Spectrum-enhanced spreading of partially coherent beams

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We investigate the effects of a finite spectral width on the propagation characteristics of partially coherent gaussian beams. Our results predict that quasi-monochromatic and broad band beams exhibit intensity profiles that are appreciably different from those of monochromatic beams. As the beam propagates in free space, the width and the shape of the intensity profile depend not only on the width and the shape of the spectral profile of the light in the source plane, but also on its coherence properties. We illustrate the general phenomenon of spectrum-enhanced spreading of partially coherent beams for several spectral lineshapes.

1. Introduction

The spreading of optical beams is traditionally studied in the context of coherent fields [1]. It has been noted by several investigators that partially coherent gaussian beams spread faster than fully coherent beams of the same intensity profile (see, e.g. refs. [2,3]). In previous investigations only effects of spatial coherence were considered for gaussian beams at a single frequency. In practice, optical beams have a finite spectral width, which raises the question whether diffractive spreading is affected by the spectrum of the light on propagation in free space. In this paper we study this problem by considering the propagation of optical beams which are partially coherent both spatially and temporally. This study is motivated by the recent discussion of the so-called Wolf effect, which describes the spectral changes occurring on propagation of partially coherent fields as a result of source correlations [4].

Our approach is based on considering the propagation of the cross-spectral density within the paraxial approximation. We find that the intensity profile of an optical beam is affected by the spectrum of the light associated with the beam. In general, beam spreading is increased and the beam profile is distorted. In particular we find that a gaussian beam does not remain gaussian when spectral effects are included. The effects are weak for quasi-monochromatic beams, for which the spectral width $\Delta\omega_0$ is much smaller than the mean frequency ω_0. Nevertheless, measurable effects are predicted to occur for beams having a relatively broad spectrum ($\Delta\omega_0/\omega_0 \sim 0.1$). Spectrum-enhanced beam spreading also depends on the shape of the spectral profile. We illustrate our results by considering a lorentzian spectrum, a gaussian spectrum, and a spectrum consisting of two narrow lorentzian lines. The spectral effects are found to be strongest for the lorentzian spectrum.

2. Propagation of partially coherent beams

We consider the propagation of polychromatic partially coherent beams in free space and in homogeneous non-dispersive media by investigating the changes in the cross-spectral density $W(\boldsymbol{r}_1, \boldsymbol{r}_2; \omega)$ on propagation. The propagation in free space is governed by the formula [5]

$$W(\boldsymbol{r}_1, \boldsymbol{r}_2; \omega) = \iint K^*(\boldsymbol{r}_1, \boldsymbol{R}_1; \omega) K(\boldsymbol{r}_2, \boldsymbol{R}_2; \omega) \times W(\boldsymbol{R}_1, \boldsymbol{R}_2; \omega)\, \mathrm{d}^2 R_1\, \mathrm{d}^2 R_2 , \quad (1)$$

where the integration is performed twice over the plane $z=0$ at which the cross-spectral density $W(\boldsymbol{R}_1, \boldsymbol{R}_2; \omega)$ is assumed to be known. In the angular spectrum representation the propagation kernel $K(\boldsymbol{r}, \boldsymbol{R}; \omega)$ has the form [6]

$$K(\boldsymbol{r},\boldsymbol{R};\omega)=\frac{k^2}{4\pi^2}\times\iint_{-\infty}^{\infty}\exp\{ik[p(x-X)+q(y-Y)+mz]\}\,dp\,dq\,, \tag{2}$$

where $k=\omega/c$ is the wavenumber, c is the speed of light in vacuum, and

$$m=\sqrt{1-p^2-q^2} \quad \text{if } p^2+q^2\leqslant 1\,,$$
$$=i\sqrt{p^2+q^2-1} \quad \text{if } p^2+q^2>1\,. \tag{3}$$

Propagation in homogeneous non-dispersive media of constant refractive index n can be studied by replacing k by nk in eq. (2). The intensity of the field at an arbitrary point $\boldsymbol{r}$ in the beam is obtained from the formula

$$I(\boldsymbol{r})=\int_{-\infty}^{\infty}W(\boldsymbol{r},\boldsymbol{r};\omega)\,d\omega\,. \tag{4}$$

