Crosstalk Penalty in Multichannel ASK Heterodyne Lightwave Systems

GOVIND P. AGRAWAL, SENIOR MEMBER, IEEE

Abstract—**This paper addresses the issue of optimum channel spacing in multichannel amplitude-shift-keying (ASK) heterodyne lightwave systems by considering the bit-error-rate (BER) degradation resulting from interchannel crosstalk. An expression for the BER of a specific channel is obtained by considering the crosstalk from both of its nearest neighbors. The general result is used to calculate the power penalty as a function of the crosstalk level. The crosstalk is shown to depend on a large number of system parameters such as the bit rates, the received powers, the channel spacing, and the response function of the bandpass filter (BPF). For the case of equal bit rates and equal received powers in all channels, the crosstalk from each channel should be below −12 dB to keep the power penalty below 1 dB. This requirement translates into a minimum channel spacing of about four or five times the bit rate, depending on whether the filter bandwidth is two or three times the bit rate. If the objective is to reduce the power penalty below 0.1 dB, the interchannel crosstalk should be less than −18 dB. This would require a minimum channel spacing of about ten times the bit rates.**

I. Introduction

ONE OF THE attractions of coherent lightwave systems [1], [2] is that they offer the potential of simultaneously transmitting a large number of closely spaced channels in the minimum-loss wavelength window of single-mode fibers by using frequency-division multiplexing techniques. An important issue for such multichannel coherent systems concerns the minimum channel spacing that must be maintained before the interchannel crosstalk degrades the system performance significantly. Although this issue has recently been addressed both theoretically and experimentally [3]–[8], the effect of crosstalk on the system performance has not been studied in detail. Such a study should be able to predict the dependence of the power penalty resulting from interchannel crosstalk on such various design parameters as the channel spacing, the bit rates, the received channel powers, and the filter bandwidth. The objective of this paper is to develop a general crosstalk model, to study the extent of power penalty, and to determine the optimum interchannel spacing for multichannel coherent lightwave systems.

Coherent lightwave systems can be classified into two broad categories, depending on whether synchronous (phase-sensitive) or nonsynchronous (phase-insensitive) demodulation techniques are employed at the receiver end [1]. Although synchronous demodulation results in an overall higher receiver sensitivity, it generally imposes stringent requirements on the laser linewidth. For this reason, nonsynchronous demodulation techniques (such as envelope detection) have attracted considerable attention [9]–[12]. In this paper, we focus on an envelope-detection heterodyne receiver designed for the amplitude-shift-keying (ASK) format. The analysis can be extended to the case of the frequency-shift-keying (FSK) format.

The paper is organized as follows. In Section II, we discuss how the received signal in a specific channel is corrupted by interference from the neighboring channels. Section III calculates the error probability in the presence of crosstalk from two nearest neighbors. The special case in which only a single neighbor contributes to the crosstalk (two-channel experiments, for example) is considered in Section IV, where we discuss how the power penalty depends on the bandwidth and the response function of the bandpass filter (BPF). Section V extends the power-penalty calculation to the general case of multichannel systems. The minimum channel spacing needed to limit the crosstalk penalty below a certain level is discussed in Section VI, where we summarize our main conclusions.

II. Received Signal

In the receiver configuration of Fig. 1, the multichannel optical input is coherently mixed with the local-oscillator field by using a 3-dB coupler. The resulting optical signal is converted to the intermediate-frequency (IF) signal by using a balanced heterodyne receiver whose output is given by

$$I(t) = 2R \sum_{n=0}^{M-1} (P_{LO}P_n)^{1/2} m_n(t) \cdot \cos(2\pi f_n t + \psi_n) + N(t) \tag{1}$$

where R is the detector responsivity, M is the number of channels, P_{LO} is the local-oscillator power, P_n is the received power in the nth channel, $m_n(t) = 1$ or 0 depending on whether a "1" or "0" is transmitted in the nth channel, and ψ_n is the optical phase. The frequency f_n is given by

$$f_n = |\nu_n - \nu_{LO}| = \begin{cases} nD_{opt} + f_{IF} \text{ for } \nu_n > \nu_{LO} \\ nD_{opt} - f_{IF} \text{ for } \nu_n < \nu_{LO} \end{cases} \tag{2}$$

where ν_n is the optical frequency of the nth channel, and ν_{LO} is the local-oscillator frequency. The local oscillator

Manuscript received March 30, 1988; revised August 1, 1988.

The author was with AT&T Bell Laboratories, Murray Hill, NJ 07974. He is now with the Institute of Optics, University of Rochester, Rochester, NY 14627.

IEEE Log Number 8930419.

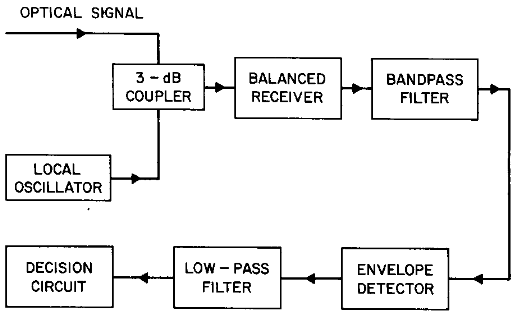

Fig. 1. Schematic diagram of the heterodyne receiver used for multichannel ASK lightwave systems.

is tuned to select a particular channel at f_{IF}. The optical-channel spacing D_{opt} is taken to be the same for all channels. However, the electrical channel spacings are not equal because the image-band channels ($\nu_n < \nu_{LO}$) interleave with the channels having frequencies $\nu_n > \nu_{LO}$. In (1), $N(t)$ accounts for the white noise (shot noise as well as thermal noise) added to the signal during the detection process.

