a one-dimensional FPE given by

$$\frac{\partial p_\nu(S_\nu; t)}{\partial t} = -\frac{\partial}{\partial S_\nu}[rn(t) + g_\nu[n(t)]S_\nu]p_\nu(S_\nu; t) + \frac{1}{2}\frac{\partial^2}{\partial S_\nu^2}[2rn(t)S_\nu]p_\nu(S_\nu; t). \quad (27)$$

The solution of (27) with the initial condition (21) is

$$p_\nu(S_\nu; t) = \frac{1}{m_\nu(t)}\exp\left[-\frac{S_\nu}{m_\nu(t)}\right] \quad (28)$$

where $m_\nu(t)$, the mean value of the photon density for the mode ν, can be analytically obtained, once some reasonable approximations are made (see Appendix A).

The solution we have obtained obeys natural boundary conditions, i.e., the PDF vanishes at infinity. As shown in a previous paper ([9]), this solution is a good approximation inside the domain D of that obtained for absorbing boundary conditions on D.

Substituting (28) into (13) one obtains (see Appendix B):

$$\Phi(S_1) = \int_0^\infty \frac{dt}{m_1(t)\,m_2(t)}\left[\frac{\dot m_1(t)}{m_1(t)}S_1 + \frac{\dot m_2(t)}{m_2(t)}(S_T - S_1)\right] \times \exp\left[-\frac{S_1}{m_1(t)} - \frac{S_T - S_1}{m_2(t)}\right] \quad (29)$$

where the dots stand for time derivatives.

The PDF $\Phi(S_1)$ is defined for $0 \le S_1 \le S_T$, and hence, it is of the order of S_T^{-1}. In order to define a PDF which is of the order of 1, it is convenient to define a scaled intensity as $s_1 = S_1/S_T$, so that the correspondent PDF is

$$\phi(s_1) = \Phi(s_1 S_T)S_T. \quad (30)$$

This PDF will be very useful in the following to provide us an analytical expression for the probability of excitation of the side mode.

Some comments on the particular expression of the PDF reported in (26) are in order. The decoupling between the PDF of the main mode and that of the side mode in (26), which is essential in our derivation of (29) and (30), could be hard to be accepted. It is generally assumed, in fact, that main and side modes are strongly anticorrelated, even during the transient (see [11]). This anticorrelation, however, is induced by the depletion of the carrier density due to the stimulated emission, which starts to be effective when the total intensity attains a value of the order of its final stationary value. For these reasons, we do not choose to follow the laser dynamics in the time domain, but we consider the laser evolution until the total value of the intensity does not overcome a small fixed value of the intensity S_T, when the mode correlation can be neglected. This approach cannot be followed when the initial biasing is above threshold (see [18]).

Q-Switching Conditions

The integral in (29) cannot be performed analytically. In order to obtain some physical insight into the transient multimode dynamics, let us consider the most simple situation of a class *A* [24] *Q*-switched laser. In these kind of lasers, the population inversion can be adiabatically eliminated, so that the well-known van der Pol equations hold:

$$\frac{dS_1}{dt} = R_1 + (g_1 - g_{11}S_1 - g_{12}S_2)S_1 + \sqrt{2R_1S_1}F_1(t) \quad (31)$$

$$\frac{dS_2}{dt} = R_2 + (g_2 - g_{22}S_2 - g_{21}S_1)S_2 + \sqrt{2R_2S_2}F_2(t) \quad (32)$$

where the Langevin noise terms have the properties given by (4). Further, $g_\nu(\nu = 1, 2)$ are the net gains of the two modes, $g_\nu = g_0 - \alpha_\nu$, where g_0 is the gain and α_ν are the losses of the two modes defined as in (16)–(17) ($\alpha_1 < \alpha_2$), while $g_{\nu\mu}$ are saturation coefficients. Let us suppose now that at $t = 0$ the laser gain g_0 is switched from a value below α_1 to a value above α_1 and α_2. Consequently, before saturation occurs the laser radiation starts growing, obeying to a linearized version of (31) and (32):

$$\frac{dS_1}{dt} = R_1 + g_1S_1 + \sqrt{2R_1S_1}F_1(t) \quad (33)$$

$$\frac{dS_2}{dt} = R_2 + g_2S_2 + \sqrt{2R_2S_2}F_2(t). \quad (34)$$

We suppose the two modes have gains of the same order ($g_1 \gtrsim g_2$) in order to have a nonvanishing probability of excitation of the side mode. Proceeding as above, we obtain an equation equal to (29), where now $m_\nu(t)$ are the mean values of the photon densities during the linearized evolution which can be obtained performing a mean on (33)–(34). The final result is

$$m_\nu(t) = \langle S_\nu(t)\rangle = \frac{R_\nu}{g_\nu}(e^{g_\nu t} - 1) + \langle S_\nu(0)\rangle e^{g_\nu t}. \quad (35)$$

For the sake of simplicity, we consider in the following, the particular case in which the mean photon number is initially zero in each of the two modes ($\langle S_\nu(0)\rangle = 0$). In order to have a rough estimate of the probability that the mode 2, the mode having the lower gain, is excited, let us consider $\phi(s_1 = 0)$. It has to be noted that imposing $s_1 = 0$ corresponds to consider the situation in which no excitation of the main mode is present when the total intensity attains the value S_T. From (29) and (30) we have

$$\phi(0) = \int_0^\infty dt\,\frac{S_T^2}{m_1(t)}\frac{\dot m_2(t)}{m_2(t)^2}\exp\left[-\frac{S_T}{m_2(t)}\right]. \quad (36)$$

Substituting (35) into (36) we obtain, after some algebra (see Appendix C)

$$\phi(0) = \frac{g_1}{g_2}\frac{R_2}{R_1}\left(\frac{R_2}{S_Tg_2}\right)^{\Delta g/g_2}\Gamma\left(\frac{g_1}{g_2} + 1\right) \quad (37)$$

where we have defined $\Delta g = g_1 - g_2$ and Γ is the Euler gamma function [25].

Let us now discuss the meaning of (37). If $\Delta g = 0$, $\phi(s_1)$ is a function only of s_1 and of the ratio R_2/R_1 of the spontaneous emission in the two modes, as can be proved by solving (29). In particular, $\phi(0)$ is equal to R_2/R_1, which means that, when the net gain of the two modes is equal, the probability of exciting only the secondary mode is proportional to the spontaneous-emission rate in that mode. When $\Delta g > 0$, (37) gives a fast exponential decreasing of $\phi(0)$ when Δg increases, because $R_2/S_Tg_2 \ll 1$ in (36) when S_T is of the order of the final stationary value (see Appendix C). Moreover, for a fixed value of

Δg, $\phi(0)$ is an increasing function of g_2, as the relevant quantity is not just Δg but the ratio $\Delta g/g_2$. This property has been obtained for a laser in which the gain is kept constant, but it extends also to the case of gain switching. In this last case, which corresponds to semiconductor lasers, g_2 has to be considered as the value of the gain of the secondary mode at the switching time of the laser radiation. As a consequence, in nearly single-mode semiconductor lasers, a relevant parameter is not only the value of Δg, but also the level the carrier density attained during the switching of the laser radiation: carrier density is proportional to the gain of the two modes at the same instant. The overshoot of the carrier density is an increasing function of the value of the injected current and the differential gain, so that the larger these two quantities are, the larger the probability of excitation of the side mode is.