Eqs. (1)-(4) govern the propagation of partially coherent fields in free space. When the field is beam like, we may simplify expression (2) by using the paraxial approximation. This amounts to assuming that only waves for which $p^2+q^2\ll 1$ contribute significantly to the field so that m can be approximated by $m\approx 1-(p^2+q^2)/2$. The propagation kernel then takes the form

$$K(\boldsymbol{r},\boldsymbol{R};\omega)=\frac{k\exp(ikz)}{2\pi iz}\times\exp\{(ik/2z)[(x-X)^2+(y-Y)^2]\}\,. \tag{5}$$

To carry out the integrations indicated in eq. (1), we need to specify $W(\boldsymbol{R}_1,\boldsymbol{R}_2;\omega)$ in the plane $z=0$ of the secondary source. We choose as the source the so-called gaussian Schell model [7] source. The cross-spectral density of the field in the source plane is then given by

$$W(\boldsymbol{R}_1,\boldsymbol{R}_2;\omega)=S^{(0)}(\omega)\times\exp[-(\boldsymbol{R}_1^2+\boldsymbol{R}_2^2)/4\sigma_I^2-(\boldsymbol{R}_2-\boldsymbol{R}_1)^2/2\sigma_g^2]\,, \tag{6}$$

where σ_I is the root-mean-square (rms) width of the intensity profile and σ_g is the effective spatial coherence length of the light in the source plane. Temporal coherence depends on the source spectrum $S^{(0)}(\omega)$, which we assume to be the same at all source points. We take the spectrum to be normalized so that

$$\int_{-\infty}^{\infty}S^{(0)}(\omega)\,d\omega=A\,, \tag{7}$$

where A is a positive constant. It is easy to verify by using eqs. (4) and (6) that the intensity profile in the source plane $z=0$ corresponds to that of a gaussian beam, i.e., that in the source plane

$$I(\boldsymbol{R})=A\exp(-R^2/2\sigma_I^2)\,. \tag{8}$$

The commonly used spot size [8] w_0 is equal to $2\sigma_I$. Other beam profiles can be considered by appropriately modifying eq. (6).

In what follows we focus our attention on gaussian beams propagation. To obtain an expression for the intensity of the beam, we substitute from eqs. (1), (5), and (6) in eq. (4). The integration over $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ can be performed analytically and one then finds that

$$I(\boldsymbol{r})=\int_{-\infty}^{\infty}d\omega\,S^{(0)}(\omega)\times\frac{1}{1+\xi^2}\exp\left(-\frac{x^2+y^2}{2\sigma_I^2}\frac{1}{1+\xi^2}\right), \tag{9}$$

where

$$\xi=z\sqrt{1+4\sigma_I^2/\sigma_g^2}/2k\sigma_I^2\,. \tag{10}$$

We see from eq. (9) that the intensity $I(\boldsymbol{r})$ depends on the frequency ω not only through the source spectrum $S^{(0)}(\omega)$ but also through the parameter ξ which depends on the wavenumber $k=\omega/c$. For a monochromatic source $S^{(0)}(\omega)=A\delta(\omega-\omega_0)$, and eq. (9) then gives

$$I(\boldsymbol{r})=\frac{A}{1+(z/z_{\text{eff}})^2}\times\exp\left(-\frac{x^2+y^2}{2\sigma_I^2}\frac{1}{1+(z/z_{\text{eff}})^2}\right), \tag{11}$$

where

$$z_{\text{eff}}=z_R\alpha/\sqrt{1+\alpha^2}\,, \tag{12}$$

with

$$z_R = 2\omega_0 \sigma_I^2 / c\,, \quad \alpha = \sigma_g / 2\sigma_I\,. \tag{13}$$

The parameter z_R is the Rayleigh range [8] or diffraction length defined for a monochromatic beam of frequency ω_0, and α is a measure of the spatial coherence of the light in the source plane. For a spatially incoherent source $\alpha \to 0$. The physical meaning of z_{eff} is evident: it is the effective diffraction length of a gaussian beam that is spatially partially coherent. It follows from eq. (12) that the effective diffraction length decreases as the beam becomes more and more incoherent. Eq. (11) is in agreement with previous investigations on fully coherent [8] and partially (spatially) coherent [3] gaussian beams.

