Effect of Gain and Index Nonlinearities on Single-Mode Dynamics in Semiconductor Lasers

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Abstract—**The finite intraband relaxation time in semiconductor lasers leads to gain saturation at high laser powers. The nonperturbative solution of the single-mode density-matrix equations shows that both the optical gain and the refractive index become intensity dependent as a result of intraband relaxation dynamics. We include the gain and index nonlinearities in the rate equations and study how the modulation response and noise characteristics of semiconductor lasers are affected by such nonlinearities. The intensity dependence of the frequency and the damping rate of relaxation oscillations leads to a fundamental limit imposed on the small-signal modulation bandwidth; our analysis provides an expression for the ultimate modulation bandwidth in terms of the material parameters. In the case of large-signal modulation the rise and fall times associated with the optical pulse increase considerably because of gain nonlinearities while the frequency chirp is affected by index nonlinearities. The intraband-relaxation effects also lead to saturation and rebroadening of the laser linewidth at higher operating powers.**

I. Introduction

The dynamic response of semiconductor lasers is generally studied by solving a set of rate equations which describe the interaction between photons and charge carriers mediated by stimulated emission occurring inside the active region of the laser cavity [1]. The modal gain in these equations is often assumed to be intensity independent. However, many dynamic features of semiconductor lasers are properly accounted for only when the modal gain g is assumed to decrease with an increase in the mode intensity [2]–[11]. In a simple approach, the gain nonlinearities are included by adopting a phenomenological functional form [3]–[7] $g = g_L(1 - \epsilon S)$, where S is the intracavity photon density and ϵ is the nonlinear gain parameter ($\epsilon \sim 1 \cdot 10^{-17}$ cm^3). Such a form is clearly valid at low power levels such that $\epsilon S << 1$. However, a different functional form is needed when the laser power is large enough to violate this condition. The form $g = g_L(1 + \epsilon S)^{-1}$ has been used in some previous work [2], [8] in direct analogy with a homogeneously broadened two-level system. This form is not appropriate for semiconductor lasers for the following reason. Even though the semiconductor gain medium is nearly homogeneously broadened owing to a short intraband relaxation time (~ 0.1 ps), it cannot generally be treated as a two-level system because of a large spread in the transition frequencies.

A correct treatment of the nonlinear effects on the dynamic response of semiconductor lasers should start with the density-matrix formalism [12]–[16] which includes the intraband relaxation effects of charge carriers within the conduction and valence bands. The complexity of the problem often forces one to make use of third-order perturbation theory whereas a nonperturbative analysis is needed to include the gain-saturation effects occurring because of intraband relaxation. Such a nonperturbative analysis can be carried out for a semiconductor laser oscillating in a single-longitudinal mode [17]. The results show that in general both the gain and the refractive index become intensity dependent. The index nonlinearities have not been considered before but must be included in the rate equations since they can affect such laser characteristics as the linewidth and the frequency chirp.

In this paper we use the exact functional form of the gain and index nonlinearities in the rate equations and study how their inclusion affects the modulation response and the noise characteristics of semiconductor lasers. The paper is organized as follows. Section II presents the modified rate equations by including the intraband-relaxation contribution to the carrier-induced susceptibility. Section III considers small-signal modulation response with particular attention paid to the intensity dependence of the relaxation oscillations and the modulation bandwidth. Large-signal modulation is considered in Section IV. Section V discusses how the laser linewidth is affected by gain and index nonlinearities while Section VI focuses on the relative intensity noise. Finally, the main results are summarized in Section VII.

II. Modified Rate Equations

The response of the semiconductor active medium to the intracavity optical field is governed by a set of density-matrix equations [12]–[17] which take into account intraband relaxation of charge carriers within the conduction and valence bands through the relaxation times τ_c and τ_v. Relaxation of the induced polarization (governed by the off-diagonal elements of the density matrix) is in-

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cluded through the relaxation time τ_{in}. Since the three relaxation times are much shorter than the photon and carrier lifetimes, the medium is often assumed to respond instantaneously to the intracavity field. The steady-state solution of the density-matrix equations in that case can be used to obtain the carrier-induced change $\Delta\chi$ in the susceptibility. $\Delta\chi$ is found to consist of a linear part and a nonlinear part such that

$$\Delta\chi = \Delta\chi_L + \Delta\chi_{NL}. \tag{1}$$

The optical gain g and the index change Δn are related to $\Delta\chi$ by the general relation

$$\Delta\chi = 2\bar{n}(\Delta n - ig/2k_0) \tag{2}$$

where $\bar{n}$ is the effective index, $k_0 = \omega_0/c = 2\pi/\lambda_0$, ω_0 is the angular optical frequency, and λ_0 is the optical wavelength.

The general expressions for $\Delta\chi_L$ and $\Delta\chi_{NL}$ are difficult to obtain as they depend on details of the band structure, among other things. Furthermore, the evaluation of $\Delta\chi_{NL}$ in the multimode case often requires the use of third-order perturbation theory [12]–[16]. A nonperturbative analysis can be carried out [17] for a laser oscillating predominantly in a single mode, such as a distributed feedback (DFB) semiconductor laser. We make use of this nonperturbative analysis and restrict our attention to a single-mode semiconductor laser. The approximate expressions for $\Delta\chi_L$ and $\Delta\chi_{NL}$ are given by [17]

$$\Delta\chi_L = 2\bar{n}(\Delta n_L - ig_L/2k_0) \tag{3}$$

$$\Delta\chi_{NL} = \frac{\bar{n}g_L}{k_0}\frac{[\beta + i(1+p)^{-1/2}]p}{1 + (1+p)^{1/2}} \tag{4}$$

where $p = |E_0|^2/I_s$, $|E_0|^2$ is the intracavity mode intensity, and the saturation intensity I_s is related to the intraband relaxation times by the definition

$$I_s = \hbar^2/[\mu^2\tau_{in}(\tau_c + \tau_v)]. \tag{5}$$

The parameter β is related to the slope of the linear gain g_L as

$$\beta = \frac{1}{g_L(\omega_0)\tau_{in}}\left(\frac{dg_L}{d\omega}\right)\omega = \omega_0. \tag{6}$$

