Modulation bandwidth of high-power single-mode semiconductor lasers: Effect of intraband gain saturation

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(Received 19 February 1990; accepted for publication 24 April 1990)

The effect of intraband gain saturation on the modulation bandwidth of single-mode semiconductor lasers is discussed by using a nonperturbative form of the optical gain that is valid at high-power levels. The small-signal analysis of the modified rate equations is used to predict the power dependence of the modulation bandwidth. The results are used to discuss the ultimate modulation bandwidth of InGaAsP distributed feedback semiconductor lasers and its dependence on various device parameters.

The high-speed performance of semiconductor lasers is often characterized by the 3 dB modulation bandwidth $f_{3\,\mathrm{dB}}$, defined as the modulation frequency at which the small-signal response of the semiconductor laser reduces by a factor of 2 relative to the zero frequency or dc response.[1-5] A simple rate equation analysis[1] predicts that $f_{3\,\mathrm{dB}}$ increases as $\sqrt{P}$ with an increase in the bias level power P provided it is not limited by the electrical parasitic elements present in any packaged semiconductor laser. Indeed, values of $f_{3\,\mathrm{dB}}$ in excess of 20 GHz have been realized in 1.3 μm InGaAsP lasers designed to reduce the effect of parasitic elements.[3-5] The main limitation on $f_{3\,\mathrm{dB}}$ is then due to the nonlinear nature of the optical gain, manifested through a decrease in the modal gain as the bias power increases. In a simple model,[2] the gain g is assumed to vary as $g = g_L(1-\epsilon_{\mathrm{NL}}P)$, where $\epsilon_{\mathrm{NL}} \sim 10$ W^{-1} for InGaAsP lasers. This assumption is obviously valid for $\epsilon_{\mathrm{NL}}P \ll 1$ but breaks down for high-power lasers (typically for $P \gtrsim 20$ mW). An alternate functional form, $g = g_L(1+\epsilon_{\mathrm{NL}}P)^{-1}$, has been used[3] in analogy with a two-level system. Its use, however, is generally not appropriate as semiconductor lasers are far from being a two-level system. In this letter we obtain an expression for $f_{3\,\mathrm{dB}}$ by using a nonperturbative functional form of the optical gain for the case in which the origin of gain nonlinearity is due to a finite intraband relaxation time.[6] This expression provides us with the accurate power dependence of $f_{3\,\mathrm{dB}}$ for a single longitudinal mode semiconductor laser. It is used to obtain the ultimate modulation bandwidth of single-mode semiconductor lasers limited by the intraband relaxation effects.

For a single mode such as a distributed feedback (DFB) semiconductor laser, the nonlinear gain can be obtained by using the density matrix equations and solving them nonperturbatively.[6] If we also assume that the small-signal gain varies linearly with the carrier density n, the optical gain at any power level is given by

$$g(n,p) = a(n-n_0)/\sqrt{1+p}\,, \tag{1}$$

where a is the differential gain coefficient, n_0 is the transparency value of the carrier density, $p = |E_0|^2/I_{\mathrm{sat}}$, $|E_0|^2$ is the intracavity mode intensity, and the intraband saturation intensity I_{sat} is defined by

$$I_{\mathrm{sat}} = \hbar^2/[\mu^2\tau_{\mathrm{in}}(\tau_c+\tau_v)], \tag{2}$$

where μ is the dipole moment, τ_c and τ_v are the population relaxation times for the conduction and valence bands, and τ_{in} is the polarization relaxation time. If the induced polarization relaxes instantaneously ($\tau_{\mathrm{in}} = 0$), I_{sat} tends to infinity, and the gain becomes power independent. Typically $\tau_{\mathrm{in}} \simeq 100$ fs for InGaAsP semiconductor lasers.