Results and Discussion

In this section we present the results of the linearized theory together with those obtained by numerical simulations of the complete set of stochastic differential equations (1)–(2). Even if the results of the simulations are not completely original (see [18] and [19]), their discussion can be useful for the comparison between the complete model and the approximate analytical theory. The simulations has been performed by using a second order Runge–Kutta method (Heun's method) [26] with an integration step of 1 ps. In our simulation, the current is supposed to change instantaneously from the bias to the *on* value. Before applying the current step, our computer program has been run for 1 ns with the current in its biasing value, to ensure that the initial distribution was the equilibrium one at the biasing state. Our simulation does not reproduce any practical modulation of a semiconductor laser used in optical communication systems, but they are useful to investigate the switching properties independently of the particular modulation scheme which is considered. In practical modulation schemes the probability of excitation of the side mode depends also on pattern effects (see [18]).

Before passing to the discussion of the obtained results, however, let us discuss briefly the meaning of the main quantity given by the theory, i.e., $\phi(s_1)$ [(30)], and its connection with the meaningful quantity from a practical point of view, that is the probability the secondary mode exceeds a given value of photon density. To this end, let us assume in the following discussion that $S_T = S_{st}/2$, S_{st} being the stationary value of the total photon density in the *on* state. The linear theory cannot be used to determine $\phi(s_1)$ if larger values of S_T are considered, because of the presence of gain saturation. Under these conditions, the quantity

$$\Pr(S_1 > \eta S_0) = \int_0^{1/(1+\eta)} \phi(s_1)\, ds_1 \qquad (38)$$

represents the probability that the side mode has a photon density S_1 larger than ηS_0 ($0 < \eta < \infty$) when the total photon density is S_T. The analytical expression of this probability is given in Appendix D.

The values of the physical parameters, which are common to all the figures, are the same as in [15] and are given in Table II. The plots of $\phi(s_1)$ obtained through numerical integrations of (29)–(30) are compared in Fig. 2 with the PDF's obtained by means of 40 000 realizations of the process. The two curves have been obtained for $\Delta g = 2.5$ cm^{-1}, an excitation current $C = 1.5C_{th}$, and a differential gain $A = \bar{A} = 5.62 \times 10^{-6}$ cm^3 s^{-1} (curve a) and $A = 2\bar{A}$ (curve b); a good agreement between theory (regular line) and simulations (wavy line) is present. The two curves have been obtained for $\chi/S_{st} = 20$, where χ is the saturation photon density; other simulations, performed with $\chi/S_{st} = 6.66$ and $\chi/S_{st} = 100$ while leaving unchanged the other parameters, show practically the same results, in agreement with the linear theory. Almost the same result (curve b of Fig. 2) is obtained for $A = \bar{A}$ and for $C = 2C_{th}$, because $\phi(s_1)$ strongly depends on A and $C - C_{th}$ only through their product $[A(C - C_{th})]$ (see Appendix A). Both theory and simulations show that $\phi(s_1)$ is independent of the initial biasing conditions up to values of the ratio C_0/C_{th} equal to 0.95 [27].

TABLE II
VALUES OF THE PARAMETERS USED IN OBTAINING THE RESULTS

Parameter	Value	Unit
η	0.5	
d	0.3	μm
W	2	μm
D_1	0.5	
D_2	0.5	
L	400	μm
τ_{sp}	2	ns
n_g	4	
γ	2.2×10^{-5}	
$\bar{A}$	5.62×10^{-6}	cm^3 s^{-1}
n_0	6.8×10^{17}	cm^{-3}
α_1	50.3	cm^{-1}

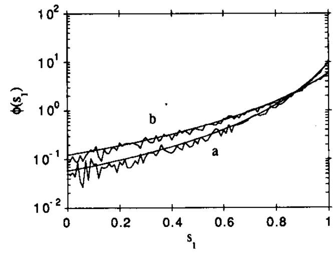


Fig. 2. Probability density function $\Phi(s_1)$ versus s_1 for $\chi/S_{st} = 20$, $\Delta g = 2.5$ cm^{-1}, $C = 1.5C_{th}$, $A = \bar{A} = 5.62 \times 10^{-6}$ cm^3 s^{-1} (curve a) and $A = 2\bar{A}$ (curve b).

In Fig. 3 $\phi(s_1)$ is shown for $A = \bar{A}$, for $C = 1.5C_{th}$ and for Δg equal to 2.5 cm^{-1} (curve a), 5 cm^{-1} (curve b), and 7.5 cm^{-1} (curve c). As expected, the larger the value of Δg, the lower the probability of excitation of the side mode is. It has to be noted that the probability of having a side mode larger than the main mode during the leading edge of the optical pulse is relatively large even for the largest value of Δg in Fig. 3. One should be careful to use such a laser in optical communication systems typically requiring a BER of $< 10^{-9}$ since each time the side mode exceeds the main-mode amplitude an error would be made. This happens in spite of the low value, at any time, of the mean photon density of the secondary mode with respect to that of the main mode, as can be seen in Fig. 4. Fig. 4 shows on a logarithmic scale the evolution of the mean values of the main and side mode photon density during the switch on, obtained by means of 20 000 realizations of the process in the same conditions of curve c of Fig. 3, together with the temporal evolution of the side-mode suppression ratio. The side-mode

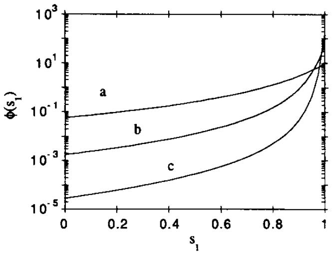


Fig. 3. Probability density function $\Phi(s_1)$ versus s_1 for $A = \overline{A}$, $C = 1.5C_{\rm th}$, $\Delta g = 2.5$ cm^{-1} (curve a), $\Delta g = 5$ cm^{-1} (curve b), $\Delta g = 7.5$ cm^{-1} (curve c).