The linear part of Δn_L of the carrier-induced index change is often related to the linear gain g_L phenomenologically by using the relation

$$\Delta n_L = -\alpha_0 g_L/2k_0 \tag{7}$$

where α_0 is the so-called linewidth enhancement factor at the mode frequency ω_0. In (5) and (6), μ is the dipole moment, and τ_c, τ_v, and τ_{in} are the three intraband relaxation times.

The total carrier-induced change in the refractive index and the gain can be obtained by substituting (3) and (4) in (1) and expressing the result in the form (2). We then obtain

$$\Delta n = -\frac{g_L}{2k_0}\left(\alpha_0 - \frac{\beta p}{1 + (1+p)^{1/2}}\right) \tag{8}$$

$$g = g_L/(1+p)^{1/2}. \tag{9}$$

Equation (9) shows that the functional form of the saturated gain is different from a homogeneously broadened two-level system for which $g = g_L/(1+p)$. The intraband origin of gain saturation is evident from (5) which shows that $p = 0$ for $\tau_{in} = 0$. Note from (8) that a finite intraband relaxation time makes the mode index also intensity dependent.

The modified rate equations are obtained by substituting (8) and (9) in the conventional rate equations given by [1]

$$\dot{P} = \Gamma v_g(g - g_{th}) + R_{sp} \tag{10}$$

$$\dot{\phi} = -\Gamma v_g k_0(\Delta n - \Delta n_{th}) \tag{11}$$

$$\dot{N} = I/q - \gamma_e N - \Gamma v_g g P \tag{12}$$

where Γ is the confinement factor, v_g is the group velocity, R_{sp} is the rate of spontaneous emission into the lasing mode, I is the current, P is the number of intracavity photons, N is the number of electrons, ϕ is the phase of the optical field, and γ_e is the electron recombination rate in the absence of stimulated emission. The dot on the variables P, ϕ, and N denotes the time derivative. By using (8) and (9) in (10)–(12) the modified rate equations take the following form

$$\dot{P} = (G_L/\sqrt{1+p} - \gamma)P + R_{sp} \tag{13}$$

$$\dot{\phi} = \frac{\alpha_0}{2}(G_L - \gamma) - \frac{\beta}{2}\frac{G_L p}{1 + \sqrt{1+p}} \tag{14}$$

$$\dot{N} = I/q - \gamma_e N - G_L P/\sqrt{1+p} \tag{15}$$

where

$$G_L = \Gamma v_g g_L = G_N(N - N_0) \tag{16}$$

$$\gamma = \Gamma v_g g_{th} = G_N(N_{th} - N_0). \tag{17}$$

We have assumed that g_L varies linearity with an increase in the electron population, an approximation often used in practice [1]. The parameter γ represents the cavity decay rate and is related to the photon lifetime by $\tau_p = 1/\gamma$. Similarly, the carrier lifetime is given by $\tau_e = 1/\gamma_e$.

The modified rate equations include the effects of intraband relaxation through the p dependent terms. The modal gain is reduced by a factor $(1+p)^{1/2}$ as a result of gain nonlinearities. The phase of the optical field is affected by the index nonlinearities entering through the last term in (14). The parameter β controls the nonlinear phase change. We can estimate β by using (6) and assuming a Gaussian spectral profile for the linear gain, i.e.,

$$g_L(\omega) = g_L(\omega_p)\exp[-(\omega - \omega_p)^2/\Delta\omega_g^2] \tag{18}$$

where ω_p is the gain-peak frequency and $\Delta\omega_g$ is the gain bandwidth. The result is

$$\beta = \frac{-2(\omega_0 - \omega_p)}{\tau_{in}\Delta\omega_g^2}. \tag{19}$$

To the first order, $\Delta\omega_g \sim 1/\tau_{in}$. Thus, $\beta \sim -2\Delta\omega/\Delta\omega_g$, where $\Delta\omega = \omega_0 - \omega_p$ is the detuning of the laser mode from the gain peak. For a Fabry–Perot laser $\beta = 0$ since the lasing mode nearly coincides with the gain peak. However, $\beta \neq 0$ for DFB lasers which can operate at wavelengths away from the gain peak as a result of the feedback provided by the built-in grating. It can be positive or negative depending on whether the DFB laser opeates on the red or the blue side of the gain peak. Typical values of β are expected to be such that $|\beta| < 1$.

The dimensionless parameter p in (13)–(15) takes into account the effects of intraband carrier relaxation. It can be related to the output power by using the relation

$$p = \frac{|E_0|^2}{I_s} = \frac{P}{P_s} = \frac{P^{\text{out}}}{P_s^{\text{out}}} \tag{20}$$

where P_s is the saturation photon number and P_s^{out} is the saturation output power. The saturation photon number P_s is related to the saturation intensity I_s by the linear relation

$$P_s = \frac{\epsilon_0 \bar{n} n_g V}{\hbar\omega_0 \Gamma} I_s \tag{21}$$

where n_g is the group index, V is the active volume, and Γ is the confinement factor. The output saturation power is related to P_s by $P_s^{\text{out}} = (\eta_d \hbar\omega_0/\tau_p) P_s$, where η_d is the differential quantum efficiency of the output facet. Even though I_s is linearly related to P_s and P_s^{out}, the proportionality constant involves a large number of device parameters such as the active-region dimensions, the confinement factor, and the facet reflectivities. It also depends on whether the semiconductor laser is Fabry–Perot or DFB type. For this reason P_s^{out} is device dependent even though I_s depends only on material parameters.