The small-signal modulation response is obtained by solving the conventional rate equations[1] written in the form

$$\frac{dp}{dt} = \left(\Gamma v_g g - \frac{1}{\tau_p}\right)p + \frac{\Gamma R_{\mathrm{sp}}}{VS_{\mathrm{sat}}}\,, \tag{3}$$

$$\frac{dn}{dt} = \frac{I}{qV} - \frac{n}{\tau_n} - v_g g\, S_{\mathrm{sat}}\, p, \tag{4}$$

where Γ is the confinement factor, v_g is the group velocity, R_{sp} is the rate of spontaneous emission into the mode, V is the active volume, τ_p and τ_n are the photon and carrier lifetimes, I is the injected current, and S_{sat} is the saturation photon density defined by

$$S_{\mathrm{sat}} = (\epsilon_0\,\bar{n}n_g/\hbar\omega_0)\,I_{\mathrm{sat}}\,. \tag{5}$$

In Eq. (5), $\hbar\omega_0$ is the photon energy, $\bar{n}$ is the effective mode index, and n_g is the group index.

The small-signal response under sinusoidal current modulation is obtained by linearizing Eqs. (3) and (4) about the bias level[1] and is given by

$$M(\omega) = \frac{\Omega_R^2+\Gamma_R^2}{|(\Omega_R - i\Gamma_R+\omega)(\Omega_R+i\Gamma_R-\omega)|}\,, \tag{6}$$

where $M(\omega)$ is normalized to its dc value $[M(0) = 1]$ and Ω_R and Γ_R are the frequency and the damping rate of relaxation oscillations. Both Ω_R and Γ_R are affected by intraband gain saturation and are approximately given by[7]

$$\Omega_R^2 = \frac{v_g a S_{\mathrm{sat}}}{2\tau_p}\,\frac{p(2+p)}{(1+p)^2}\,, \tag{7}$$

$$\Gamma_R = \frac{1}{\tau_p}\,\frac{p/4}{(1+p)^{3/2}}\,. \tag{8}$$

The 3 dB modulation bandwidth is obtained by requiring that $M(2\pi f_{3\,\mathrm{dB}}) = \frac{1}{2}$ and is given by

$$f_{3\,\mathrm{dB}} = \frac{1}{2\pi}\{\Omega_R^2 - \Gamma_R^2 + 2[\Omega_R^2(\Omega_R^2+\Gamma_R^2)+\Gamma_R^4]^{1/2}\}^{1/2}. \tag{9}$$

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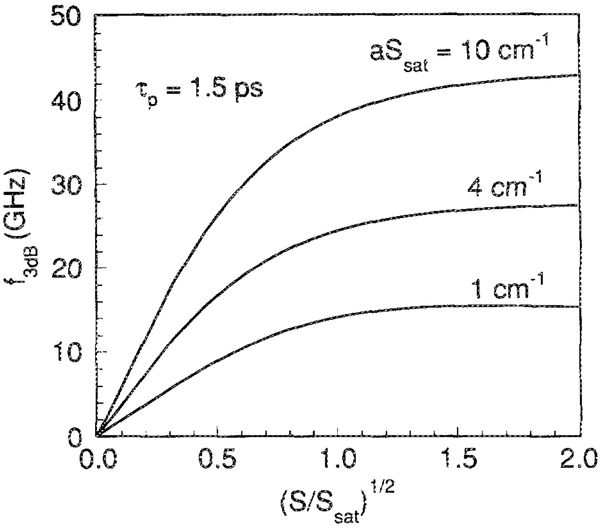

FIG. 1. Modulation bandwidth $f_{3\,dB}$ vs $\sqrt{p}$ for three different values of the parameter aS_{sat}.

