Effect of Cross Saturation on Frequency Fluctuations in a Nearly Single-Mode Semiconductor Laser

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Abstract—**The effect of side-mode cross saturation on the frequency noise of the main mode is investigated by considering the Langevin rate equations which contain the nonlinear gain with both the self-saturation and cross-saturation terms. When cross saturation is stronger than self saturation, the frequency noise of the main mode is found to be significantly enhanced in the low-frequency regime (< 1 GHz). An increase of more than 20 dB is predicted due to a side mode suppressed by 15–20 dB. The enhanced frequency noise manifests as broadening of the laser linewidth and may affect the performance of coherent optical communication systems.**

I. Introduction

There has been considerable interest recently in the effects of nonlinear gain on the characteristics of both single mode and multimode semiconductor lasers [1]–[11]. A self-saturation term is added to the usual rate equations to account for the reduction in gain caused by the mode itself. Such gain compression has been studied thoroughly for single-mode lasers [1]. In a multimode laser, cross saturation should also be included because the main mode gain is reduced by the presence of side modes. Although cross saturation has been included in several theoretical studies [2]–[9], its impact on the laser noise characteristics is not well understood. In this letter we investigate the effect of cross saturation on the frequency noise spectrum (FNS) of the main mode in a nearly-single-mode laser, such as a distributed feedback (DFB) laser. The theory treats the side mode on the same footing as the main mode and therefore is valid for any value of mode-suppression ratio (MSR). We find that the FNS of the main mode can be enhanced by more than 20 dB due to cross saturation by a side mode whose power is negligible compared with the main mode power (MSR = 15–20 dB).

II. Theory

The rate equations for the photon numbers of the two modes may be written as [3]

$$\dot{P}_i = (G_i - \gamma_i)P_i + R_{sp} + F_i(t) \qquad (1)$$

where P_i is the photon number and G_i and γ_i are the respective gain and loss of the main mode ($i = 1$) and the side mode ($i = 2$). The dot signifies the time derivative. R_{sp} is the rate of spontaneous emission into each mode (assumed to be the same for both modes) and $F_i(t)$ is a Langevin noise source that accounts for fluctuations induced by spontaneous emission. The gain G_i depends in general on carrier population as well as photon number of both modes in order to account for self and cross saturation. For a single-mode laser, the corresponding gain has been derived exactly [1]; for the two-mode laser the following gain expression (generally valid at power levels below 10 mW) is used:

$$G_i = A(N - N_0) - \beta_i P_i - \theta_{ij} P_j \quad (i, j = 1, 2;\ i \neq j). \qquad (2)$$

A is the linear gain coefficient, N_0 is the number of electrons required to reach transparency, and β_i, θ_{ij} are the self- and cross-saturation coefficients, respectively. The functional form of β_i and θ_{ij} need not be specified. An asymmetric form of the nonlinear gain corresponds to $\theta_{12} \neq \theta_{21}$ [9].

The rate equation for the total carrier number is given by

$$\dot{N} = I/q - \gamma_e N - (G_1 P_1 + G_2 P_2) + F_n(t) \qquad (3)$$

where I is the injection current, q is the electron charge, γ_e is the electron recombination rate and $F_n(t)$ is the Langevin source term. In order to calculate the main mode FNS, we also need the phase equation:

$$\dot{\Phi} = \frac{1}{2}\alpha[A(N - N_0) - \gamma_1] + F_\phi(t) \qquad (4)$$

where α is the linewidth enhancement factor responsible for the *carrier*-induced index change. The *intensity*-induced index change, discussed in [1] for a single-mode laser, is neglected here.