We shall present our results in terms of the dimensionless parameter p in order to make them device independent. However, it is useful to provide an order-of-magnitude estimate of P_s^{out}. We consider a 1.55 μm buried-heterostructure laser as the gain-saturation effects are stronger at longer wavelengths. The parameter I_s is estimated from (5) by using $\mu = 9 \cdot 10^{-29}$ mC, $\tau_{in} = 0.1$ ps, $\tau_c = 0.3$ ps, and $\tau_v = 0.07$ ps. It is given by $I_s \approx 3.3 \cdot 10^{13}$ (V/m)^2. The energy density is related to I_s by $\epsilon_0 \bar{n} n_g I_s$ and is estimated to be 3.8 mJ/cm^3 by using $\bar{n} = 3.3$ and $n_g = 4$. For a mode volume $V/\Gamma = 2 \cdot 10^{-10}$ cm^3, the saturation photon number $P_s = 5 \cdot 10^6$. The output saturation power can range from ~20–100 mW depending on what fraction of the intracavity photons escapes from the output facet. It is generally smaller for longer wavelength lasers as I_s scales as λ^{-2}. It is also expected to be smaller for quantum-well semiconductor lasers for which I_s is reduced as a result of the quantum-confinement effects [11].

III. Small-Signal Modulation

The small-signal modulation response is obtained by solving the rate equations (13)–(15) with the injected current

$$I = I_b + I_m \exp(i\omega_m t) \tag{22}$$

such that $I_m << I_b - I_{th}$, where I_{th} is the threshold current, I_b is the bias current, I_m is the modulation current, and ω_m is the modulation frequency. The dynamic variables follow the sinusoidal variation approximately and have a solution of the form

$$P(t) = \bar{P} + \delta P \exp(i\omega_m t) \tag{23}$$

with similar relations for N and ϕ. The steady-state values $\bar{P}$, $\bar{N}$, and $\bar{\phi}$ correspond to the bias level I_b. The modulation response is obtained by linearizing (13)–(15) in terms of δP, δN, and $\delta\phi$. The result for δP is [1] (overbar over the steady-state quantities is dropped for notational simplicity)

$$\delta P(\omega_m) = \frac{G_N P (I_m/q)(1+p)^{-1/2}}{(\Omega_R + \omega_m - i\Gamma_R)(\Omega_R - \omega_m + i\Gamma_R)} \tag{24}$$

where Ω_R and Γ_R are the frequency and the damping rate of relaxation oscillations and are given by

$$\Omega_R = \left[G_L G_N P \frac{1 + p/2}{(1+p)^2} - \frac{1}{4}(\Gamma_p - \Gamma_N)^2 \right]^{1/2} \tag{25}$$

$$\Gamma_R = (\Gamma_N + \Gamma_p)/2 \tag{26}$$

$$\Gamma_N = \gamma_e + N\frac{\partial \gamma_e}{\partial N} + \frac{G_N P}{(1+p)^{1/2}} \tag{27}$$

$$\Gamma_P = \frac{R_{sp}}{P} + \frac{G_L}{2}\frac{p}{(1+p)^{3/2}}. \tag{28}$$

The effect of gain nonlinearities is included through $p = P/P_s$. The nonlinear index affects only the modulated phase that is given by

$$\delta\phi = \frac{\delta P}{2i\omega_m P}\left[\alpha_0(\Gamma_p + i\omega_m) - \frac{\beta G_L p}{2(1+p)^{1/2}} \right]. \tag{29}$$

Equations (25)–(28) show that relaxation oscillations are strongly affected by gain nonlinearities. By keeping only the dominant term, the frequency and the damping rate of relaxation oscillations are found to depend on the mode intensity through

$$\Omega_R^2 \cong \frac{G_N P_s}{\tau_p}\frac{(1 + p/2)\,p}{(1+p)^2} \tag{30}$$

$$\Gamma_R \cong \frac{1}{\tau_p}\frac{p/4}{(1+p)^{3/2}} \tag{31}$$

where we used $\tau_p = 1/G_L$ to express the results in terms of the photon lifetime. Typically, τ_p = 1–2 ps for semiconductor lasers while $G_N P_s \sim 10^{10}\ \mathrm{s}^{-1}$. It is easy to verify that Ω_R takes its maximum value for $p \rightarrow \infty$ whereas Γ_R peaks at $p = 2$. The maximum values of Ω_R and Γ_R are given by

$$\Omega_{\max} = \left(\frac{G_N P_s}{2\tau_p}\right)^{1/2}, \quad \Gamma_{\max} = \frac{1}{6\sqrt{3}}\frac{1}{\tau_p}. \tag{32}$$

By using $G_N P_s = 1 \cdot 10^{11}\ \mathrm{s}^{-1}$ and $\tau_p = 1$ ps, we estimate that $\Omega_{\max}/2\pi \cong 35$ GHz and $\Gamma_{\max} \simeq 1 \cdot 10^{11}\ \mathrm{s}^{-1}$.