Equations (7)–(9) show that $f_{3\,dB}$ depends only on a few device parameters. The most important parameter is the combination aS_{sat} as it depends only on the material parameters that cannot be easily tailored. Typical values of aS_{sat} for InGaAsP lasers are ~ 1 cm^{-1}. The other important parameter is the photon lifetime $\tau_p \sim 1$ ps. For definiteness, we choose $\tau_p = 1.5$ ps and $n_g = 4$. Figure 1 shows the variation of $f_{3\,dB}$ with $\sqrt{p}$ for several values of aS_{sat}. The dimensionless parameter

$$p = \frac{|E_0|^2}{I_{sat}} = \frac{S}{S_{sat}} = \frac{P_{out}}{P_{sat}} \tag{10}$$

is directly proportional to the intracavity photon density S or the output power P_{out}. Figure 1 shows that, as also observed experimentally, $f_{3\,dB}$ increases linearly with $\sqrt{p}$ for $p \ll 1$ but begins to saturate for $p > \frac{1}{2}$. Depending on the value of aS_{sat}, $f_{3\,dB}$ is nearly maximum for p in the range ~ 1–2; further increase in the bias level does not increase $f_{3\,dB}$ significantly but may decrease it slightly when aS_{sat} is relatively small.

The limiting value of $f_{3\,dB}$ depends strongly on the value of aS_{sat} and can exceed 40 GHz for $aS_{sat} = 10$ cm^{-1}. It is possible to obtain an analytic expression for the limiting value by taking the limit $p \gg 1$ in Eq. (9). It is given by

$$f_{max} = (3v_g aS_{sat}/8\pi^2\tau_p)^{1/2} \tag{11}$$

and shows that f_{max} scales as $\sqrt{aS_{sat}}$. If we use Eqs. (2) and (5) to express S_{sat} in terms of the fundamental material parameters, f_{max} can be written as

$$f_{max} = [3\epsilon_0\hbar\bar{n}\lambda_0 a/16\pi^3\mu^2\tau_p\tau_{in}(\tau_c + \tau_v)]^{1/2}, \tag{12}$$

where λ_0 is the laser wavelength. Equation (12) represents one of the main results of this letter. It shows how f_{max} depends on the photon lifetime τ_p and the intraband relaxation times τ_{in}, τ_c, and τ_v. If the carriers were able to relax instantaneously, f_{max} would be infinite as intraband gain saturation is then absent.

Let us examine f_{max} for 1.55 μm InGaAsP DFB lasers because of their importance for high-speed optical communication systems. Only an order of magnitude estimate can be provided since the intraband relaxation times are not known very accurately for the InGaAsP active material; we choose $\tau_{in} = 0.1$ ps, $\tau_c = 0.3$ ps, and $\tau_v = 0.07$ as the representative values.[7,8] The dipole moment μ depends on the laser wavelength[8] and is approximately $\mu \simeq 9 \times 10^{-29}$ mC at $\lambda_0 = 1.55$ μm. By using these values in Eq. (2), $I_s \simeq 3.3 \times 10^{13}$ (V/m)^2. The saturation photon density S_{sat} is estimated from Eq. (5) by using $\bar{n} = 3.3$, $n_g = 4$, and is given by $S_{sat} \simeq 2.5 \times 10^{16}$ cm^{-3}. The differential gain coefficient a depends on many material parameters with a typical value given by $a = 2 \times 10^{-16}$ cm^2. The parameter aS_{sat} is thus $\simeq 5$ cm^{-1} for 1.55 μm DFB lasers. From Eq. (11) or Fig. 1 we conclude that $f_{max} \simeq 30$ GHz if we use the representative values $\tau_p = 1.5$ ps and $n_g = 4$. The value of f_{max} is expected to be larger for 1.3 μm InGaAsP lasers since the dipole moment μ is smaller. In fact, by noting[8] that μ scales linearly with λ_0, f_{max} is found to scale with the wavelength as $\lambda_0^{-1/2}$ provided other parameters are assumed to be wavelength independent in Eq. (12). If we further assume that these parameters remain unchanged for $\lambda_0 = 1.3$ μm, Eq. (12) predicts only a 10% increase in the value of f_{max} for 1.3 μm InGaAsP lasers. By contrast, a 40% increase is possible if the differential gain a increases by a factor of 2; this can occur for quantum well lasers.