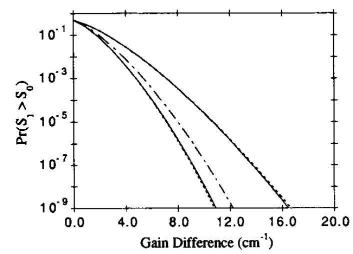


Fig. 5. Probability that $S_1 > S_0$ when $S_T = S_{\rm st}/2$ versus the gain difference Δg between the two modes. Upper solid line: $A = 2\overline{A}$; $C - C_{\rm th} = \overline{C} = 0.5C_{\rm th}$; upper dashed line: $A = \overline{A}$; $C - C_{\rm th} = 2\overline{C}$; dashed-dotted line: $A = \overline{A}$; $C - C_{\rm th} = \overline{C}$; lower solid line: $A = 0.75\overline{A}$; $C - C_{\rm th} = \overline{C}$; lower dashed line: $A = \overline{A}$; $C - C_{\rm th} = 0.75\overline{C}$.

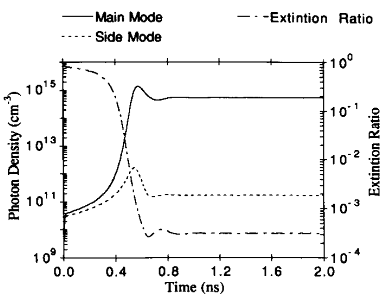


Fig. 4. Evolution of the average photon density of the main mode and the side mode under conditions of curve c in Fig. 3. SMSR is also shown as a function of time during switching.

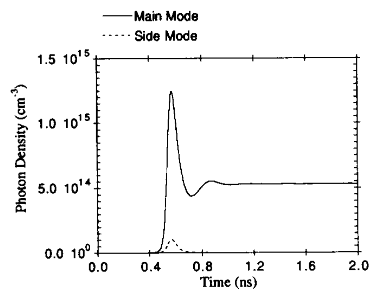


Fig. 6. Evolution of the mean photon densities of main and side modes for the case of curve a in Fig. 2.

suppression ratio is 35 dB in the stationary conditions corresponding to the *on* state of the laser, and 29.3 dB at the peak of the overshoot of the secondary mode, which seem to be good values for applications in the optical transmission system. The high probability of excitation of the secondary mode under this condition is also confirmed by the observation that, during the 20 000 realizations, the side mode exceeded once a level of 50% of the stationary value of the photon density of the main mode is in the *on* state.

Fig. 5 shows the probability that the side mode has a photon density larger than that of the main mode when the total photon density is one half the main mode photon density in the stationary state, as a function of the gain difference Δg between the two modes. The curves have been obtained through a numerical integration of (6) of Appendix D and show the dependence on Δg of the probability of excitation of the side mode. The dotted-dashed line corresponds to $A = \overline{A}$ and $C - C_{\rm th} = \overline{C} = 0.5C_{\rm th}$, while the upper and lower solid lines have been obtained for $A = 2\overline{A}$ and $A = 0.75\overline{A}$, respectively, (leaving unchanged the injected current). The upper and lower dashed lines corresponds to $C - C_{\rm th} = 2\overline{C}$ and $C - C_{\rm th} = 0.75\overline{C}$, and $A = \overline{A}$. As can be seen, there is a small difference between curves obtained for different values of A and $C - C_{\rm th}$ when $[A(C - C_{\rm th})]$ is constant. The dependence of the probability of excitation of the side mode upon the value of $[A(C - C_{\rm th})]$ can be explained by noting that, when this product increases, the overshoot of the carrier density and, consequently, the gain at the time corresponding to the leading edge of the optical pulse emission also increases. As a consequence, according to (37) which can be considered a rough estimate of the probability of the excitation of the side mode also for gain-switching conditions, this probability increases due to a decrease in the ratio $\Delta g/g_2$.

The high values of the probability of the excitation of the side mode that we have obtained cannot be due to the photon fluctuations at the bias state. It has been already shown both theoretically [8] and experimentally [9] that the photon fluctuations at the bias state cannot explain the spreading of the first passage time statistics of gain-switched semiconductor lasers biased below threshold. Our analysis extends this earlier results to the side-mode excitation probability in the same driving conditions. The initial photon distribution $S_\nu(0)$ appears in $\phi(s_1)$ only through its mean value $\langle S_\nu(0) \rangle$ which is always multiplied by a vanishing exponential [see (12.A) and (13.A)]. The large values of the side-mode excitation probability is the consequence of the spreading of the field distribution of both modes when their gains become positive and the phase of the field has not yet attained a well-defined value (i.e., is the field is yet incoherent).

Fig. 5 confirms that $\Delta g = 7.5$ cm^{-1} is not enough, in general, to ensure dynamical single-mode operation in laser diodes used in high-speed optical transmission systems.

In Figs. 6 and 7 the evolution of mean photon densities of the main and secondary modes is shown for the two cases of curve (a) and (b), respectively, of Fig. 2. These curves have been obtained by means of numerical simulations on 10 000 realizations of the process. Fig. 8 shows the same quantity of Fig. 6 obtained by decreasing the value of $\chi/S_{\rm st}$ down to 6.66. We have already shown that $\phi(s_1)$ is practically independent of the saturation photon density while, on the contrary, Figs. 6 and 8 show a strong dependence of the mean value of the side-mode photon density on this quantity. In order to understand

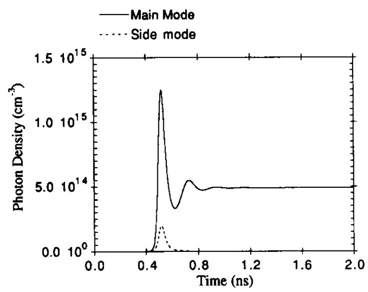


Fig. 7. The same as in Fig. 6, for the case of curve b in Fig. 2.

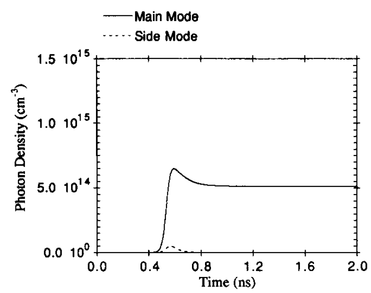


Fig. 8. The same as in Fig. 6, but with $\chi/S_{st} = 6.66$.

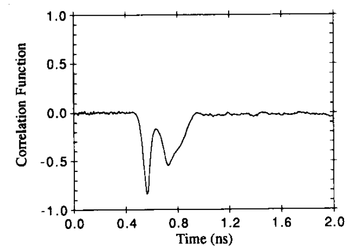


Fig. 9. Correlation function between the main and the side modes for the conditions of curve a in Fig. 2.