A BPF centered at f_{IF} is used to filter the electrical signal. The filtered output is obtained from

$$I_F(t) = \int_{-\infty}^{\infty} \tilde{I}(f)\, H(f - f_{IF}) \exp(-2\pi i f t)\, df \quad (3)$$

where $H(f)$ is the amplitude response of the BPF, and $\tilde{I}(f)$ is the Fourier transform of $I(t)$ given by (1). The envelope detector provides the envelope $|I_F(t)|$ to the decision circuit through a low-pass filter. Using (1)–(3), the signal used by the decision circuit is given by

$$r = \left|I_F(t)\right| = \left|\sum_{n=0}^{M-1} m_n A_n \exp(i\psi_n') + (N_c + iN_s)\right| \quad (4)$$

where N_c and N_s are the quadrature components of the narrow-band noise, and A_n is given by

$$A_n = 2R(P_{LO}P_n)^{1/2} \left| T_n \int_{-\infty}^{\infty} H(f) \cdot \operatorname{sinc}\left[\pi T_n (f + f_n - f_{IF})\right] \exp(2\pi i f t_d)\, df \right|. \quad (5)$$

Here, $\operatorname{sinc}(x) = \sin(x)/x$, $T_n = B_n^{-1}$, and B_n is the bit rate of channel n. The sin c function results from our assumption of a rectangular pulse; other pulse shapes can be easily accounted for with only minor changes. We assume for simplicity that the decision instant $t_d = 0$ in (5). This amounts to neglecting the effect of intersymbol interference. In general, the phase of A_n can vary from bit to bit. This is accounted for by assuming that the phase ψ_n' in (4) consists of two parts: $\psi_n' = \psi_n + \phi_n^D$, where ψ_n is the optical phase, and ϕ_n^D is an additional phase shift.

Equation (4) shows how the neighboring channels interfere with the detection process of a single channel. When the local oscillator is tuned to select a particular channel, all channels on both sides of that channel contribute to the sum in (4) and generate the crosstalk signal that interferes with the detection process. In general, the crosstalk signal is different for different channels, depending on the relative position of the channel of interest. In practice, however, the dominant contribution to the crosstalk signal comes from the nearest channels located at $D_{opt} - 2f_{IF}$ and D_{opt} since the filter bandwidth W is usually chosen to be a fraction of the channel spacing ($W < D_{opt}$). In the following calculation of the error probability, we consider the crosstalk from the nearest neighbors only. A distinction should be made for the boundary channels since they have only one such neighbor.

The calculation of the error probability is quite involved in the general case of arbitrary laser linewidths [13] since ψ_n' can vary considerably over the duration of a single bit in (4). To simplify the analysis, we make the critical assumption that ψ_n' is constant over one bit period. This amounts to assuming that the laser linewidths $\Delta\nu$ are significantly smaller than the bit rates of interest ($\Delta\nu T_n << 1$). For simplicity, we neglect the effect of laser linewidths on the error probability by assuming that the phases ψ_n are constant.

For the calculation of the error probability, we consider a specific channel $n = 0$ as the channel of interest and write (4) in the following form:

$$r = \left\{\left[m_0 A_0 + N_c + (m_1 A_1 \cos\phi_1 + m_2 A_2 \cos\phi_2)\right]^2 + \left[N_s + (m_1 A_1 \sin\phi_1 + m_2 A_2 \sin\phi_2)\right]^2\right\}^{1/2} \quad (6)$$

where we have replaced the subscript -1 by 2 for notational convenience. The phases ϕ_1 and ϕ_2 are the relative phases of the interfering channels

$$\phi_n = \psi_n' - \psi_0' \qquad (n = 1, 2). \quad (7)$$

Using (2) and (5), the signal A_0 and the crosstalk signals A_1 and A_2 are given by

$$A_0 = 2R(P_{LO}P_0)^{1/2} \left| \frac{1}{B_0} \int_{-\infty}^{\infty} H(f) \operatorname{sinc}(\pi f / B_0)\, df \right| \quad (8)$$

$$A_n = 2R(P_{LO}P_n)^{1/2} \left| \frac{1}{B_n} \int_{-\infty}^{\infty} H(f) \cdot \operatorname{sinc}\left[\pi (f + D_n)/B_n\right] df \right| \quad (9)$$

where the electrical-domain channel spacings are related to D_{opt} by

$$D_1 = D_{opt} - 2f_{IF}, \qquad D_2 = D_{opt}. \quad (10)$$

It should be noted that the relative phases ϕ_1 and ϕ_2 in (6) are not expected to remain constant from bit to bit even when the laser phase fluctuations are neglected. In the following calculation of the error probability, we average over ϕ_1 and ϕ_2 with the assumption that they are uniformally distributed. This procedure can be extended to include the effect of laser linewidths if one assumes that the optical phase does not fluctuate significantly over a single bit period [11] and averages the error probability over the appropriate distribution. We also note that the effect of laser phase noise in ASK systems can be reduced by increasing the BPF bandwidth [11]–[13]. In this paper, we neglect the laser phase noise but consider the effect of increasing the BPF bandwidth on the crosstalk penalty.

