Effect of Gain Nonlinearities on the Dynamic Response of Single-Mode Semiconductor Lasers

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Abstract—The effect of gain nonlinearities on the dynamic response is studied by using a nonperturbative form of the nonlinear gain in the signal-mode rate equation. The nonperturbative form is different from that used previously and leads to qualitative and quantitative differences in the intensity dependence of the important dynamic parameters such as the frequency and the damping rate of relaxation oscillations. The results are important for the design of high-speed semiconductor lasers and suggest that the new form of the nonlinear gain should be used for modeling the modulation response in both the small-signal and large-signal regimes.

THE DYNAMIC response of semiconductor lasers is governed by a set of rate equations which describe the transient interaction between the photons and the charge carriers mediated by stimulated emission [1]. The modal gain in these equations is often assumed to be intensity independent. However, many dynamic features of semiconductor lasers are properly explained only when the gain G is assumed to decrease with an increase in the mode intensity [2]–[10]. At low power levels, gain nonlinearities are included by assuming $G = G_L(1 - \epsilon I)$, where ϵ is the nonlinear gain parameter [3]–[7]. An alternative phenomenological functional form $G = G_L(1 + I/I_s)^{-1}$ has been used [2], [8] in analogy with a two-level system, to deal with high-power semiconductor lasers. This form of nonlinear gain may, however, be misleading as the two-level model is generally not appropriate for semiconductor lasers. It was shown recently [11] that a nonperturbative analysis of the nonlinear gain can be carried out for a single-mode semiconductor laser when the gain nonlinearities are due to intraband relaxation of charge carriers [3]. In this paper, we use this exact form of the nonlinear gain in the rate equations and study its effect on the frequency and the damping rate of relaxation oscillations. Our results show that the intensity dependence of the relaxation-oscillation parameters is qualitatively different from that predicted previously. The results are important for the design of high-speed semiconductor lasers and suggest that the new form of the nonlinear gain should be used for modeling both the small-signal and large-signal modulation responses.

The effects of intraband relaxation on the gain are included through a density-matrix analysis [11] in which the induced polarization is calculated by integrating over the band states with an appropriate density of states. Although a perturbative approach is necessary for the multimode lasers, an approximate nonperturbative calculation can be carried out for the single-mode case with the modal gain given by [11]

$$G = G_L/\sqrt{1 + I/I_s} \tag{1}$$

where I is the intracavity mode intensity and the linear gain G_L depends on the carrier density and the band structure details. The saturation intensity I_s depends on the intraband relaxation times, among other things, and is estimated to be $\sim$10 MW/cm^2 for InGaAsP lasers [11]. If we assume a cross-section mode area $\sim 1\ \mu m^2$ for an index-guided device, the intracavity saturation power is $\sim$100 mW. The output saturation power would be lower by a factor that depends on the facet reflectivities and the laser design. This estimate shows that a nonperturbative form of the nonlinear gain is necessary to use for lasers operating at power levels $\sim$20 mW.

The single-mode rate equations can be written in their conventional form [1]

$$\dot{P} = (G - \gamma)P + R_{sp} \tag{2}$$

$$\dot{N} = C - \gamma_e N - GP \tag{3}$$

where G is the gain or net stimulated emission rate, γ is the cavity decay rate, R_{sp} is the rate of spontaneous emission into the laser mode, C is the carrier injection rate, and γ_e is the spontaneous carrier recombination rate. The dynamic variables P and N stand for the number of photons and carriers, respectively, inside the active region of the laser. The linear G_L is often approximated by $G_L = A(N - N_0)$, where A is the gain coefficient and N_0 is the number of injected carriers needed for transparency [1]. By using (1), the nonlinear gain can be written in the form

$$G(N, P) = \frac{A(N - N_0)}{\sqrt{1 + P/P_s}} \tag{4}$$

where P_s is the photon number corresponding to I_s.