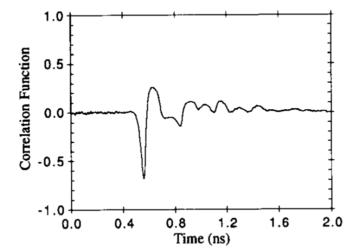


Fig. 10. The same as in Fig. 10, but with $\chi/S_{st} = 100$.

this apparent contradiction let us look at the different meaning of an increase of A or of χ/S_{st}. The first case corresponds to an increase of the probability of having the side mode larger than the main one. The second one, on the contrary, corresponds to an increase of the overshoot of both the main- and side-mode intensity during the first relaxation peak. To verify this analysis, we computed the side-mode suppression ratio (SMSR) at the maximum of the overshoot of the side mode for three different values of χ/S_{st}, namely 6.66, 20, and 100, and a constant value of $A = \bar{A}$, obtaining the almost constant values 12.46, 11.94, and 11.76, respectively. The same has been performed for a constant $\chi/S_{st} = 20$, and a larger value of differential gain $A = 2\bar{A}$, obtaining for the SMSR the smaller value 6.29.

In spite of their different effect on the laser dynamics, the two parameters play a similar role in determining the performance of digital transmission systems where an error is made once the side mode exceeds a threshold value. This is evident if we calculate the number of times the photon density of the side mode reaches the value $S_{st}/2$ during 10 000 realizations. We have obtained the frequencies 0.129, 0.292, and 0.033 for the case of Figs. 6, 7, and 8, respectively.

Fig. 9 shows the correlation function between the main and side modes for the case of Fig. 6. A strong negative correlation between the two modes around the switching time is shown, which vanishes for long times. This behavior of the correlation function can be intuitively explained by considering that the side mode rises when random fluctuations due to spontaneous emission give rise to a delay of the main-mode switching. As a consequence, a strong excitation of the side mode always corresponds to a negative fluctuation on the main mode, as usually happens for mode-partition noise. For longer times, on the contrary, the large suppression ratio has the consequence to decouple the fluctuations of the side mode from those of the main mode [28]. An interesting feature of the correlation between the two modes is shown in Fig. 10, which is the correlation function obtained by increasing the saturation photon density up to one hundred the stationary photon density, and for $A = \bar{A}$ and $C - C_{th} = \bar{C}$. The correlation function during the transient switches from negative to positive values. This behavior, different from that of mode-partition noise in which negative correlations are always present, can be explained by considering the stochastic dynamics of the switching of a two mode laser. The realizations which show an excitation of the side mode are associated with a delay of the switching of the main mode, which reaches its maximum later with respect to most of the realizations. If the relaxation oscillations are undamped, this maximum is larger than the mean value at the same time, so that in this region positive fluctuations of the side mode are associated to positive fluctuations of the main mode, which explains the positive correlations.

Experiment

The measurements of transients bimodal dynamics were performed on several buried heterostructure DFB lasers, using an experimental technique similar to that reported in [9] and [11]. The experimental setup shown in Fig. 11 allowed us to measure the pulse shape, the output spectrum averaged on different realizations of the process, and the statistical distribution of the intensity in the main mode and the most intense side mode at different time delays with respect to the laser excitation. In Fig. 11, the dashed portion was used only in a part of the measurements. The risetime and the width of the electrical pulses were 100 ps and 2 ns, respectively, and the repetition frequency was 20 kHz. The bias current I_b was varied between 0 and the threshold current I_{th} and the pulse step had a maximum amplitude I_p = 50 mA. In all lasers the SMSR under CW operation was better than 35 dB for currents larger than $1.5I_{th}$. In spite of this large value of SMSR, the gain difference between the main mode and the most intense side mode, measured for currents

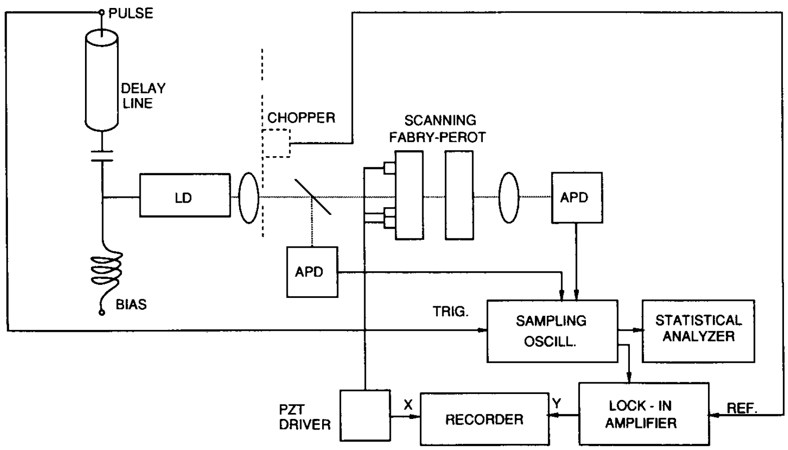


Fig. 11. Experimental setup.

just below I_{th} was quite different for various devices. A minimum value of 4 cm^{-1} was found for one device, while for other devices it was always larger than 8 cm^{-1}. We were able to observe the transient bimodal operation only in one device, a Fujitsu FLD 150 F1 laser operating at a wavelength of 1555.2 nm and having a threshold current of 18 mA. This device was the one with the minimum gain difference between main and side modes, even though a SMSR of 34 dB at 20 mA and 44 dB at 60 mA was measured under CW operation.

The optical spectra, averaged for different realizations and for different time delays t_d measured from the turn-on instant (defined as the instant at which the optical intensity reaches 10% of its stationary value in the *on* state), is shown in Fig. 12(a) and (b) for $I_b = 0.01$ and 12 mA, respectively, and for $I_p = 50$ mA. The broadening of the optical spectrum is due to frequency chirping occurring during the integration time of our photodiode (≈ 30 ps). The spectral structure is due to relaxation oscillations of the laser. The statistical distribution of intensity at the side-mode wavelength, obtained under the same driving conditions of Fig. 12(b), is shown in Fig. 13 for two values of t_d. The small dots refer to the instrumental noise measured at zero output intensity. Defining the dynamical SMSR as the ratio between the powers emitted in the two most intense modes in a time interval centered at t_d and of length equal to the integration time of the photodiode, it is interesting to note that in Fig. 12(b) it has a minimum value of 13 dB for $t_d = 40$ ps (this quantity cannot be measured at $t_d = 0$) and a value >20 dB for $t_d = 200$ ps. In spite of the large values of SMSR, a tail of the side-mode intensity distribution towards the larger values of the intensity is evident. This underlines the nature of transient bimodality that shows up as rare but intense events which, though being unnoticed in the average spectrum, can introduce errors in high bit rate systems.

A quantitative agreement between theory and the experimental results is hard to establish because of the noise of our experimental apparatus that leads to a broadening of the intensity distribution which partially masks the structure of the investigated phenomenon. One can, however, extract the following conclusions:

1) The average amplitude of the side mode increases with an increase in the pulse amplitude. This is evident from Fig. 14 where the average output spectrum is shown for the same bias of Fig. 12(b) but for $I_p = 25$ mA.