III. Error Probability

The system performance of a digital communication system is measured through the bit-error rate (BER), which is defined as the probability of incorrect identification of a single bit. In this section, we calculate the error probability P_e when the received signal r given by (6) is corrupted by white noise as well as by the crosstalk from the neighboring channels. Since the crosstalk signal that interferes with the decision process depends on the bit pattern of the neighboring channels, it is necessary to consider all possible combinations of m_1 and m_2. Because there are four such combinations, the error probability is given by

$$P_e = \tfrac{1}{4} \sum_{i=0}^{1} \sum_{j=0}^{1} P_{ij} = \tfrac{1}{4}(P_{11} + P_{10} + P_{01} + P_{00}) \quad (11)$$

where we assumed that "1" and "0" bits are equally likely to occur. The error probability P_{ij} for a specific bit combination of the interfering channels is calculated by using the standard communication theory [14]–[16]. If r_T is the decision threshold, P_{ij} is given by

$$P_{ij} = \tfrac{1}{2} \int_0^{r_T} P_{ij,1}(r)\, dr + \tfrac{1}{2} \int_{r_T}^{\infty} P_{ij,0}(r)\, dr \quad (12)$$

where $P_{ij,k}(r)$ is the probability density function (PDF) of r given by (6) with i, j, and k taking values 0 and 1. For all eight combinations of i, j, and k, the PDF is given by a Ricean distribution [14]. The explicit expressions for PDF are

$$P_{ij,1}(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + q_{ij}^2}{2\sigma^2}\right) I_0\left(\frac{q_{ij} r}{\sigma^2}\right) \quad (13)$$

where σ^2 is the noise variance, I_0 is the zeroth-order modified Bessel function, and q_{ij} is obtained using

$$q_{11}^2 = A_0^2 + A_{eff}^2 + 2A_0 A_{eff} \cos(\phi_{eff}) \quad (14)$$

$$q_{10}^2 = A_0^2 + A_1^2 + 2A_0 A_1 \cos(\phi_1) \quad (15)$$

$$q_{01}^2 = A_0^2 + A_2^2 + 2A_0 A_2 \cos(\phi_2) \quad (16)$$

$$q_{00}^2 = A_0^2. \quad (17)$$

In (14), A_{eff} and ϕ_{eff} are given by

$$A_{eff} = \left[A_1^2 + A_2^2 + 2A_1 A_2 \cos(\phi_1 - \phi_2)\right]^{1/2} \quad (18)$$

$$\phi_{eff} = \tan^{-1}\left(\frac{A_1 \sin \phi_1 + A_2 \sin \phi_2}{A_1 \cos \phi_1 + A_2 \cos \phi_2}\right). \quad (19)$$

The expressions for $P_{ij,0}(r)$ can be obtained from (13)–(17) by setting $A_0 = 0$. The goal is to evaluate the integrals in (12) using (13)–(19), average over the uniform distribution of ϕ_1 and ϕ_2, and then use (11) to obtain the error probability.

The evaluation of P_e by following the above prescription is fairly involved and requires a numerical approach. In the following analysis, we obtain an approximate analytic expression for P_e. The approximation is based on the observation that the contribution of the first integral in (12) is smaller by more than a factor of A_0/σ compared with that of the second integral [15]. Since the signal-to-noise ratio (SNR) $A_0/\sigma \gg 1$ under typical operating conditions, we can approximate (12) by

$$P_{ij} \simeq \tfrac{1}{2} \int_{r_T}^{\infty} P_{ij,0}(r)\, dr \quad (20)$$

where

$$P_{ij,0}(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + q_{ij}^2}{2\sigma^2}\right) I_0\left(\frac{q_{ij} r}{\sigma^2}\right). \quad (21)$$

As mentioned before, q_{ij} is obtained from (14)–(17) by setting $A_0 = 0$ and is given by

$$q_{11} = A_{eff}, \quad q_{10} = A_1, \quad q_{01} = A_2, \quad q_{00} = 0 \quad (22)$$

where A_{eff} is given by (18). The physical meaning of these coefficients is clear; q_{ij} represents the crosstalk signal leaking through the BPF in the channel of interest.

The integral in (20) can be expressed in terms of the Marcum's Q function [16]–[19] with the result

$$P_{ij} = \frac{1}{2} Q\left(\frac{q_{ij}}{\sigma}, \frac{r_T}{\sigma}\right) \quad (23)$$

where

$$Q(\alpha, \beta) = \int_{\beta}^{\infty} x \exp\left(-\frac{x^2 + \alpha^2}{2}\right) I_0(\alpha x)\, dx. \quad (24)$$

The Q function can be evaluated by using the series (see Appendix)

$$Q(\alpha, \beta) = \exp\left(-\frac{\alpha^2 + \beta^2}{2}\right) \sum_{n=0}^{\infty} \left(\frac{\alpha}{\beta}\right)^n I_n(\alpha\beta) \quad (25)$$

where I_n is the nth order modified Bessel function.