Equations (2)–(4) can be used to study the effect of gain nonlinearities on the modulation response of semiconductor lasers both in the small-signal and large-signal modulation regimes. Two parameters which play an important role in characterizing the modulation response are the frequency Ω_R and the damping rate of Γ_R of relaxation oscillations [6], [8],

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[10]. They can be obtained by performing a linear stability analysis of the rate equations about the steady state. Since the procedure is straightforward [1], we write the result directly:

$$\Omega_R^2 = \left(G + \frac{\partial G}{\partial P}P\right)\frac{\partial G}{\partial N}P - \frac{1}{4}(\Gamma_N - \Gamma_P)^2, \quad (5)$$

$$\Gamma_R = \frac{1}{2}(\Gamma_N + \Gamma_P) \quad (6)$$

where

$$\Gamma_N = \gamma_e + \frac{\partial \gamma_e}{\partial N}N + \frac{\partial G}{\partial N}P, \quad (7)$$

$$\Gamma_P = \frac{R_{sp}}{P} - \frac{\partial G}{\partial P}P. \quad (8)$$

The effect of gain nonlinearities is included through the derivative $\partial G/\partial P$. We evaluate Ω_R^2 and Γ_R by using (4) in (5)-(8). By keeping only the dominant term, the resulting expressions can be approximated to provide

$$\Omega_R^2 \simeq G_L A P_s \frac{(1+p/2)p}{(1+p)^2} \quad (9)$$

$$\Gamma_R \simeq G_L \frac{p/4}{(1+p)^{3/2}} \quad (10)$$

where $p = P/P_s$ is a normalized parameter that can be interpreted as the output power relative to the saturation output power. It is easy to verify that Ω_R takes its maximum value for $p \to \infty$, whereas Γ_R peaks at $p = 2$. The maximum values of Ω_R and Γ_R are

$$\Omega_{max} = \sqrt{G_L A P_S/2}, \qquad \Gamma_{max} = G_L/6\sqrt{3}. \quad (11)$$

By using typical parameter values for InGaAsP lasers, we estimate that $\Omega_{max}/2\pi \sim 30$-$50$ GHz and $\Gamma_{max} = 4$-6×10^{10} s^{-1}.

It is interesting to compare (9) and (10) with those obtained when (4) for the nonlinear gain is replaced by

$$G(N, P) = \frac{A(N - N_0)}{1 + P/P_s}. \quad (12)$$

As discussed earlier, this form has been used previously [2], [8] for semiconductor lasers in analogy with a two-level system. The approximate expressions for Ω_R^2 and Γ_R take the form

$$\Omega_R^2 \simeq G_L A P_S \frac{p}{(1+p)^3}, \quad (13)$$

$$\Gamma_R \simeq G_L \frac{p/2}{(1+p)^2}, \quad (14)$$

and should be compared to (9) and (10).

The qualitative dependence of the relaxation-oscillation frequency Ω_R on the output power is quite different for the two forms of the nonlinear gain. Fig. 1 shows the variation of Ω_R/Ω_{max} with $\sqrt{p}$ for the two cases. Ω_R increases linearly with $\sqrt{p}$ for $p \ll 1$. This linear dependence has been observed in many experiments [8], [10]. A sublinear increase with $\sqrt{p}$ begins to occur for $p > 0.1$. The nonlinear gain of (12) predicts that Ω_R would peak at $p = 1/2$ and then decrease with a further increase in p. By contrast, our use of more realistic expression (4) shows that Ω_R continues to increase sublinearly and approaches Ω_{max} asymptotically for $p \gg 1$. The maximum value is larger by about a factor of 2 in the latter case. The damping rate Γ_R of relaxation oscillation plays an important role in characterizing both the small-signal and the large-signal dynamic response of semiconductor lasers. Fig. 2 shows the variation of Γ_R with p for the two models of the nonlinear gain. For $p \ll 1$, both models predict a linear increase of Γ_R with p, as also observed experimentally. However, the growth rate is smaller by a factor of 2 in the case of the model based on (4). Γ_R peaks at $p = 2$ for the case of (4) whereas it peaks at $p = 1$ when (12) is