It is interesting to compare (30) and (31) with the corresponding expressions obtained when (9) is replaced by

$$g = g_L/(1 + p). \tag{33}$$

This form of the nonlinear gain has been used in some previous work [2], [8] in analogy with a two-level system. The relaxation-oscillation frequency Ω_R and the damping rate Γ_R are then given by

$$\Omega_R^2 \cong \frac{G_N P_s}{\tau_p}\frac{p}{(1+p)^3} \tag{34}$$

$$\Gamma_R \cong \frac{1}{\tau_p}\frac{p/2}{(1+p)^2} \tag{35}$$

and should be compared with (30) and (31). Figs. 1 and 2 show the variation of $\Omega_R/\Omega_{\max}$ and $\Gamma_R/\Omega_{\max}$ with p for the two different functional forms of the nonlinear gain. Ω_R varies linearly with $\sqrt{p}$ for $p << 1$ in both cases. This linear dependence has been observed in many experiments [8], [10]. A sublinear increase with $\sqrt{p}$ begins to occur for $p > 0.1$ for both nonlinear-gain models. However, the sublinear behavior is more pronounced when the phenomenological model (33) is used. In fact, Ω_R peaks at $p = 0.5$ and then begins to decrease according to this model. By contrast, our model shows that Ω_R continues to increase sublinearly at all power levels and attains its maximum value asymptotically for $p >> 1$. The maximum value is larger by about a factor of 2 than the peak value predicted by (34).

The intensity dependence of the damping rate Γ_R is also quite different for the two models. Both models predict a linear increase of Γ_R with p for $p << 1$, as also observed experimentally [10]. However, the growth rate is smaller by a factor of 2 when (9) is used for the nonlinear gain. The peak value is also smaller by about 30% in this case. These differences can be used to verify experimentally which nonlinear-gain model is appropriate for semiconductor lasers. Experimental verification would require high-power DFB semiconductor lasers. In one experiment [18] the relaxation oscillation frequency was measured as a function of the output power for a 1.2 μm DFB laser. The data showed a linear increase of Ω_R with $\sqrt{p}$ up to about 20 mW and a slightly sublinear increase in the range 20–40 mW. The output saturation power for this laser is estimated to be about 100 mW. The experimentally observed behavior supports the use of (9) in place of (33) since the saturation of Ω_R with $\sqrt{p}$ is less pronounced than predicted by (33) [curve (b) of Fig. 1].

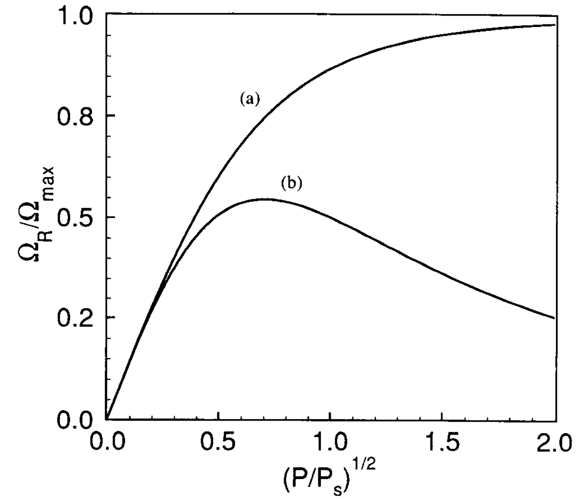

Fig. 1. Variation of the relaxation-oscillation frequency with $(P/P_s)^{1/2}$ for (a) $g = g_L/\sqrt{1+p}$ and (b) $g = g_L/(1+p)$ as the functional form of the nonlinear gain. The parameter $p = P/P_s$.

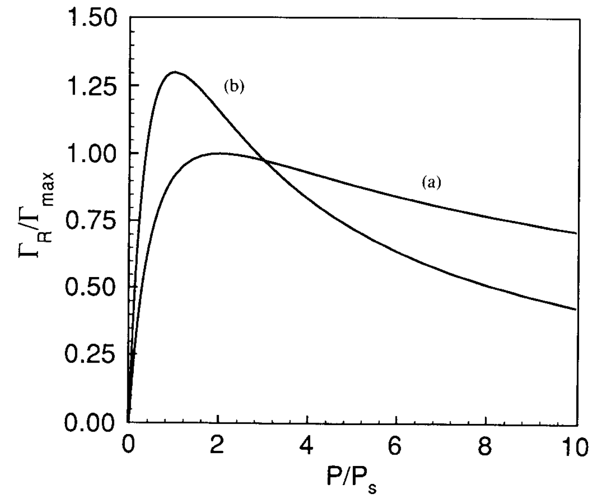

Fig. 2. Variation of the damping rate of relaxation oscillations with $p = P/P_s$ for (a) $g = g_L/\sqrt{1+p}$ and (b) $g = g_L/1 + p)$ as the functional form of the nonlinear gain.

The quantity of interest from a practical standpoint is not the relaxation-oscillation frequency Ω_R but the 3 dB bandwidth $\Delta\omega_{3\mathrm{dB}}$, defined as the modulation frequency at which the modulation response drops by a factor of 2 from its zero-frequency value, i.e.,

$$\left|\delta P_m(\Delta\omega_{3\mathrm{dB}})/\delta P_m(0)\right| = \tfrac{1}{2}. \tag{36}$$

By using (24) and (36), $\Delta\omega_{3\mathrm{dB}}$ is found to be related to the relaxation-oscillation parameters by the relation

$$\Delta\omega_{3\mathrm{dB}}^2 = \Omega_R^2 - \Gamma_R^2 + 2\left[\Omega_R^2(\Omega_R^2 + \Gamma_R^2) + \Gamma_R^4\right]^{1/2}. \tag{37}$$

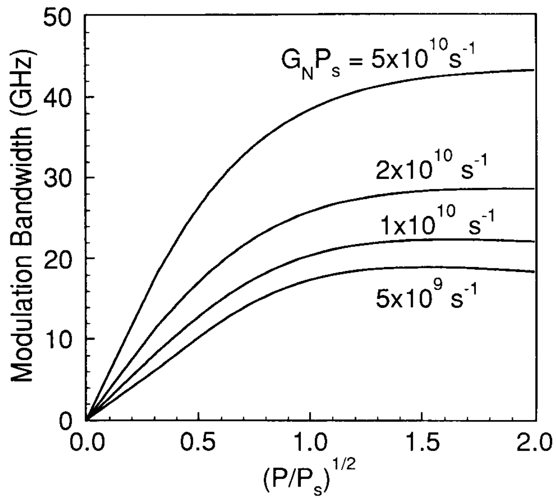

Fig. 3. Variation of the 3 dB modulation bandwidth with $(P/P_s)^{1/2}$ for several values of $G_N P_S$. The photon lifetime $\tau_P = 1$ ps.