3. Spectrum-enhanced beam spreading

It is evident from eq. (9) that the beam profile depends on the parameter ξ and on the shape and the width of the source spectrum $S^{(0)}(\omega)$. To illustrate this dependence, we assume that the source spectrum consists of a single lorentzian line of width $\Delta\omega_0$ centered at ω_0, viz.,

$$S^{(0)}(\omega) = \frac{A}{\pi} \frac{\Delta\omega_0}{(\omega - \omega_0)^2 + \Delta\omega_0^2}\,. \tag{14}$$

Other spectral profiles will be considered later. We substitute $S^{(0)}(\omega)$ from eq. (14) into eq. (9) and evaluate the integral numerically. It is convenient to define the relative linewidth through the parameter $\delta = \Delta\omega_0/\omega_0$. In the limit as $\delta \to 0$, $S^{(0)}(\omega) \to A\delta(\omega - \omega_0)$, and the gaussian beam becomes monochromatic. For quasi-monochromatic beams $\delta \ll 1$.

Fig. 1 presents a comparison of the intensity profiles for the case when $z/z_{eff} = 2$ and $\delta = 0$ and $\delta = 0.1$. The intensity profile is significantly distorted as a result of the finite spectral width. In particular, the intensity profile does not remain gaussian. This observation is significant because within the accuracy of the paraxial approximation [9] coherent gaussian beams are known to retain their gaussian intensity profile on propagation [1,8]. The beams retain the gaussian intensity profile even when they are spatially partially coherent. However, the profiles depart from the gaussian shape when spectral effects are incorporated. The extent of the beam distortion

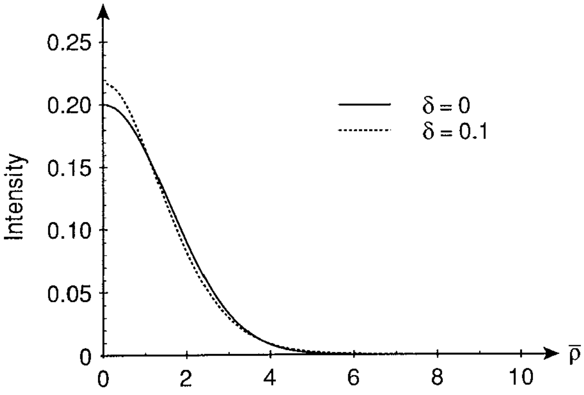

Fig. 1. Intensity profiles of partially coherent gaussian beams at a propagation distance $z/z_{eff} = 2$. The radial distance is normalized so that $\bar{\rho} = \rho/\sqrt{2}\sigma_I$. The intensity is normalized to the peak intensity at $z = 0$. The parameter $\delta = \Delta\omega_0/\omega_0$ is a measure of the relative linewidth of the lorentzian spectrum.

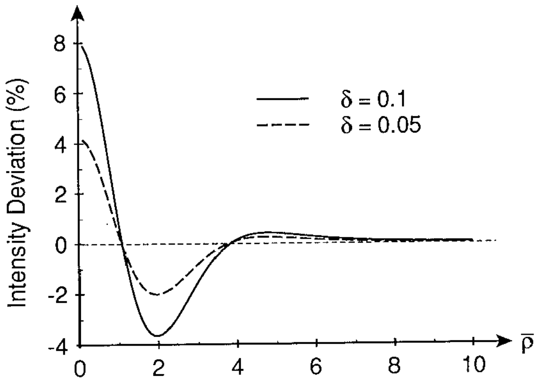

Fig. 2. Intensity deviation $d(\rho)$ defined by eq. (15) for two values of δ and for a propagation distances $z = 2z_{eff}$.

and its dependence on δ is shown in fig. 2, where the intensity deviation

$$d(\rho) = [I(\rho) - I_0(\rho)]/I(0)\,, \tag{15}$$

is plotted as a function of the radial distance ρ for $\delta = 0.05$ and $\delta = 1$. Here $I_0(\rho)$ is the intensity profile of a monochromatic beam ($\delta = 0$).

The beam distortion depends of course, also on the spectral lineshape. Fig. 3 compares the extent of beam distortion by plotting the deviation $d(\rho)$ for a lorentzian spectrum, a gaussian spectrum, and a two-peak spectrum consisting of two narrow lorentzian lines ($\delta = 0.01$) located at $\omega_{1,2} = \omega_0(1 \pm 0.1)$.