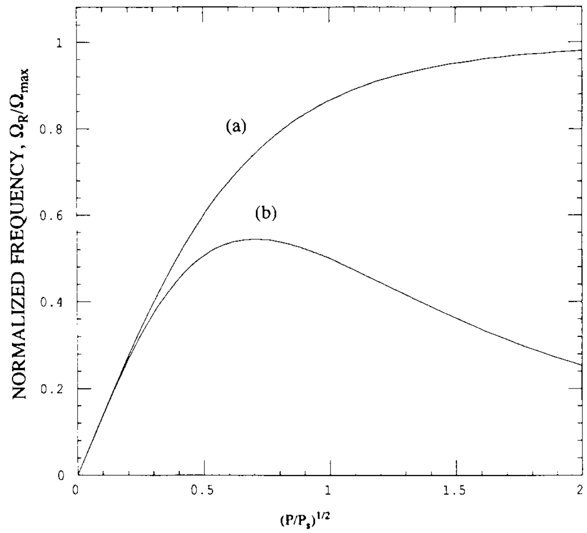

Fig. 1. Power dependence of the relaxation-oscillation frequency. (a) Nonlinear gain from (4). (b) Nonlinear gain from (12).

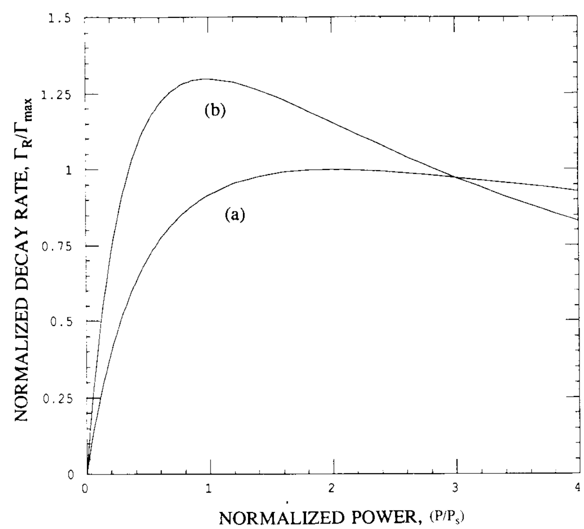

Fig. 2. Power dependence of the relaxation-oscillation decay rate. (a) Nonlinear gain from (4). (b) Nonlinear gain from (12).

used. The peak value is larger by about 30 percent in the latter case. These features can be used to verify the validity of the nonlinear gain model of (1). By noting that $G_L \simeq \gamma = 1/\tau_p$, where τ_p is the photon lifetime, and using (11), the damping time of relaxation oscillations is found to have a lower bound of $10\tau_p$ (15–20 ps for typical InGaAsP lasers).

The experimental observation of the nonlinear gain effects discussed here would require the use of high-power distributed feedback (DFB) lasers. In one experiment [12], the relaxation-oscillation frequency was measured as a function of the output power for a 1.3 μm DFB laser. The data showed a linear increase of Γ_R with $\sqrt{p}$ up to about 20 mW and a slightly sublinear increase in the range 20–40 mW. This behavior is consistent with that shown in Fig. 1 provided the output saturation power P_s is taken to be about 100 mW. Such a high value of P_s for this laser is due to two reasons. First, the use of anti-reflection coating on the output facet implies that P_s is nearly equal to the intracavity saturation power. Second, P_s is larger at 1.3 μm than at 1.55 μm as it scales as λ^{-2}. The use of a 1.55 μm DFB laser with high-reflection facet coatings would permit the observation of nonlinear gain effects at much lower output powers. Note that the data of [12] actually support the use of (4) in place of (12) since the observed sublinearity is less pronounced than predicted by (12) [curve (b) of Fig. 1].

In conclusion, the functional form of the nonlinear gain plays an important role in determining the dynamic response of semiconductor lasers. When the gain nonlinearities are due to intraband relaxation effects, it is possible to obtain an analytic form of the nonlinear gain [11] that is different from a previously used functional form [2], [8]. Our results suggest that this form should be used in comparing experiments with theory related with the dynamic response both in the small-signal and large-signal modulation regimes. The results have applications in the design of high-speed distributed feedback lasers. The effect of gain nonlinearities is particularly important for quantum-well lasers for which the saturation intensity is generally reduced as a result of quantum confinement [13].

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