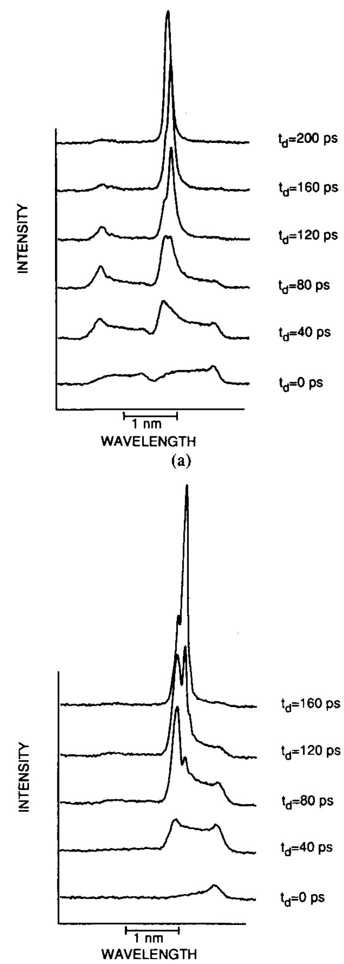


Fig. 12. Optical spectra averaged on different realizations of laser swtich on. The driving conditions are: $I_b = 0.01$ mA, $I_p = 50$ mA (a); $I_b = 12$ mA, $I_p = 50$ mA (b). t_d is the time delay measured from the instant at which the output intensity reaches 10% of its steady-state on-state value.

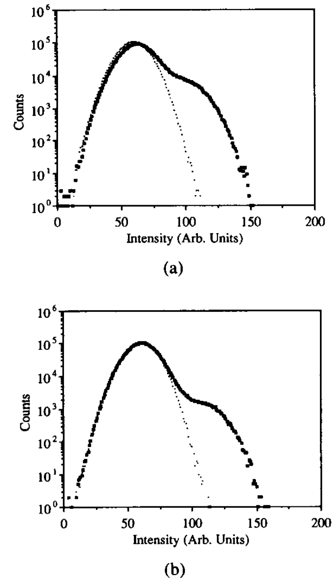


Fig. 13. Experimental statistical distribution of side-mode intensity. The driving conditions are: I_b = 12 mA, I_p = 50 mA, while the time delay is t_d = 100 ps (a) and t_d = 200 ps (b). The small dots represent the instrumental zero.

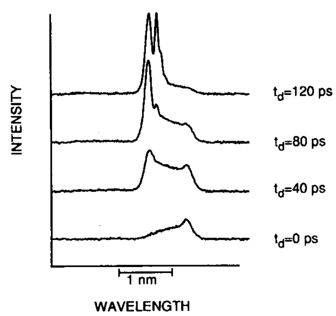


Fig. 14. Optical spectra averaged for different realizations of laser switching. The driving conditions are: I_b = 12 mA, I_p = 25 mA.

2) The value of SMSR increases for increasing t_d as shown in Fig. 15 where the dynamical SMSR is shown for I_b = 6 mA and I_p = 50 mA. This increase is due to the gain difference between the two modes which, for increasing values of t_d, tends to promote the main mode even in those realizations in which it starts later than the side mode. This is in agreement with the dependence of $\phi(0)$ on the value of S_T.

3) SMSR increases by increasing I_b for a constant value of I_p as shown in Fig. 16 where the dynamical SMSR is shown as a function of I_b for I_p = 50 mA at t_d = 40 ps.

Our theoretical results showed that the side-mode excitation is independent of the bias conditions and depends only on the total current, when the bias is sufficiently below threshold [see (15.A)]. If the current step is constant in the experiment, the bias increase is equivalent to an increase of the excitation in the *on* state and, from the theory, it should correspond to a decrease in the dynamical SMSR. Experimentally SMSR increased with an increase in the bias current (Fig. 16). This contradiction between theoretical and experimental results could be explained by taking into account a different mechanism which determines

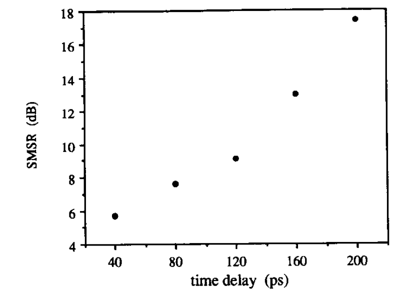


Fig. 15. Dynamical side-mode suppression ratio versus time delay. The driving conditions are: I_b = 6 mA, I_p = 50 mA.

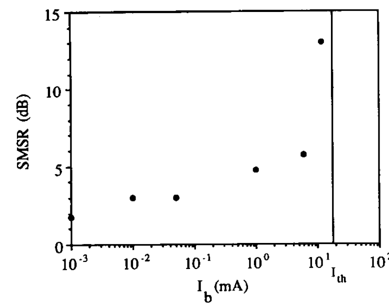


Fig. 16. Dynamical side-mode suppression ratio versus bias current. The conditions are I_p = 50 mA, t_d = 40 ps.

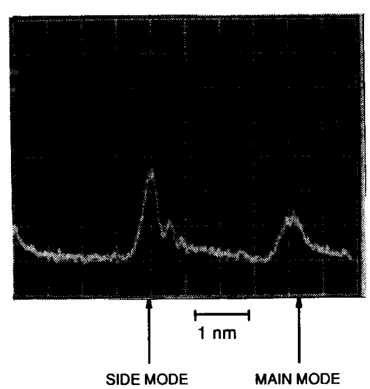


Fig. 17. Optical spectra averaged for different realizations of laser switching and obtained for I_b = 0.1 mA, I_p = 50 mA, and $t_d \leq 0$ ps.

the bias dependence of the dynamical SMSR. The presence of this different mechanism can be inferred by looking at Fig. 17 in which the dynamical spectrum obtained at I_b = 0.1 mA and I_p = 50 mA for t_d slightly below 0 is reported (under these driving conditions the main mode is always dominant for t_d > 0). The figure shows that the side mode can be larger than the mode that becomes the main mode under stationary conditions. This behavior is clearly not allowed in the framework of the theory developed in this paper and can be explained by only supposing that the gain of the side mode overcomes the gain of the main mode during switching. It has to be noted that a larger value of the spontaneous-emission rate on the side mode cannot explain a larger excitation of the side mode due to a weak dependence of the side-mode excitation on this parameter [see

(37)]. An inhomogeneous distribution of the carrier density, which modifies the longitudinal properties of the grating during switching and induces a larger gain on the side mode, can explain this phenomenon. After switching, diffusion can make the carrier density more homogeneous along the cavity, thus raising the gain of the main mode in the expected way.