For a laser under CW operation, (1)–(4) are linearized about the steady state resulting in the following equations [3]:

$$\dot{p}_j = -\Gamma_j p_j + (AP_j)n - (\theta_{jk}P_j)p_k + F_j(t) \qquad (j, k = 1, 2;\ j \neq k) \quad (5)$$

$$\dot{n} = -\Gamma_n n - G_1 P_1 - G_2 P_2 + F_n(t) \qquad (6)$$

$$\dot{\phi} = \frac{1}{2}\alpha A n + F_\phi(t). \qquad (7)$$

Here n, p_1, p_2, and ϕ are the fluctuations in N, P_1, P_2 and Φ, respectively. The decay constants Γ_n, Γ_1, and Γ_2 are given by

$$\Gamma_n = \gamma_e + (d\gamma_e/dN)N + A(P_1 + P_2) \qquad (8)$$

$$\Gamma_j = R_{sp}/P_j + \beta_j P_j \quad (j = 1, 2). \qquad (9)$$

In (5)–(9), N, P_1, and P_2 represent average values. Equations (5) and (6) can be solved explicitly for p_1, p_2, and n in the Fourier domain. The results are used to calculate the FNS of the main mode defined as $S_f(\omega) = \langle \omega^2 |\tilde{\phi}(\omega)|^2 \rangle$ where $\tilde{\phi}(\omega)$ is the

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Fourier transform of $\phi(t)$. For simplicity we neglect F_n and assume $G_1 = G_2$. The result is

$$S_f(\omega) = \frac{R_{sp}}{2P_1}\left\{1 + \alpha^2 \frac{(GAP_1)^2}{|L(\omega)|^2} \cdot \left[|H_{21}^{(2)}(\omega)|^2 + |H_{12}^{(1)}(\omega)|^2 \frac{P_2}{P_1} \right]\right\} \quad (10)$$

where

$$L(\omega) = (\Gamma_n + i\omega)B(\omega) + GAP_1 H_{12}^{(2)}(\omega) + GAP_2 H_{21}^{(1)}(\omega) \quad (11a)$$

$$H_{km}^{(j)}(\omega) = \Gamma_j + i\omega - \theta_{km} P_j \quad (j, k, m = 1, 2;\ k \neq m) \quad (11b)$$

$$B(\omega) = (\Gamma_1 + i\omega)(\Gamma_2 + i\omega) - \theta_{12}\theta_{21}P_1P_2. \quad (11c)$$

The last term in (10) represents the side-mode contribution to the FNS. This contribution can be considerably enhanced, as discussed below, because of the dependence of $L(\omega)$ on cross saturation.

III. Discussion

First we note that the expression (10) for the FNS is valid for either symmetric or asymmetric form of the nonlinear gain. Since self saturation usually arises from symmetric processes such as spectral hole burning, we take $\beta_1 = \beta_2 = \beta$. For definiteness, we focus on the case of symmetric cross saturation and take $\theta_{12} = \theta_{21} = \theta$. The case of $\theta_{12} \neq \theta_{21}$ does not appear to lead to new qualitative features. The parameter values used in the plots are taken from reference [3], with a total photon number of 2×10^5. We plot the FNS in Fig. 1 for 4 values of the ratio θ/β. All curves in Fig. 1 pertain to an MSR of 15 dB. For $\theta < \beta$ cross saturation in (2) is of minor importance, and the small enhancement in the FNS is due to the carrier-induced mode coupling [see (3)]. The case $\theta = \beta$ has been considered before [3], [7] and leads to no enhancement; this is part of the reason that cross-saturation effects were thought to be of minor importance. For $\theta > \beta$ the low-frequency part of the spectrum is enhanced by as much as 20 dB, even though the side mode is suppressed by 15 dB.

The low-frequency enhancement seen in Fig. 1 is expected to depend considerably on the MSR. Fig. 2 shows the dc ($\omega = 0$) enhancement in the FNS as a function of MSR for six different values of θ/β. When $\theta = \beta$ there is negligible enhancement for any value of MSR. A slight enhancement occurs for $\theta/\beta < 1$ as was discussed in relation to Fig. 1. However, for $\theta/\beta > 1$ the enhancement peaks sharply at a particular value of MSR.

The results shown in Figs. 1 and 2 can be understood by looking closely at (10) for the FNS. The side-mode contribution to the FNS is magnified by the large amount whenever $|L(\omega)|^2$ in the denominator becomes small. By using (9) and (11), the dc value of $L(\omega)$ is given by

$$L(0) = \Gamma_n P_1[\beta R_{sp}/P_2 + (\beta^2 - \theta^2)P_2] + GAP_1[R_{sp}/P_2 + 2(\beta - \theta)P_2] \quad (12)$$

where we used the excellent approximation $\Gamma_1 \approx \beta P_1$. Obviously, the FNS will be enhanced whenever either of the terms in $L(0)$ go to zero. The second term in (12) is the dominant one,

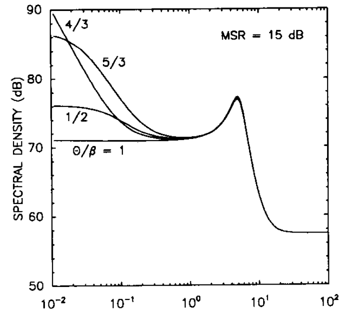

Fig. 1. $S_f(\omega)/2\pi$ versus frequency for an MSR of 15 dB and four different values of θ/β.