How do the theoretical estimates compare with the experimental data on InGaAsP lasers? We restrict ourselves to the measurements of $f_{3\,dB}$ made on DFB lasers since our theory assumes single longitudinal mode operation. Unfortunately DFB lasers with high output powers such that $p > 1$ are not easy to fabricate. Most of the experimental data[9,10] on $f_{3\,dB}$ appear to be limited to power levels such that $p < \frac{1}{2}$. We should note that the saturation output power P_{sat}, although related linearly to S_{sat}, depends on many device parameters such as the active volume, the confinement factor, and the differential quantum efficiency. It is thus difficult to estimate p accurately for the devices used in the experiments unless all parameters are specified. Nonetheless, a rough comparison can be made. We first consider the data of Kamite *et al.*[9] for 1.3 μm DFB lasers. A specific DFB laser whose ouput facet was antireflection (AR) coated ($\simeq 5\%$) exhibited $f_{3\,dB} = 13$ GHz at $P_{out} = 16$ mW. The saturation output power P_{sat} for this laser is $\simeq 100$ mW because of the use of AR coating, resulting in $p \simeq 0.16$. If we take $aS_{sat} = 4$ cm^{-1} for this laser, Fig. 3 shows $f_{3\,dB} \simeq 14$ GHz in good agreement with the experimental value. Hirayama *et al.*[10] presented the data on a 1.5 μm DFB laser with $f_{3\,dB} = 13$ GHz at $P_{out} = 12$ mW. For this laser P_{sat} is estimated to be $\simeq 75$ mW because the facets were not AR coated. This is the reason why 13 GHz bandwidth could be achieved at a lower output power. The magnitude of $f_{3\,dB}$ is again in good agreement with the theory. Uomi *et al.*[11] recently presented the data on a λ/4-shifted DFB laser with 17 GHz bandwidth at $P_{out} = 5$ mW, a power level at which $p \simeq 0.1$. The large bandwidth for this laser is due to an increase in the differential gain a (realized by detuning the laser wavelength from the gain peak) and a reduction in the photon lifetime τ_p (caused by a large internal loss).

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We can use Eq. (1) to provide an expression for the often used parameter ϵ defined as $g = g_L\ (1 - \epsilon S)$,[2-4] where $g_L = a(n - n_0)$ is the linear gain. By using Eqs. (1), (2), (5), and (10) together with $p \ll 1$, we obtain

$$\epsilon = \frac{1}{2S_{\text{sat}}} = \frac{\mu^2 \omega_0 \tau_{in} (\tau_c + \tau_v)}{2\epsilon_0 \hbar \bar{n} n_g} . \qquad (13)$$

This expression differs somewhat from a corresponding result obtained by using third-order perturbation theory.[12] In particular, the values of ϵ predicted by Eq. (13) are larger by about a factor of 2. If we use our estimated value $S_{\text{sat}} \simeq 2.5 \times 10^{-16}$ cm^{-3} in Eq. (13), we find that $\epsilon \simeq 2 \times 10^{-17}$ cm^3 for InGaAsP lasers. This value is in good agreement with other estimates of ϵ, particularly if we keep in mind its dependence on the three intraband relaxation times.

In conclusion, we have used a nonperturbative analysis of the density matrix equations to study the effect of intraband gain saturation on the small signal modulation response. The result is used to obtain an expression for the modulation bandwidth that is valid at all power levels of a laser oscillating in a single mode. This expression, in turn, is used to estimate the ultimate limit on the modulation bandwidth of InGaAsP DFB lasers. For 1.55 μm DFB lasers the ultimate room-temperature bandwidth ~30 GHz is predicted. This value can be enhanced by using a quantum well design. The differential gain coefficient is typically enhanced by a factor of ~2–3. The saturation photon density is, however, expected to reduce by a factor in the range 1–2 since the dipole moment μ becomes smaller for quantum well lasers. Thus, the modulation bandwidth of quantum well lasers is expected to be larger by at most 50% compared with that obtained for conventional semiconductor lasers.

This research is supported by the U. S. Army Research Office and the Joint Services Optics program.

[1] See, for example, G. P. Agrawal and N. K. Dutta, *Long-Wavelength Semiconductor Lasers* (Van Nostrand–Reinhold, New York, 1986), Chap. 6.

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