Fig. 3 shows the variation of $\Delta\omega_{3\,\text{dB}}/2\pi$ as a function of $\sqrt{p}$ for several values of $G_N P_S$ with $\tau_p = 1$ ps. For relatively small values of $G_N P_S$, $\Delta\omega_{3\,\text{dB}}$ exhibits a shallow maximum in the vicinity of $p \sim 1$. In most cases of practical interest, however, $\Delta\omega_{3\,\text{dB}}$ is expected to saturate to a limiting value. The limiting value is obtained from (37) and is given by

$$\Delta\omega_{3\text{dB}}^{\max} \cong \sqrt{3}\,\Omega_{\max} = \left(\frac{3}{2}\frac{G_N P_s}{\tau_p}\right)^{1/2} \tag{38}$$

where we used (32) and assumed that $\Gamma_R << \Omega_R$. We can write (38) in terms of the material parameters by using $G_N = \Gamma v_g a/V$, where a is the differential gain (the derivative of g_L with respect to the carrier density). If we use P_s from (21), the result is

$$\Delta\omega_{3\text{dB}}^{\max} = \left(\frac{3\epsilon_0 c\bar{n} a I_s}{2\hbar\omega_0 \tau_p}\right)^{1/2}. \tag{39}$$

Except for the photon lifetime τ_p and the photon energy $\hbar\omega_0$, $\Delta\omega_{3\text{dB}}^{\max}$ depends only on the material parameters. The most crucial parameter is the differential gain coefficient a that depends on the density of states and can be enhanced by using a quantum-well structure. Consider the case of a 1.55 μm InGaAsP laser with a photon lifetime $\tau_p = 1.5$ ps. In the case of a conventional double-heterostructure design, $a = 2 \cdot 10^{-16}$ cm^2. Using the value of $I_s \simeq 3 \cdot 10^{13}$ $(\text{V/m})^2$ estimated in Section II, the 3 dB bandwidth is found to be limited to about 32 GHz. This value increases to 40 GHz if the photon lifetime reduces to 1 ps. Similarly, it can increase considerably if the differential gain a is enhanced. For a quantum-well design, a is expected to be enhanced by a factor 2–3. The saturation intensity I_s also depends on the density of states through the dipole moment μ, and is generally reduced [11] for a quantum well by a factor in the range 1–2. Thus, the limiting 3 dB bandwidth is expected to be larger for quantum-well lasers but by not more than 50% of that of the conventional lasers.

IV. Large-Signal Modulation

In the case of large-signal modulation the semiconductor laser is biased close to threshold and modulated with nearly rectangular pulses of large amplitude. The optical pulse is obtained by solving numerically the modified rate equations (13)–(15) with the injected current

$$I(t) = I_b + I_m f(t) \tag{40}$$

where I_b is the bias current, I_m is the modulation current, and $f(t)$ governs the shape of the current pulse. We use a super-Gaussian model to represent a nearly rectangular current pulse and choose

$$f(t) = \exp\left[-\left(\frac{2t}{T_b}\right)^{T_b/T_r}\right] \tag{41}$$

where T_r is the rise time and T_b is the pulse duration or the bit slot at the bit rate $B = 1/T_b$. We use $T_r = 0.2\,T_b$ as a representative value.

Fig. 4 shows the simulated optical pulse shapes for a 1.55 μm semiconductor laser biased at threshold ($I_b = I_{\text{th}}$) and modulated at 2 Gb/s ($T_b = 0.5$ ns) with $I_m = 5I_{\text{th}}$ to provide about 12 mW of on-state output power P_{on}. The effect of gain nonlinearity is demonstrated by changing the output saturation power P_s^{out} in the range 10–30 mW. In terms of the dimensionless parameter $p = P_{\text{on}}/P_s^{\text{out}}$, for $p < 0.5$ the main effect of nonlinear gain is to suppress relaxation oscillations. However, for larger values of p the rise and fall times associated with the optical pulse increase considerably. The increase in rise time reduces the modulation efficiency since the peak power never reaches its on-state value expected in the absence of gain nonlinearities. The increase in fall time results in a long trailing edge. In an actual communication system the energy in the trailing edge would appear in the neighboring bits and would affect the system performance through intersymbol interference. The fall-time effects become more severe at higher bit rates. This is shown in Fig. 5 where optical pulse shapes are shown for $B = 5$ Gb/s ($T_b = 0.2$ ns) while keeping other parameters identical to those of Fig. 4. The trailing edge begins to extend to the third neighboring bit for $p \sim 1$. The main conclusion is that gain nonlinearities limit the on-state power well below the saturation-power level to avoid the detrimental effect of long fall times.