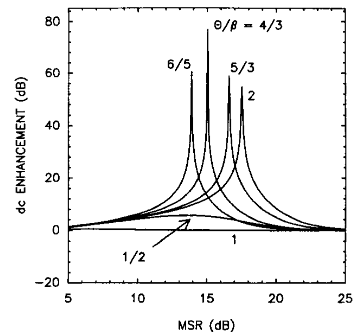

Fig. 2. Zero-frequency enhancement of the frequency fluctuations $S_f(0)$, normalized by the corresponding value in the absence of a side mode, as a function of MSR for six different values of θ/β.

and it becomes zero when

$$\frac{\theta}{\beta} = 1 + \frac{R_{sp}}{2\beta P_2^2}. \quad (13)$$

For $\theta < \beta$ the equation is never satisfied. However for $\theta/\beta > 1$, a side-mode power exists which will satisfy (13). Using $\theta/\beta = 4/3$, $\beta = 5 \times 10^4\ s^{-1}$ and $R_{sp} = 1.5 \times 10^{12}\ s^{-1}$ as typical values, the largest enhancement occurs for a side-mode photon number of about 6700. Since the conversion of intracavity photon number to output power depends on many device parameters, this corresponds to a side-mode power in the 100 μW range.

It is important to investigate whether values of $\theta > \beta$ are possible in real lasers. The ratio θ/β depends on several properties of the gain medium, including both spectral and spatial hole burning, and can range from 0–2 depending on the laser. In semiconductor lasers cross saturation has its origin in the modulation of the intraband carrier distribution (population pulsations) at the beat frequency of the two modes. The estimates based on the density-matrix approach [2], [10] indicate that θ can exceed β for both Fabry–Perot and DFB lasers. In particular, $\theta/\beta \geq 4/3$ for DFB lasers, where the exact value depends on the wavelength spacing between the main and side modes [10]. On the experimental side, there is considerable evidence for $\theta > \beta$. Examples include mode-hopping and associated optical bistability [9], enhanced relative intensity noise at low frequencies [4], and linewidth saturation in DFB lasers [12]. Our analysis shows that the wavelength of DFB lasers is likely to exhibit considerable low-frequency fluctuations because of cross-saturation effects associated with the presence of a weak side mode with an MSR of 15–20 dB.

What are the implications of side-mode enhanced frequency noise for lightwave systems? The performance of coherent communication systems operating at bit rates below 1 Gb/s will certainly be affected. This can be avoided by requiring an MSR of 30 dB or more (see Fig. 2). Accordingly, one must ensure that the MSR does not degrade during the system lifetime and approach a value near which frequency noise is enhanced in Fig. 2. The low-frequency enhancement of the FNS will also lead to a broadening of the laser linewidth. Qualitatively speaking, the enhancement factor in Fig. 2 is an approximate measure of the linewidth broadening since the linewidth is related to the dc value of the FNS. In fact, our theory predicts saturation and rebroadening of the laser linewidth at high operating powers as a result of the side-mode cross-saturation phenomenon and may explain a similar behavior observed experimentally in many DFB lasers [12]. A more quantitative approach should calculate the spectral lineshape by taking the Fourier transform of the field autocorrelation function.

In conclusion we have investigated the effect of cross saturation on the main mode FNS of a nearly-single-mode laser. Depending on the ratio of cross- to self-saturation coefficient, we find that the low-frequency part (< 1 GHz) of the FNS can be enhanced by greater than 20 dB due to cross saturation by a side mode that is suppressed by 15–20 dB.

Note Added in Proof: A paper reaching similar conclusions [13] appeared after submission of this paper.

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