SUMMARY

This paper has discussed how often a side mode may be excited transiently in nearly single-mode semiconductor lasers designed such that the average side-mode amplitude remains negligible (SMSR > 30 dB). The problem is of considerable practical interest for optical communication systems whose BER depends on such transient excitation of side modes. Starting from the Langevin rate equations we obtain a FPE by considering the main mode and the most intense side mode. We have solved this FPE analytically under the assumption that the laser is biased below threshold. The resulting solution is used to calculate the side-mode excitation probability. The validity of the approximate solution is verified through Monte Carlo simulations of the transient laser dynamics. Our results show the relevance of the carrier-density overshoot during laser turn-on in determining the side-mode excitation probability. They also confirm that a much larger value of the gain margin is required to ensure side-mode suppression under transient conditions compared with the value that would ensure a high SMSR on an average basis. We have performed experiments by using the DFB lasers and have found qualitative agreement with theory. The experiments on the bias dependence of the dynamical SMSR indicate that the inhomogeneous nature of the carrier-density distribution along the cavity axis may be important for understanding the transient multimode dynamics in DFB lasers.

APPENDIX A

In this Appendix, an analytical expression for $m_\nu(t)$, the mean value of the photon density of the two modes, is found when the simplified stochastic differential equations (14)–(15) hold.

From (15), we find $m_\nu(t)$ as the solution of the following differential equation:

$$\frac{dm_\nu(t)}{dt} = rn(t) + g_\nu[n(t)]\, m_\nu(t), \qquad m_\nu(0) = \langle S_\nu(0)\rangle. \quad (1.A)$$

The integration of (1.A) from 0 to t gives

$$m_\nu(t) = \langle S_\nu(0)\rangle \exp[G_\nu(0;t)] + \int_0^t rn(t')\cdot \exp[G_\nu(t';t)]\,dt'$$
$$= \left\{\langle S_\nu(0)\rangle \exp[G_\nu(0;\tau)] + \int_0^t rn(t') \cdot \exp[G_\nu(t';\tau)]\,dt'\right\} \times \exp[G_\nu(\tau;t)] \quad (2.A)$$

where

$$G_\nu(t_1;t_2) = \int_{t_1}^{t_2} g_\nu[n(t)]\,dt \quad (3.A)$$

and τ is any fixed time between 0 and ∞. By using (16) and (3.A), we have

$$n(t') = \frac{1}{a}\left[-\frac{d}{dt'}G_\nu(t';\tau) + an_{\rm th} + \Delta g_\nu\right] \quad (4.A)$$

and (2.A) becomes

$$m_\nu(t) = \left\{\langle S_\nu(0)\rangle \exp[G_\nu(0;\tau)] + \frac{r}{a}[\exp[G_\nu(0;\tau)] - \exp[-G_\nu(\tau;t)]] + \frac{r}{a}(an_{\rm th} + \Delta g_\nu) \cdot \int_0^t \exp[G_\nu(t';\tau)]\,dt'\right\} \exp[G_\nu(\tau;t)] \quad (5.A)$$

where we have used the property $G_\nu(t_1, t_2) = -G_\nu(t_2, t_1)$. In order to give an explicit expression for G_ν, it is convenient to write [see (24) and (25)]

$$g_\nu(n(t)) = a[\tau_{\rm sp}(C - C_{\rm th})]\left\{1 - \exp\left(-\frac{t - t_{\rm th}}{\tau_{\rm sp}}\right)\right\} - \Delta g_\nu \quad (6.A)$$

where

$$t_{\rm th} = \tau_{\rm sp} \ln \frac{C - C_0}{C - C_{\rm th}} \quad (7.A)$$

is the time at which the threshold is reached. From the definition (3.A), we then obtain

$$G_\nu(t_1;t_2) = a[\tau_{\rm sp}(C - C_{\rm th})] \cdot \left\{(t_2 - t_1) + \tau_{\rm sp}\left[\exp\left(-\frac{t_2 - t_{\rm th}}{\tau_{\rm sp}}\right) - \exp\left(-\frac{t_1 - t_{\rm th}}{\tau_{\rm sp}}\right)\right]\right\} - \Delta g_\nu(t_2 - t_1). \quad (8.A)$$

It is difficult to integrate (5.A) when (8.A) is inserted into it. In order to get to an analytical expression for $m_\nu(t)$, let us note that the quantity between braces has the meaning of an effective initial condition at time τ, so that it is convenient to chose $\tau = t_{\rm th}$. Furthermore, $\exp[G_\nu(t_1, t_2)]$ has a sharp maximum for a fixed t_2 when the gain g_ν, which is the derivative of G_ν, change sign, i.e., for t_1 equal to

$$\tau_\nu = \tau_{\rm sp} \ln \frac{C - C_0}{C - C_{\rm th} - \dfrac{\Delta g_\nu}{a\tau_{\rm sp}}}. \quad (9.A)$$

Thus, one can approximate the integral into (5.A) with the steepest descent method [29] (it has to be noted that τ_ν for $\nu = 1$, i.e., for the main mode, corresponds to $t_{\rm th}$). With this aim, let us perform a second order expansion of $G_\nu(t', t_{\rm th})$ around τ_ν and write

$$G_\nu(t', t_{\rm th}) \approx G_\nu(\tau_\nu, t_{\rm th}) - B_\nu(t' - \tau_\nu)^2 \quad (10.A)$$

where

$$B_\nu = \frac{1}{2}\frac{\partial^2}{\partial t'^2}[G_\nu(t';t_{\rm th})]_{t'=\tau_\nu} = \frac{1}{2}a_\nu\left(C - C_{\rm th} - \frac{\Delta g_\nu}{a_\nu \tau_{\rm sp}}\right). \quad (11.A)$$

Inserting (10.A) into (5.A), we obtain

$$m_\nu(t) = R_\nu(t)\exp[G_\nu(t_{\rm th}, t)] \quad (12.A)$$

where

$$R_\nu(t) = \langle S_\nu(0)\rangle \exp[G_\nu(0, t_{th})] + \frac{r}{a}\{\exp[G_\nu(0; t_{th})] - \exp[-G_\nu(t_{th}; t)]\} + \frac{r}{a}(an_{th} + \Delta g_\nu) \cdot \exp[G_\nu(\tau_\nu; t_{th})] I_\nu(t) \quad (13.A)$$

$$I_\nu(t) = \left(\frac{\pi}{4B_\nu}\right)^{1/2} \{\mathrm{erf}[(B_\nu)^{1/2}\tau_\nu] + \mathrm{erf}[(B_\nu)^{1/2}(t - \tau_\nu)]\}. \quad (14.A)$$

In the special case in which the bias current is sufficiently below threshold ($C_0/C_{th} \lesssim 0.95$ (see [8] and [9]), (13.A) tends to

$$R_\nu(\infty) = \frac{r}{a}(an_{th} + \Delta g_\nu)\left(\frac{\pi}{B_\nu}\right)^{1/2}. \quad (15.A)$$