The decision threshold $r_T \simeq A_0/2$ when the channel operates by itself without interference from the neighboring channels. In multichannel systems, r_T may be different if the performance is optimized for the desired channel. Although r_T must be optimized when comparing theory with the experiments, this is not essential for the purpose of estimating the channel spacings. For the following discussion, we therefore set $r_T = A_0/2$. Using (11), (22), and (23), the error probability in the presence of crosstalk is given by

$$P_e = \frac{1}{8}\left[Q\left(\frac{A_{eff}}{\sigma}, \frac{A_0}{2\sigma}\right) + Q\left(\frac{A_1}{\sigma}, \frac{A_0}{2\sigma}\right) + Q\left(\frac{A_2}{\sigma}, \frac{A_0}{2\sigma}\right) + Q\left(0, \frac{A_0}{2\sigma}\right)\right]. \tag{26}$$

Equation (26) is our main result. The four terms in this equation can be identified as the contribution to the error probability when a) both interfering channels receive bit "1", b) channel 1 receives bit "1" with bit "0" in channel 2, c) channel 2 receives bit "1" with bit "0" in channel 2, and d) both interfering channels receive bit "0".

IV. Single-Neighbor Crosstalk

Equation (26) gives the error probability for the general case when the nearest neighbors on each side of the desired channel contribute to the crosstalk. The special case of a single neighbor is also of considerable interest. Since this case is relatively simple and useful for understanding several important concepts, we concentrate on it in this section. The results obtained here are applicable to a) the experiments in which only two channels are used to study the effect of interchannel crosstalk [3], [7], b) the boundary channels of a multichannel system, and c) the intermediate channel of a multichannel system whose nearest neighbor is inactive (not used for data transmission).

The error probability for the case of a single-neighbor crosstalk can be easily obtained using the results of Section III. The only difference is that there are only two combinations that should be considered in (11), i.e.

$$P_e = \tfrac{1}{2}(P_{10} + P_{00}). \tag{27}$$

If we use the general result (23) to obtain P_{10} and P_{00}, the error probability is given by

$$P_e = \frac{1}{4}\left[Q\left(\frac{A_1}{\sigma}, \frac{A_0}{2\sigma}\right) + Q\left(0, \frac{A_0}{2\sigma}\right)\right] \tag{28}$$

where A_0 is the signal and A_1 is the interference signal due to crosstalk from the neighboring channel. A_0 and A_1 can be obtained by using (8) and (9), and they depend on the system parameters such as the received powers, the bit rates, the channel separation, and the BPF bandwidth. Their explicit expression are, however, not needed to understand the effect of crosstalk on system performance if we note that A_0^2/σ^2 and A_1^2/σ^2 represent, respectively, SNR and the interference-to-noise ratio (INR).

Fig. 2 shows the bit-error rate (BER) calculated by using (28) as a function of A_0/σ for three values of A_1/σ. The Marcum's Q function $Q(\alpha, \beta)$ was evaluated using the series (25). Because of the relatively large values of SNR, only one or two terms are generally needed to obtain $Q(\alpha, \beta)$ to the desired accuracy. For the special case

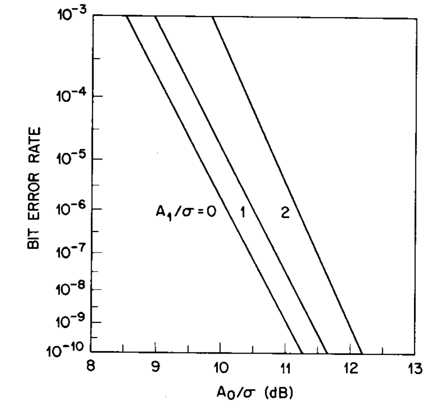

Fig. 2. Calculated dependence of the BER of a specific channel on the A_0/σ for three values of the A_1/σ. Only a single neighboring channel is assumed to contribute to the crosstalk signal A_1.

of $A_1 = 0$ (no crosstalk), we obtain the simple analytic expression

$$P_e = \frac{1}{2}Q\left(0, \frac{A_0}{2\sigma}\right) = \frac{1}{2}\exp\left(-\frac{A_0^2}{8\sigma^2}\right). \tag{29}$$

This is the well-known result [14]–[16] for the error probability of a single isolated ASK-modulated channel (under the assumption that the contribution of the first integral in (12) is negligible).

It is easy to see from Fig. 2 how much the BER of a lightwave system is degraded even by small amounts of crosstalk from a neighboring channel. In the absence of crosstalk ($A_1 = 0$), a BER of 1×10^{-9} can be obtained for $A_0/\sigma \simeq 12.65$ or 11 dB. However, the BER degrades to 4×10^{-6} for $A_1/\sigma = 2$, i.e., when the crosstalk-induced interfering signal is only twice the noise level. From the system standpoint, the relevant issue is not the extent of BER degradation but the increase in the SNR needed to offset the effect of crosstalk in order to maintain the same BER. Fig. 2 shows that when $A_1/\sigma = 2$, the required $A_0/\sigma \simeq 12$ dB instead of the 11 dB that would have been sufficient in the absence of crosstalk.