Index nonlinearities affect only the frequency chirp imposed on the optical pulse as a result of the time-varying optical phase. Fig. 6 shows the frequency chirp for $\beta = -1$, 0 and 1 for $\alpha_0 = 5$, $B = 2$ Gb/s and $P_s^{\text{out}} = 20$ mW. The other parameters are the same as in Fig. 4. For $\beta = 0$, the chirp is solely due to carrier-induced index changes. For $\beta \neq 0$, index nonlinearities [the last term in (14)] increase the chirp for negative β and decrease it for positive β. The contribution of the nonlinear index to the fre-

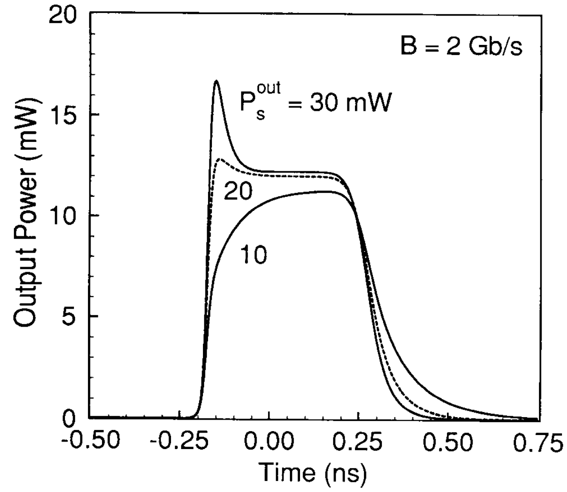

Fig. 4. Simulated pulse shapes for several values of the saturation output power under strong-signal modulation at 2 Gb/s. The semiconductor laser is biased at threshold and modulated five times above threshold.

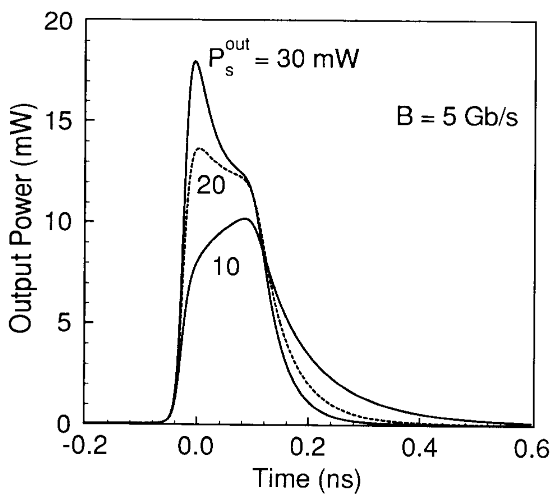

Fig. 5. Same as for Fig. 4 except that the bit rate B = 5 Gb/s.

quency chirp is generally small as $\beta << 1$ under typical operating conditions. In particular, this contribution is absent for Fabry–Perot lasers operating at the gain peak. For DFB lasers the index contribution depends on the detuning of the laser wavelength from the gain peak. Note, however, that the linewidth enhancement factor α_0 itself depends on the detuning $\Delta\omega = \omega_0 - \omega_p$. It is well known [19] that α_0 increases for $\Delta\omega < 0$ and decreases for $\Delta\omega > 0$. Thus, the frequency chirp can be reduced by detuning a DFB laser toward the high-frequency side of the gain peak. Since β is negative under those conditions, the nonlinear index contribution to the frequency chirp is positive. As a result, the chirp would be enhanced from its low-power value. This result can be understood from (8) by noting that the two contributions to the index change are opposite in sign.

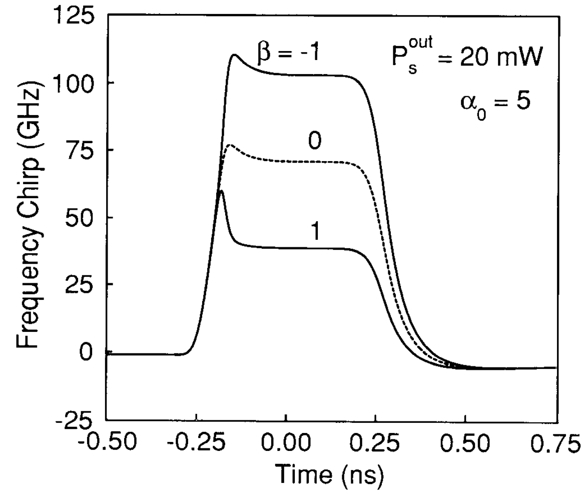

Fig. 6. Frequency chirp profiles for three values of β with α_0 = 5 and P_s^{out} = 20 mW showing the effect of index nonlinearities.

V. Laser Linewidth

The gain and index nonlinearities resulting from intraband relaxation of charge carriers also affect the noise characteristics of semiconductor lasers. In this section we show how such nonlinearities can lead to saturation and rebroadening of the laser linewidth with an increase in the optical power and give rise to the linewidth floor, a phenomenon that has attracted considerable attention recently [20]–[25].

For the discussion of the laser linewidth the modified rate equations (13)–(15) are supplemented with the Langevin noise sources and take the form

$$\dot{P} = (G_L/\sqrt{1+p} - \gamma)P + R_{sp} + F_P(t) \tag{42}$$

$$\dot{\phi} = \frac{\alpha_0}{2}(G_L - \gamma) - \frac{\beta}{2}\frac{G_L p}{1 + \sqrt{1+p}} + F_\phi(t) \tag{43}$$

$$\dot{N} = I/q - \gamma_e N - G_L P/\sqrt{1+p} + F_N(t) \tag{44}$$

where the random noise variables F_p, F_ϕ, and F_N are assumed to have zero mean and taken to be delta-correlated in the Markoffian approximation, i.e.,

$$\langle F_i(t) F_j(t') \rangle = 2D_{ij}\delta(t - t'). \tag{45}$$

The diffusion coefficients D_{ij} for $i, j = P, \phi, N$ are given by [1], [26]

$$D_{PP} = R_{sp}P, \qquad D_{\phi\phi} = R_{sp}/4P, \quad D_{P\phi} = 0 \tag{46}$$

$$D_{NN} = R_{sp}P + \gamma_e N, \quad D_{PN} = -R_{sp}P, \quad D_{N\phi} = 0. \tag{47}$$