Appendix B

In order to derive (29), let us substitute (26)–(28) into (6) and (13) and obtain

$$\Phi(S_1) = \sqrt{2}\int_0^\infty dt \int_0^\infty dn\, J^T(S_1; S_T - S_1; n; t)e = \int_0^\infty dt \left\{\left[rn(t) + g_1[n(t)]S_1 - rn(t)\frac{\partial}{\partial S_1}S_1\right] \cdot p_1(S_1; t)\, p_2(S_2; t) + \left[rn(t) + g_2[n(t)]S_2 - rn(t)\frac{\partial}{\partial S_2}S_2\right] p_1(S_1; t)\, p_2(S_2; t)\right\} \quad (1.B)$$

where $S_2 = S_T - S_1$. Integrating (27) from 0 to S_ν and consid ering that (as can be easily checked by direct differentiation)

$$\left\{\left[rn(t) + g_\nu(n(t))S_\nu - rn(t)\frac{\partial}{\partial S_\nu}S_\nu\right] p_\nu(S_\nu; t)\right\}_{S_\nu = 0} = 0 \quad (2.B)$$

one obtains

$$\left[rn(t) + g_\nu[n(t)]S_\nu - rn(t)\frac{\partial}{\partial S_\nu}S_\nu\right] p_\nu(S_\nu; t) = -\frac{\partial}{\partial t}\int_0^{S_\nu} p_\nu(S; t)\, dS \quad (3.B)$$

and, hence

$$\Phi(S_1) = \int_0^\infty \left\{\left[-\frac{\partial}{\partial t}\int_0^{S_1} p_1(S; t)\, dS\right] p_2(S_T - S_1; t) + \left[-\frac{\partial}{\partial t}\int_0^{S_T - S_1} p_2(S; t)\, dS\right] p_1(S_1; t)\right\} dt. \quad (4.B)$$

From (28) one finally obtains (29).

Appendix C

Here an analytical expression for $\phi(0)$ is derived for the case of a laser switched with constant gains. From (35), when $\langle S_\nu(0)\rangle = 0$, $m_1(t)$ is given by

$$m_1(t) = \frac{R_1}{g_1}\left\{\left[\frac{g_2}{R_2}m_2(t) + 1\right]^{g_1/g_2} - 1\right\}. \quad (1.C)$$

With the substitution

$$y = \frac{S_T}{m_2(t)} \quad (2.C)$$

and considering that, from (35), $t = 0 \Rightarrow y = \infty$ that $t \to \infty \Rightarrow y = 0$, and substituting into (29)–(30), we obtain

$$\phi(0) = \int_0^\infty dy \frac{S_T g_1}{R_1}\left(\frac{R_2 y}{S_T g_2}\right)^{g_1/g_2}\left[\left(1 + \frac{R_2 y}{S_T g_2}\right)^{g_1/g_2} - \left(\frac{R_2 y}{S_T g_2}\right)^{g_1/g_2}\right]^{-1} e^{-y}. \quad (3.C)$$

The quantity $R_2/S_T g_2$ in (3.C) is much less than one for macroscopic values of S_T for typical values of the physical parameters.

The quantity between square brackets into (3.C) is different from one only for values of y such that

$$y \gtrsim \left(\frac{R_2}{S_T g_2}\right)^{-1}$$

that is for values which do not contribute to the integral because of the term exp $(-y)$. For this reason, we can substitute the quantity between square brackets into (3.C) with one, in so obtaining

$$\phi(0) = \int_0^\infty dy \frac{S_T g_1}{R_1}\left(\frac{R_2 y}{S_T g_2}\right)^{g_1/g_2} e^{-y}. \quad (4.C)$$

The integral can be now performed analytically, and the result is

$$\phi(0) = \frac{S_T g_1}{R_1}\left(\frac{R_2}{S_T g_2}\right)^{g_1/g_2} \Gamma\left(\frac{g_1}{g_2} + 1\right) \quad (5.C)$$

where Γ is the Euler gamma function [21]. Defining $\Delta g = g_1 - g_2$, we finally obtain (37).

Appendix D

In this Appendix the analytical expression of Pr $(S_1 > \eta S_0)$ is given. Equation (30) can be written as

$$\phi(s_1) = \int_0^\infty dt\, [A(t)s_1 + B(t)] \exp[-C(t)s_1 - D(t)] \quad (1.D)$$

where

$$A(t) = \frac{1}{m_1(t)\, m_2(t)}\left[\frac{\dot{m}_1(t)}{m_1(t)} - \frac{\dot{m}_2(t)}{m_2(t)}\right] \quad (2.D)$$

$$B(t) = \frac{1}{m_1(t)\, m_2(t)}\frac{\dot{m}_2(t)}{m_2(t)} \quad (3.D)$$

$$C(t) = S_T\left[\frac{1}{m_1(t)} - \frac{1}{m_2(t)}\right] \quad (4.D)$$

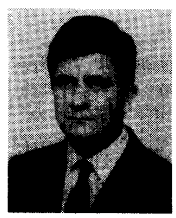
$$D(t) = \frac{S_T}{m_2(t)}. \quad (5.D)$$

$\Pr(S_1 > \eta S_0)$ can be easily obtained by integration of (1.D):

$$\Pr(S_1 > \eta S_0) = \int_0^{1/(1+\eta)} \phi(s_1)\, ds_1$$
$$= S_T \int_0^{\infty} dt\, e^{-D(t)} \left\{ \left[\frac{A(t)}{C(t)^2} + \frac{B(t)}{C(t)} \right] \cdot [1 - \exp[-C(t)/(1+\eta)]] - \frac{A(t)}{C(t)(1+\eta)} \exp[-C(t)/(1+\eta)] \right\}. \tag{6.D}$$