For a given receiver noise σ, the SNR can be increased only by increasing the signal A_0. If we use (8), we may conclude that the SNR or A_0 can be increased simply by increasing the local-oscillator power P_{LO}. However, it is evident from (9) that this approach will also increase A_1 or the INR by the same amount, resulting in no improvement in the system performance. Therefore, the only way to counteract the effect of interchannel crosstalk is to increase the received signal power. The required increase is generally quantified as the power penalty defined (in decibels) by the expression

$$\Delta = 10 \log (P_0/\overline{P}_0) = 20 \log (A_0/\overline{A}_0) \tag{30}$$

where $\overline{A}_0$ is the received signal in the absence of crosstalk ($\overline{A}_0 = 12.65\ \sigma$ for a BER of 1×10^{-9}). In a similar manner, the level of crosstalk can be quantified by using the definition

$$C = 10 \log (A_1/A_0) \tag{31}$$

where A_1 is the interfering signal leaked through the BPF and can be obtained from (9). Note that the crosstalk in (31) is defined using the ratio of optical powers. In terms of the electrical powers, $|C|$ should be multiplied by a factor of two.

Fig. 3 shows the power penalty Δ as a function of the crosstalk C at a BER of 1×10^{-9}. To ensure a power penalty below 1 dB, the crosstalk should be < -10 dB. However, if the system is designed to permit a crosstalk penalty $\Delta < 0.1$ dB, it is necessary to reduce C below -17 dB. The curve shown in Fig. 3 is universal in the sense that the power penalty depends on a single parameter C. Of course, the value of C depends on a large number of system parameters. We now turn to study this dependence. By using (8), (9), and (31), the explicit expression for C is

$$C = 10 \log \left[\left(\frac{P_1}{P_0}\right)^{1/2} \frac{B_0}{B_1} \cdot \left| \frac{\int_{-\infty}^{\infty} H(f) \operatorname{sinc}[\pi(f+D)/B_1]\, df}{\int_{-\infty}^{\infty} H(f) \operatorname{sinc}(\pi f/B_0)\, df} \right| \right]. \tag{32}$$

As expected, C depends on the received powers, channel bit rates, and the electrical channel spacing D. It also depends on the BPF response function $H(f)$. Since the choice of a BPF is somewhat arbitrary, it is difficult to present the results in a general manner. In practice, the crosstalk C will depend not only on the BPF bandwidth but also on the filter type.

To illustrate the dependence of the crosstalk C on the filter parameters, we consider two specific kinds of filters, known as the Butterworth and the Chebyshev filters. In general, each type of filter can be further specified by its order n that takes integer values $n = 1, 2, \cdots$. The response functions $H(f)$ for various values of n are well known for both Butterworth and Chebyshev filters. For illustration purposes, we consider second-order filters ($n = 2$) with the response functions [18]

$$H_B(f) = (1 + 1.414s + s^2)^{-1} \tag{33}$$

$$H_C(f) = (3.314 + 2.372s + s^2)^{-1} \tag{34}$$

where $s = (if/f_c)$, and f_c is the cutoff frequency. In the case of the Chebyshev filter, the response function $H_C(f)$ corresponds to a filter with a 0.1-dB ripple. To compare the performance of Butterworth and Chebyshev filters, both filters should have the same 3-dB bandwidth W defined by

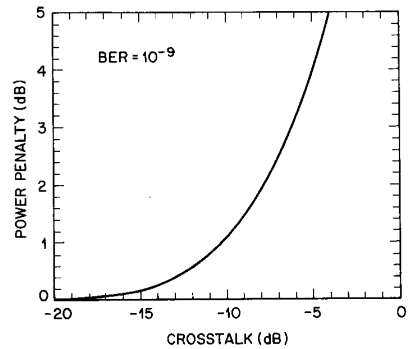

Fig. 3. Calculated power penalty as a function of the crosstalk level at a BER of 10^{-9} when a single neighboring channel contributes to the crosstalk.

$$\left| \frac{H(W/2)}{H(0)} \right|^2 = \frac{1}{2}. \tag{35}$$

Using (33)–(35), we find that $W = 2f_c$ for the Butterworth filter, whereas $W = 3.926 f_c$ for the Chebyshev filter ($n = 2$, in both cases).

The crosstalk C is calculated by substituting (33) and (34) in (32) and evaluating the integrals numerically. Fig. 4 shows the variation of crosstalk C with the channel spacing D for the two kinds of filters. For simplicity, we assume equal bit rates ($B_0 = B_1 = B$) and equal received powers ($P_0 = P_1 = P$) for the two channels. The performance of the Butterworth and Chebyshev filters is compared in Fig. 4 for two different bandwidths $W = 2B$ and $W = 3B$. The crosstalk C is, generally speaking, comparable for the two filters, the difference being -1 dB or so in the range of interest. Interestingly enough, the crosstalk is lower for the Butterworth filter when $W = 2B$, even though it is lower for the Chebyshev filter when $W = 3B$. This suggests that if a wider bandwidth of the BPF is employed to reduce the effect of laser phase noise [11]–[13], a Chebyshev filter is expected to produce less crosstalk penalty than a Butterworth filter.