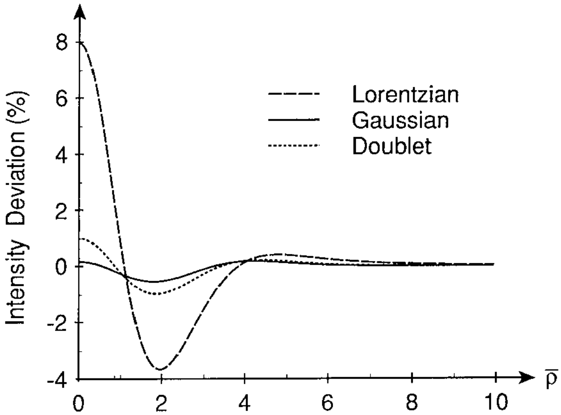

Fig. 3. Comparison of the intensity deviation $d(\rho)$ for different spectral lineshapes at a propagation distance $z=2z_{\text{eff}}$. The parameter $\delta=0.1$ for lorentzian and gaussian lineshapes. The two lines in the case of the doublet are separated by $\omega_1-\omega_2=0.1$ and are relatively narrow ($\delta=0.01$).

To quantify the effect of the spectrum on the intensity profile, one should consider how the beam width is affected by the finite linewidth. Since the beam does not remain gaussian, the conventional measures of the beam width such as the full width at half maximum (fwhm) may not be appropriate for this purpose. In fact, the fwhm may be reduced (see fig. 1) even though the beam spreads over a larger region. For this reason we consider the rms width $\sigma(\delta)$ associated with the beam and defined by

$$\sigma^2(\delta)=\frac{\int \rho^2 I(\rho,\delta)\,\mathrm{d}^2\rho}{\int I(\rho,\delta)\,\mathrm{d}^2\rho}, \tag{16}$$

where the integration is carried over the entire ρ-plane. In fig. 4 we show a comparison of the spreading factor, defined as $\sigma(\delta)/\sigma_{\mathrm{I}}$, for monochromatic ($\delta=0$) and quasi-monochromatic ($\delta=0.1$) beams, as a function of the propagation distance for the case when the source spectrum consists of a single lorentzian line. The effect of partial coherence is illustrated by considering two different values of α. The curves for $\alpha=100$ correspond to the case of spatially coherent beam whereas the curves for $\alpha=1$ correspond to the case of a relatively incoherent beam.

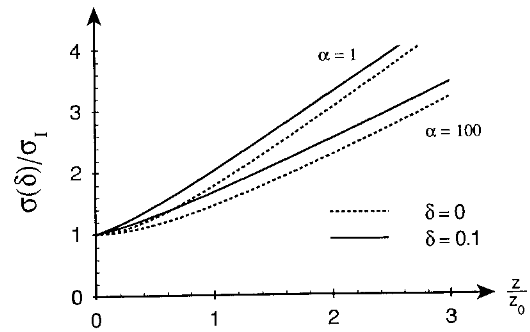

Fig. 4. Variation of rms intensity width with the normalized propagation distance z/z_0, for partially coherent gaussian beams of two states of spatial coherence. In each case solid and dashed lines correspond to a spectrum of finite width ($\delta=0.1$) and a sharp-line spectrum ($\delta=0$).

4. Discussion

The conventional theory of gaussian beam propagation [8] is based on the assumption that the radiation is monochromatic and hence, in a sense, fully coherent. Although effects of partial spatial coherence on gaussian-beam propagation have recently been studied, the temporal coherence aspects have been ignored up to now. In this note we showed that when the spectrum of the beam is taken into account, appreciable changes appear in the intensity profile of the beams. In particular, the spreading of the beam on propagation is enhanced, and the shape of the intensity profile does not remain gaussian for beams whose spectrum is broad. This phenomenon is related to the Wolf effect [4], namely the change in the spectrum of emitted light arising from spatial coherence of the source. In the case that we have considered, it is the temporal coherence of the source, manifested through a finite spectral width, that affects the intensity profile of the beam.

According to the analysis presented in this paper, the width and the shape of the intensity profile are affected by the width and the shape of the spectral profile associated with the beam. We illustrated our results by comparing a lorentzian spectrum, a gaussian spectrum, and a spectrum consisting of two sharp spectral lines. Eq. (9) can, however, be used to calculate the intensity profile for an arbitrary spectral profile. Similarly, although we have considered beams with a gaussian profile, other beam profiles can be analyzed by suitably modifying eq. (6). Different functional forms of the source correlations can also be studied by modifying eq. (6).

The general conclusion is that when a beam generated by a partially coherent source propagates in free space or in a homogeneous non-dispersive medium its intensity profile is affected both by the coherence properties of the source and by its spectrum. The spectral enhancements are negligible for a relatively narrow spectral line such that $\delta \ll 1$, but become significant for quasi-monochromatic light for which $\delta \geqslant 0.1$.

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