References

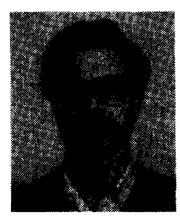
[1] F. Haake, J. W. Haus, and R. Glauber, "Passage-time statistics for the decay of unstable equilibrium states," *Phys. Rev. A*, vol. 23, pp. 3255–3271, 1981.
[2] C. W. Gardiner, *Handbook of Stochastic Methods*. Berlin: Springer-Verlag, 1983.
[3] F. Haake, J. W. Haus, H. King, and G. Schröder, "Delay-time statistics and inhomogeneous line broadening in superfluorescence," *Phys. Rev. Lett.*, vol. 45, pp. 558–561, 1980.
[4] —, "Delay-time statistics of superfluorescent pulses," *Phys. Rev. A*, vol. 23, pp. 1322–1333, 1981.
[5] F. T. Arecchi and A. Politi, "Transient fluctuations in the decay of an unstable state," *Phys. Rev. Lett.*, vol. 45, pp. 1219–1222, 1980.
[6] F. de Pasquale, P. Tartaglia, and P. Tombesi, "New expansion technique for the decay of an unstable state," *Phys. Rev. A*, vol. 25, pp. 466–471, 1982.
[7] M. C. Torrent and M. San Miguel, "Stochastic-dynamics characterization of delayed laser threshold instability with swept control parameters," *Phys. Rev. A*, vol. 38, pp. 245–251, 1988.
[8] A. Mecozzi, S. Piazzolla, A. D'Ottavi, and P. Spano, "Passage time statistics in semiconductor laser turn on," *Phys. Rev. A*, vol. 38, pp. 3136–3139, 1988.
[9] P. Spano, A. D'Ottavi, A. Mecozzi, B. Daino, and S. Piazzolla, "Experimental measurements and theory of first passage time in pulse-modulated semiconductor lasers," *IEEE J. Quantum Electron.*, vol. 25, pp. 1440–1449, 1989.
[10] F. T. Arecchi, R. Meucci, and J. A. Roversi, "Determination of a small photon number by statistical amplifying transients," *Europhys. Lett.*, vol. 8, pp. 225–231, 1989.
[11] M. M. Choy, P. L. Liu, and S. Sasaki, "Origin of modulation-induced mode partition and Gb/s system performance of highly single-mode 1.5 μm distributed feedback lasers," *Appl. Phys. Lett.*, vol. 52, pp. 1762–1764, 1988.
[12] F. Koyama, Y. Suematsu, S. Arai, and T. Tawee, "1.5 μm GaInAsP/InP dinamic-single-mode (DSM) lasers with distributed bragg reflector," *IEEE J. Quantum Electron.*, vol. QE-19, pp. 1042–1051, 1983.
[13] L. S. Posadas, K. Komori, Y. Tomori, S. Arai, and Y. Suematsu, "Spectral behavior of 1.5 μm BIG-DBR-DSM Lasers," *IEEE J. Quantum Electron.*, vol. QE-23, pp. 769–803, 1987.
[14] S. Tsui, M. Okai, H. Nakano, N. Chinone, and M. M. Choy, "Dynamic mode stability in gain-switched quarter wave shifted distributed feedback lasers," *IEEE J. Quantum Electron.*, vol. 25, pp. 1333–1338, 1989.
[15] N. H. Jensen, H. Olesen, and K. E. Stubkjær, "Partition noise in semiconductor lasers under CW and pulsed operation," *IEEE J. Quantum Electron.*, vol. QE-23, pp. 71–79, 1987.
[16] P. L. Liu and M. M. Choy, "Modeling rare turn-on events of injection lasers," *IEEE J. Quantum Electron.*, vol. 25, pp. 1767–1770, 1989.
[17] —, "Effect of bias on transient mode partition—An analysis based on photon statistics," *IEEE J. Quantum Electron.*, vol. 25, pp. 854–857, 1989.
[18] S. E. Miller, "On the injection laser contribution to mode partition noise in fiber telecommunication systems," *IEEE J. Quantum Electron.*, vol. 25, pp. 1771–1781, 1989.
[19] —, "On the prediction of the mode partitioning floor in injection lasers with multiple side modes at 2 and 10 Gb/s," *IEEE J. Quantum Electron.*, vol. 26, pp. 242–249, 1990.
[20] S. E. Miller and D. Marcuse, "On fluctuations and transients in injection lasers," *IEEE J. Quantum Electron.*, vol. QE-20, pp. 1032–1044, 1984.
[21] C. H. Henry, "Theory of phase noise and power spectrum of a single mode injection laser," *IEEE J. Quantum Electron.*, vol. QE-19, pp. 1391–1397, 1983.
[22] G. P. Agrawal and N. K. Dutta, *Long Wavelength Semiconductor Lasers*. New York: Van Nostrand Reinhold, 1986.
[23] M. Lax, "Quantum noise X. Density-matrix treatment of field and population fluctuations," *Phys. Rev.*, vol. 157, pp. 213–231, 1967.
[24] F. T. Arecchi, G. L. Lippi, G. P. Puccioni, and J. R. Tredicce, "Deterministic chaos in laser with injected signal," *Opt. Commun.*, vol. 51, pp. 308–314, 1985.
[25] M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions*. New York: Dover, 1970.
[26] W. Rümelin, "Numerical treatment of stochastic differential equations," *SIAM J. Numer. Anal.*, vol. 19, pp. 604–613, 1982.
[27] A. Mecozzi, P. Spano, A. D'Ottavi, and S. Piazzolla, "Analysis of transients in pulse modulated semiconductor lasers biased near threshold," *Appl. Phys. Lett.*, vol. 55, pp. 769–771, 1989.
[28] C. H. Henry, P. S. Henry, and M. Lax, "Partition fluctuations in nearly single-longitudinal-mode lasers," *J. Lightwave Technol.*, vol. LT-2, pp. 209–216, 1984.
[29] M. Born and E. Wolf, *Principles of Optics*, V ed. New York: Pergamon, 1975.



Antonio Mecozzi was born in Rome, Italy, on March 19, 1959. He received the Laurea degree in chemical engineering from the University of Rome, Rome, Italy, in 1983.

In 1984 he worked with the Physical Department of the University of Rome on transience in chemical reactive systems. He joined the Fondazione Ugo Bordoni, Rome, Italy, in 1986 where he has been working on quantum optics and transient phenomena in lasers.

A. Sapia, photograph and biography not available at the time of publication.



Paolo Spano was born in Rome, Italy, on June 8, 1952. He received the Laurea degree in physics in 1975 from the University of Rome, Rome, Italy.

In 1978 he joined the Fondazione Ugo Bordoni, Rome, Italy, where he has studied semiconductor laser physics. In 1979, he spent four months at the Centre National d'Etudes des Télécommunications, Lannion, France, working on coherence properties of semiconductor laser radiation.

Govind P. Agrawal (M'83–SM'86) was born in Kashipur, India. He received the M.S. and Ph.D. degrees from the Indian Institute of Technology, New Delhi, in 1971 and 1974, respectively.

After spending several years at the Ecole Polytechnique, Palaiseau, France, and the City University of New York, New York, he became a member of the technical staff at AT&T Bell Laboratories. He joined the faculty of the Institute of Optics at the University of Rochester, Rochester, NY, in January 1989. His research interests focus on quantum electronics, nonlinear optics, and laser physics. In particular, he has contributed significantly to the fields of semiconductor lasers, nonlinear fiber optics, and optical communications. He is an author or coauthor of more than 150 research papers, several review articles, and two books entitled *Long-Wavelength Semiconductor Lasers* (Van Nostrand Reinhold Co., 1986) and *Nonlinear Fiber Optics* (Academic, 1989).

Dr. Agrawal is a Fellow of the Optical Society of America and a member of the American Physical Society.