Figs. 3 and 4 can be used to estimate the minimum channel spacing required to keep the crosstalk penalty below a certain level. Consider first the case of $\Delta < 1$ dB. Fig. 3 then shows that the crosstalk C should be < -10 dB. If we now use Fig. 4, we find that $D \gtrsim 3.5B$ is required for $W = 2B$, whereas D should exceed $4.5B$ for a wider BPF bandwidth $W = 3B$. The required channel spacing increases significantly when either the filter bandwidth W is increased or the amount of tolerable crosstalk penalty is reduced. For example, C should be below -15 dB for $\Delta \le 0.2$ dB. For $W = 3B$, this level of crosstalk can be achieved with a Chebyshev filter by choosing $D \gtrsim 6B$. Some reduction in the minimum channel spacing is possible by choosing higher order filters. The qualitative behavior, however, remains the same.

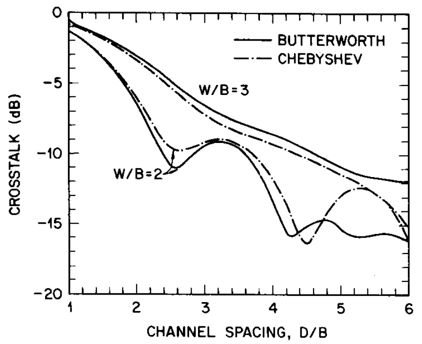

Fig. 4. Variation of the crosstalk with the channel spacing for second-order Butterworth and Chebyshev filters with a 3-dB bandwidth W chosen to be two and three times the bit rate B.

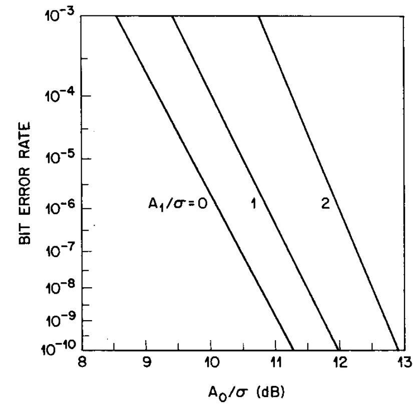

Fig. 5. Same as in Fig. 2 except that both interfering channels contribute to the crosstalk.

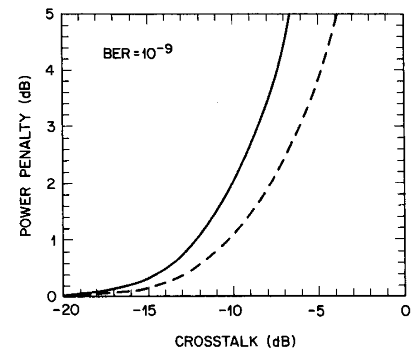

Fig. 6. Calculated power penalty as a function of the crosstalk level at a BER of 10^{-9} when both neighboring channels contribute to the crosstalk. Dashed curve compares the smaller penalty (Fig. 3) occurring when a single channel contributes to the crosstalk.

V. Multiple-Neighbor Crosstalk

The preceding calculation of the crosstalk penalty is for the case when a single neighbor contributes to the crosstalk signal. We now consider the general case when two nearest neighbors on each side of the desired channel interface with the received signal. The error probability is given by (25) and depends not only on the received signals A_1 and A_2 in the interfering channels but also on the relative phase $\phi_1 - \phi_2$ through (18). Since the relative phase is likely to fluctuate from bit to bit, we average the error probability by assuming that the relative phase is uniformly distributed over its entire range 0–2π. For simplicity, we also assume equal received powers in the two interfering channels and set $A_1 = A_2$. The results can be easily extended for the case of unequal received powers. With these simplifications, and by using (18) and (25), the error probability becomes

$$P_e = \frac{1}{8}\left[\left\langle Q\left(\frac{2A_1 \cos\phi}{\sigma}, \frac{A_0}{2\sigma}\right)\right\rangle + 2Q\left(\frac{A_1}{\sigma}, \frac{A_0}{2\sigma}\right) + Q\left(0, \frac{A_0}{2\sigma}\right)\right] \quad (36)$$

where $\phi = (\phi_1 - \phi_2)/2$ and angle brackets denote the averaging operation with respect to ϕ. In the absence of the interfering signal ($A_1 = 0$), (36) reduces to the error probability for a single-channel detection given by (29).

As in the single-neighbor case of Section IV, the error probability or BER depends on the INR. Fig. 5 shows the BER calculated by using (36) as a function of A_0/σ for several values of A_1/σ. Fig. 5 should be compared with Fig. 2 to see how the presence of a second neighboring channel affects the system performance. For a given amount of INR, the increase in SNR to maintain a certain BER is larger compared with the single-neighbor case. This translates into a larger power penalty for the two-neighbor case.