The spectrum of the emitted light is obtained by following a standard procedure [1], [26]. Although straightforward in its implementation, the calculation is lengthy and requires a numerical approach in the general case in which the effect of relaxation oscillations is included. Relaxation oscillations are known to give rise to a number of weak satellites at the multiples of the relaxation-oscillation fre-

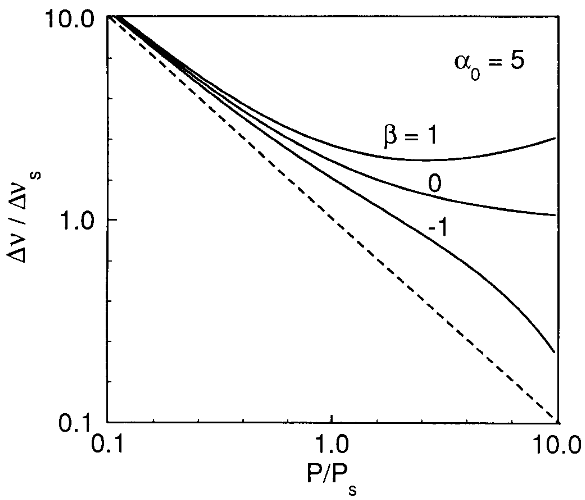

Fig. 7. Variation of the laser linewidth with normalized output power P/P_s for three values of β. Dashed line shows the expected behavior in the absence of gain and index nonlinearities. The case $\beta = 0$ corresponds to a laser oscillating at the gain peak for which index nonlinearities are absent.

quency [26]. The laser linewidth is usually defined as the fullwidth at half maximum (FWHM) of the central peak. When the contribution of weak satellites to the spectrum is ignored, the central peak is found to be Lorentzian with a FWHM given by [1]

$$\Delta\nu = S_{\dot\phi}(0)/2\pi \tag{48}$$

where

$$S_{\dot\phi}(\omega) = \langle |\omega\delta\tilde{\phi}(\omega)|^2 \rangle \tag{49}$$

and $\delta\tilde{\phi}(\omega)$ is the Fourier transform of the phase fluctuation $\delta\phi(t)$ obtained by solving (42)–(44). When (42)–(44) are linearized around the steady state and the short-noise contribution is neglected by setting $F_N = 0$, $\delta\tilde{\phi}(\omega)$ is found to be given by

$$\delta\tilde{\phi}(\omega) = \frac{1}{i\omega}\left[\tilde{F}_\phi(\omega) - \frac{\alpha_0 G_L G_N(2+p)/(1+p) + \beta G_L(\Gamma_N + i\omega)/P_s}{4\sqrt{1+p}\,(\Omega_R + \omega - i\Gamma_R)(\Omega_R - \omega + i\Gamma_R)}\tilde{F}_p(\omega)\right] \tag{50}$$

where $\tilde{F}_\phi$ and $\tilde{F}_p$ are the Fourier transforms of F_ϕ and F_p, respectively. Equation (50) shows that gain and index nonlinearities appear only in the second term that represents the contribution to the phase fluctuations occurring as a result of fluctuations in the refractive index. By using (45)-(50), the linewidth is found to be given by

$$\Delta\nu \cong \frac{R_{sp}}{4\pi P}(1 + \alpha_{eff}^2) \tag{51}$$

where the effective linewidth enhancement factor is given by

$$\alpha_{eff} = \alpha_0\sqrt{1+p} + \beta p(1+p)/(2+p). \tag{52}$$

Two approximations were made in obtaining (51): Γ_N was assumed to be dominated by the last term in (27) and Ω_R was assumed to be much larger than Γ_R. Equation (52) differs from the corresponding expression in [25] where a complete Langevin analysis was not carried out [27]. The two results agree only for $\beta = 0$.

The physical meaning of (51) is clear. Since α_{eff} is itself power dependent, the inverse dependence of $\Delta\nu$ on the output power occurs only for lower power levels such that $P << P_s$. When the output power exceeds the saturation level, the linewidth $\Delta\nu$ is also expected to saturate and may even exhibit rebroadening depending on the value of the parameter β. In order to show these features quantitatively, we define $\Delta\nu_s$ as the expected linewidth at $P = P_s$ in the absence of gain and index nonlinearities, i.e.,

$$\Delta\nu_s = \frac{R_{sp}}{4\pi P_s}(1 + \alpha_0^2) \tag{53}$$

and plot the ratio

$$\frac{\Delta\nu}{\Delta\nu_s} = \frac{1 + \alpha_{eff}^2}{1 + \alpha_0^2}\frac{1}{p} \tag{54}$$

in Fig. 7 as a function of $p = P/P_s$ for $\alpha_0 = 5$ and three different values of β. Consider first the case $\beta = 0$ corresponding to a laser oscillating at the gain peak. As evident in Fig. 7, the linewidth $\Delta\nu$ saturates to a value approximately given by $\Delta\nu_s$ for $p >> 1$. In other words, $\Delta\nu_s$ represents the power-independent contribution to the linewidth. Note that this power-independent contribution is solely due to gain saturation induced by a finite intraband relaxation time of charge carriers in semiconductor lasers. Indeed, $\Delta\nu_s$ can be written in terms of the material parameters by using (5) and (21) in the form

$$\Delta\nu_s = \frac{R_{sp}\omega_0\mu^2\tau_{in}(\tau_c + \tau_v)(1 + \alpha_0^2)}{4\pi\epsilon_0\hbar\bar{n}n_g(V/\Gamma)}. \tag{55}$$

This equation clearly shows that $\Delta\nu_s = 0$ if $\tau_{in} = 0$. Note also that $\Delta\nu_s$ can be reduced by increasing the mode volume V/Γ. Typical values of $\Delta\nu_s$ are in the range ~1–10 MHz and agree with the experimentally observed values [20], [23].