To calculate the power penalty, we follow a procedure identical to that of Section IV. In particular, the power penalty Δ and the crosstalk C are defined by (30) and (31), respectively. Fig. 6 shows the calculated power penalty Δ as a function of the crosstalk C at a BER of 10^{-9}. Dashed curve shows, for comparison, the power penalty for the single-neighbor case discussed in Section IV (see Fig. 3). As expected, Δ is larger for the two-neighbor case for a given value of C. For example, Δ increases from 1 to 2 dB for $C = -10$ dB. Note that the crosstalk C defined by (32) governs the contribution of a single interfering signal.

The minimum channel spacing for the multichannel case can be estimated with the help of Figs. 4 and 6. Let us assume that the system design restricts the crosstalk penalty below 1 dB. From Fig. 6, $\Delta < 1$ dB requires that $C < -12$ dB. If we now use Fig. 4, we find that the minimum channel spacing $D \simeq 3.8B$ for $W = 2B$ but increases to $D \simeq 5B$ if the filter bandwidth is increased to $W = 3B$. For comparison, the corresponding channel spacings for the single-neighbor case are $D \simeq 3.5B$ and $D \simeq 4.2B$, respectively. The optical-domain channel spacing $D_{opt} = D + 2f_{IF}$ if f_{IF} is selected to be closest to the desired channel.

VI. Conclusion

This paper has addressed the issue of optimum channel spacing in multichannel ASK heterodyne lightwave systems. We obtained an expression for the error probability or the BER in one of the channels of such a multichannel system after considering the crosstalk from the two nearest neighbors. The general result was used to calculate the power penalty as a function of the crosstalk level from each neighbor. The level of crosstalk is shown to depend on a large number of system parameters such as the bit rates, received channel powers, the channel separation, and the BPF response. In particular, the crosstalk depends on the filter bandwidth as well as the filter type. This dependence is illustrated by considering the Butterworth and Chebyshev filters.

The main results of the paper can be summarized as follows. For the case of equal bit rates and equal received powers in all channels, the crosstalk from each channel should be below -12 dB to keep the power penalty below 1 dB (Fig. 6). If the filter bandwidth is kept at $W = 2B$, a minimum channel spacing of about four times the bit rate B is required (Fig. 4). If the filter bandwidth is increased to three times the bit rate, the minimum channel spacing increases to about five times the bit rate. These results are in agreement with a recent time-domain analysis [6] where a minimum channel spacing $D = 4B$ is found to be necessary to keep the power penalty below 1 dB. This work also found that D increases rapidly if the design objective is to keep the power penalty to a negligible level: for example $D \geq 10B$ for $\Delta \leq 0.1$ dB. This is again in agreement with our results. A detailed comparison is not possible since it is not clear what assumptions were made regarding the BPF response in [6]. Our analysis shows that the system performance depends on the bandwidth and the type of BPF used for demodulating the channel of interest.

Appendix
Series Expansion for Marcum's Q Function

The calculation of the error probability for many communication systems [14]–[16] requires an evaluation of the integral defined by

$$Q(\alpha, \beta) = \int_\beta^\infty x \exp\left(-\frac{x^2 + \alpha^2}{2}\right) I_0(\alpha x)\, dx. \quad \text{(A1)}$$

The function $Q(\alpha, \beta)$ is known as the Marcum's Q function [17]. Some of its properties are listed in Appendix A of [16] and [18]. One way to evaluate $Q(\alpha, \beta)$ numerically is to make use of the series expansion (25). Since the derivation of (25) is not readily available, a brief derivation is given here. By expanding $I_0(\alpha x)$ in a power series [21], (A1) can be written as

$$Q(\alpha, \beta) = \sum_{k=0}^{\infty} \frac{\exp(-\alpha^2/2)}{(k!)^2} \int_\beta^\infty \left(\frac{\alpha x}{2}\right)^{2k} x e^{-x^2/2}\, dx. \quad \text{(A2)}$$

By making the substitution $x^2/2 = y$, the integral in (A2) can be expressed in terms of the incomplete gamma function [21]

$$Q(\alpha, \beta) = \sum_{k=0}^{\infty} \frac{\exp(-\alpha^2/2)}{(k!)^2} \left(\frac{\alpha^2}{2}\right)^k \Gamma(k+1, \beta^2/2). \quad \text{(A3)}$$

Since $\beta >> 1$ in most applications of the Q functions, it is beneficial to use the asymptotic expansion [21] of the incomplete gamma function in (A3) and obtain

$$Q(\alpha, \beta) = \exp\left(-\frac{\alpha^2 + \beta^2}{2}\right) \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \cdot \frac{(\alpha\beta/2)^{2k}}{k!(k-n)!} \left(\frac{2}{\beta^2}\right)^n. \quad \text{(A4)}$$

The sum over k can be carried out by using the series expansion of nth-order modified Bessel function $I_n(\alpha\beta)$. The result is

$$Q(\alpha, \beta) = \exp\left(-\frac{\alpha^2 + \beta^2}{2}\right) \sum_{n=0}^{\infty} \left(\frac{\alpha}{\beta}\right)^n I_n(\alpha\beta). \quad \text{(A5)}$$

This series expansion is particularly useful for $\alpha << \beta$, a condition satisfied for all Q functions appearing in the error-probability expression (26). As a result, only a few terms in (A5) are needed to evaluate $Q(\alpha, \beta)$ to an accuracy $\sim 10^{-6}$.