We now discuss the effect of index nonlinearities on the linewidth for $\beta \neq 0$. Fig. 7 shows that the linewidth $\Delta\nu$ is reduced below $\Delta\nu_s$ for negative values of β while rebroadening of the linewidth occurs for positive values of β. Equation (19) shows that β is positive for DFB lasers operating on the low-frequency (red) side of the gain peak. Thus, DFB lasers detuned on the red side of the gain peak are expected to exhibit a slight rebroadening of the linewidth. By contrast, DFB lasers detuned on the blue side can have a line width $\Delta\nu < \Delta\nu_S$. It should be noted that the parameter α_0 itself depends on detuning of the laser wavelength from the gain peak; it is generally lower for lasers detuned on the blue side [19]. It is clear from the above discussion that high-power DFB lasers should be detuned on the blue side of the gain peak for the purpose of reducing the laser linewidth.

VI. Relative Intensity Noise

In this section we briefly consider how the RIN of single-mode semiconductor lasers is affected by the gain and index nonlinearities. The RIN is defined by [1]

$$\mathrm{RIN} = \langle |\delta \tilde{P}(\omega)|^2 \rangle / P^2 \tag{56}$$

where P is the average power and $\delta\tilde{P}(\omega)$ is the Fourier transform of the power fluctuation $\delta P(t)$. Equations (42)–(44) can be used to obtain $\delta\tilde{P}(\omega)$ by linearizing them around the steady state and the result is

$$\delta\tilde{P}(\omega) = \frac{(\Gamma_N + i\omega)\tilde{F}_p + G_N P(1+p)^{-1/2}\tilde{F}_N}{(\Omega_R + \omega - i\Gamma_R)(\Omega_R - \omega + i\Gamma_R)}. \tag{57}$$

By using (45)–(47) and (57) in (56), we obtain the following result

$$\mathrm{RIN} = \frac{2R_{sp}[(\Gamma_N^2 + \omega^2) + G_N^2 P^2 (1+p)^{-1}(1 + \gamma_e N / R_{sp} P)]}{P[(\Omega_R^2 - \omega^2)^2 + (2\omega\Gamma_R)^2]}. \tag{58}$$

For $p \ll 1$, the above expression reduces to the well-known expression of the RIN obtained by ignoring intraband gain saturation [1]. For larger values of p, the power dependence of RIN is quite different since Γ_N, Ω_R, and Γ_R also depend on p through (25)–(27). Fig. 8 shows the power dependence of RIN for three different values of P_s by using typical parameter values of semiconductor lasers. The RIN nearly saturates when the output power exceeds the saturation power level. The limiting value is larger for lasers with a smaller output saturation power indicating that gain nonlinearities increase the RIN. It should be emphasized that index nonlinearities have no effect on RIN as they affect only the phase of the optical field.

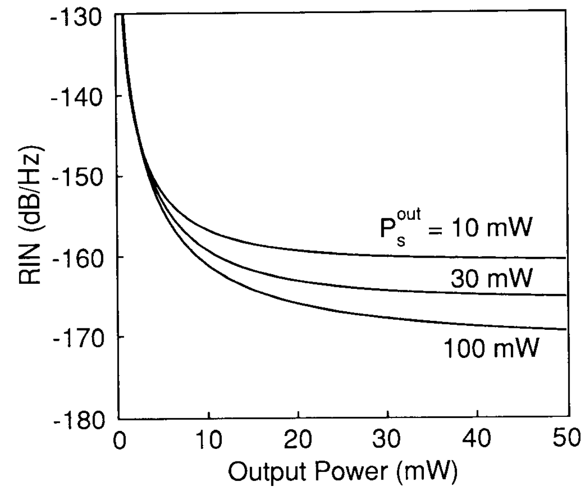

Fig. 8. Variation of RIN with output power for three values of the saturation output power P_S^{out}.

VII. Summary

This paper has addressed how the intraband relaxation time τ_{in}, although small ($\tau_{in} \simeq 0.1$ ps) but finite, affects the performance of semiconductor lasers. At high laser powers when the rate of stimulated emission becomes comparable to τ_{in}^{-1}, the optical gain begins to saturate as charged carriers cannot relax as fast as required by laser dynamics. This intraband gain saturation is accompanied by a change in the modal refractive index when the laser is operating at a wavelength detuned from the gain peak. The gain and index nonlinearities, occurring as a result of a finite intraband relaxation time, affect a large number of laser characteristics. We have studied the effect of gain and index nonlinearities by deriving a set of modified rate equations. These equations are used to discuss the modulation response and noise characteristics of semiconductor lasers.

Intraband gain saturation affects both the frequency and the decay rate of relaxation oscillations. In particular, the 3 dB small-signal modulation bandwidth is limited to a maximum value determined by the material parameters. The laser linewidth is also found to have a power-independent part as a result of intraband gain saturation that manifests as linewidth saturation at high operating powers. Linewidth rebroadening can occur when the laser operates on the red side of the gain peak so that index nonlinearities increase the effective linewidth enhancement factor. It should be remarked that many other physical factors can contribute to linewidth saturation [21]–[24]. We also note that the effect of gain nonlinearities is particularly important for quantum-well lasers for which the intraband saturation intensity is reduced as a result of quantum confinement [11].

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[27] In [25] β should be replaced by $-\beta$ in (14) and Fig. 1. The linewidth enhancement factor α in (15) should be identified with (52) of this paper to obtain the correct result.

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