Acknowledgment

The author thanks Y. K. Park for bringing this problem to his attention. He is also thankful to L. J. Greenstein and G. J. Foschini for helpful discussions.

References

[1] T. Kimura, "Coherent optical fiber transmission," *J. Lightwave Technol.*, vol. LT-5, pp. 414–428, Apr. 1987 (see also other paper in this special issue on coherent communications).
[2] D. W. Smith, "Techniques in multigigabit coherent optical transmission," *J. Lightwave Technol.*, vol. LT-5, pp. 1466–1478, Oct. 1987.
[3] E.-J. Bachus *et al.*, "Two-channel, heterodyne-type transmission experiment," *Electron. Lett.*, vol. 21, pp. 35–36, Jan. 1985.
[4] P. Healey, "Effect of intermodulation in multichannel optical heterodyne systems," *Electron. Lett.*, vol. 21, pp. 101–103, Jan. 1985.
[5] L. G. Kazovsky, "Multichannel coherent optical communication systems," *J. Lightwave Technol.*, vol. LT-5, pp. 1095-1103, Aug. 1987.
[6] L. G. Kazovsky and J. L. Gimlett, "Sensitivity penalty in multichannel coherent optical communications," *J. Lightwave Technol.*, vol. 6, pp. 1353-1365, Sept. 1988.
[7] Y. K. Park *et al.*, "Crosstalk and prefiltering in a two-channel ASK heterodyne detection system without the effect of laser phase noise," *J. Lightwave Technol.*, vol. 6, pp. 1312-1320, Aug. 1988.
[8] G. P. Agrawal, "Evaluation of crosstalk penalty in multichannel ASK heterodyne optical communication systems," *Electron. Lett.*, vol. 23, pp. 908-908, Aug. 1987.
[9] K. Emura *et al.*, "System design and long-span transmission experiments on a optical FSK heterodyne single filter detection system," *J. Lightwave Technol.*, vol. LT-5, pp. 469-477, Apr. 1987.
[10] G. Nicholson, "Transmission performance of an optical FSK heterodyne system with a single-filter envelope-detection receiver," *J. Lightwave Technol.*, vol. LT-5, pp. 501-508, Apr. 1987.
[11] I. Garrett and G. Jacobsen, "The effect of laser line linewidth on

coherent optical receivers with nonsynchronous demodulation," *J. Lightwave Technol.*, vol. LT-5, pp. 551-560, Apr. 1987.
[12] Y. K. Park *et al.*, "Performance of ASK heterodyne detection with various laser linewidths," *Electron. Lett.*, vol. 22, pp. 283-284, 1986.
[13] G. J. Foschini, L. J. Greenstein, and G. Vannucii, "Noncoherent detection of coherent optical pulses corrupted by phase noise and additive Gaussian noise," *IEEE Trans. Commun.*, vol. 36, pp. 306-314, Mar. 1988.
[14] M. Schwartz, *Information Transmission, Modulation, and Noise*. New York: McGraw Hill, ch. 6, 1980.
[15] L. W. Couch II, *Digital and Analog Communication Systems*. New York: MacMillan, ch. 7, 1983.
[16] S. Benedetto, E. Biglieri, and V. Castellani, *Digital Transmission Theory*. Englewood Cliffs: Prentice-Hall, ch. 4, 1987.
[17] J. I. Marcum, "A statistical theory of target detection by pulsed radar," *IRE Trans. Inform. Theory*, vol. IT-6, pp. 59-267, 1960.
[18] M. Schwartz, W. R. Bennett, and S. Stein, *Communication Systems and Techniques*. New York: McGraw-Hill, 1966.
[19] M. Pent, "Orthogonal polynomial approach for the Marcum Q-function numerical computation," *Electron. Lett.*, vol. 4, pp. 563-564, 1968.
[20] G. S. Mosehytz and P. Horn, *Active Filter Design Handbook*. New York: Wiley, ch. 7, 1981.
[21] M. Abramowitz and I. A. Stegun, Eds., *Handbook of Mathematical Functions*. New York: Dover, 1972.

*

Govind P. Agrawal (M'83-SM'86) was born in Kashipur, India. He received the M.S. and Ph.D. degrees from the Indian Institute of Technology, New Delhi, India, in 1971 and 1974, respectively.

After spending several years at the Ecole Polytechnique, Palaiseau, France, the City University of New York, New York, NY, and Quantel, Orsay, France, he joined AT&T Bell Laboratories, Murray Hill, NJ, in 1982 as a member of the technical staff. Since January 1989, he has been with the Institute of Optics at the University of Rochester. His research interests have been in the fields of quantum electronics, nonlinear optics, and laser physics. He is an author or coauthor of more than 100 research papers, several review articles, and two books entitled *Long-Wavelength Semiconductor Lasers* (Academic Press, 1989) and *Nonlinear Fiber Optics* (Van Nostrand Reinhold Co., 1986).

Dr. Agrawal is a Fellow of the Optical Society of America and a member of the American Physical Society.