Intensity Dependence of the Linewidth Enhancement Factor and Its Implications for Semiconductor Lasers

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Abstract—**The linewidth enhancement factor is shown to become intensity dependent when the intraband relaxation effects responsible for nonlinear gain and index changes are incorporated in the theory of semiconductor lasers. The intensity dependence of the linewidth enhancement factor infuences many laser characteristics. In particular, it leads to a power-independent contribution to the laser linewidth and predicts a rebroadening of the linewidth under certain conditons.**

SEMICONDUCTOR lasers differ from most other lasers in the respect that changes in the optical gain are invariably accompanied by significant changes in the refractive index. The refractive-index variations are included in the theory of semiconductor lasers through a dimensionless parameter α, known as the linewidth enhancement factor (LEF) [1]–[3] as it is responsible, among other things, for an enhancement of the laser linewidth [3]. The LEF affects many other laser characteristics such as the frequency chirp, the modulation response, the injection-locking range, and the effect of external feedback [1]. The LEF is generally treated as a constant in analyzing the laser behavior. In this letter, we show that the LEF becomes intensity dependent when the intraband relaxation effects, responsible for spectral hole burning, are included in the derivation of the susceptibility [4]–[7]. These effects become important when the laser output power is ~ 10 mW. The power dependence of the LEF has many implications for semiconductor lasers. As an example, we show how it can lead to linewidth saturation and rebroadening as the laser power increases, a phenomenon that has attracted considerable attention recently [8]–[11].

The LEF is defined by the general relation

$$\alpha = \frac{\partial \chi_{\text{re}}/\partial N}{\partial \chi_{\text{im}}/\partial N}, \tag{1}$$

together with the complex susceptibility

$$\chi = \chi_{\text{re}} + i\chi_{\text{im}}. \tag{2}$$

The carrier density N is assumed to be constant inside the active region of the semiconductor laser. Since α depends only on the carrier-induced change in the susceptibility, χ denotes only the carrier-induced part in the following discussion. It is related to the index change Δn and the optical gain g as

$$\chi = 2\bar{n}(\Delta n - ig/2k_0) \tag{3}$$

where $k_0 = 2\pi/\lambda_0$, λ_0 is the wavelength, and $\bar{n}$ is the effective mode index of the passive waveguide.

The evaluation of χ requires a density-matrix approach [4]–[7]. In the case of multimode semiconductor lasers, it is necessary to use third-order perturbation theory [4]–[6]. A nonperturbative strong-signal theory has been developed [7] for the case in which the laser oscillates in a single mode. The results show that χ consists of a linear part and a nonlinear part such that

$$\chi = \chi_L + \chi_{NL} \tag{4}$$

where the linear part χ_L is responsible for the carrier-induced gain g_L and the index change Δn_L, i.e.,

$$\chi_L = 2\bar{n}(\Delta n_L - ig_L/2k_0). \tag{5}$$

The nonlinear part has its origin in the intraband relaxation effects and depends on the intensity of the laser mode. It is approximately given by

$$\chi_{NL} = \frac{\bar{n}g_L}{k_0} \frac{[\beta + i(1+I)^{-1/2}]I}{1+(1+I)^{1/2}} \tag{6}$$

where $I = |E_0|^2/I_s$, $|E_0|^2$ is the intracavity mode intensity, and the saturation intensity

$$I_s = \hbar^2/[\mu^2\Gamma\tau_{\text{in}}(\tau_c + \tau_v)]. \tag{7}$$

The parameter β is related to the slope of the linear gain as

$$\beta = \frac{1}{g_L(\omega_0)\tau_{\text{in}}} \left(\frac{dg_L}{d\omega}\right)_{\omega=\omega_0} \tag{8}$$

where ω_0 is the mode frequency. In (7) and (8), μ is the dipole moment, Γ is the confinement factor, and τ_c, τ_v, and τ_{in} are the intraband relaxation times for electrons, holes, and polarization, respectively.

The real and the imaginary parts of the susceptibility are obtained using (4) and (6) and are given by

$$\chi_{\text{re}} = -\frac{\bar{n}}{k_0} g_L \left(\alpha_0 - \frac{\beta I}{1+\sqrt{1+I}}\right), \tag{9}$$

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$$\chi_{\text{im}} = -\frac{\bar{n}}{k_0}\frac{g_L}{\sqrt{1+I}} \tag{10}$$

where α_0 is the LEF when the intraband effects are ignored by setting $\chi_{NL} = 0$. The linear gain g_L is often approximated by [1]

$$g_L(N) = a(N - N_0) \tag{11}$$

where a is the gain coefficient and N_0 is the carrier density required for transparency. If we use (9)–(11) in (1), the intensity-dependent LEF is given by

$$\alpha = \alpha_0\sqrt{1+I} - \frac{\beta I}{1 + 1/\sqrt{1+I}}. \tag{12}$$

Equation (12) is the main result of the paper. It shows that when the mode intensity becomes comparable to the saturation intensity [see (7)] the LEF deviates from its low-intensity value α_0. The nonlinear gain enhances the LEF by a factor of $\sqrt{1+I}$ whereas the nonlinear index [the last term in (12)] may increase or decrease it depending on the sign of the parameter β. We can estimate β by using (8) and by assuming a parabolic frequency dependence of the linear gain for simplicity, i.e.,

$$g_L(\omega) = g_L(\omega_p)[1 - (\omega - \omega_p)^2/\Delta\omega_g^2] \tag{13}$$

where $\Delta\omega_g$ is the gain bandwidth and ω_p is the frequency at which g_L peaks. By using (13) in (8), we obtain

$$\beta = \frac{2(\omega_0 - \omega_p)}{\tau_{\text{in}}\Delta\omega_g^2}. \tag{14}$$

Equation (14) shows that $\beta = 0$ when the laser operates at the gain peak. This is the case for Fabry–Perot lasers. However, the distributed-feedback (DFB) lasers can operate away from the gain peak as the lasing frequency ω_0 is determined by the built-in grating. The parameter β can be positive or negative depending on whether the DFB laser operates on the high-frequency or the low-frequency side of the gain peak. Its numerical value is however generally small ($|\beta| < 1$) if we assume a gain bandwidth of 3 THz and use a typical value $\tau_{\text{in}} = 0.1$ ps.

To illustrate the significance of the intensity dependence of the LEF, we consider the linewidth of a single-mode semiconductor laser given by the expression [3]

$$\Delta\nu = \frac{R_{\text{sp}}(1+\alpha^2)}{4\pi N_{\text{ph}}}, \tag{15}$$

where R_{sp} is the rate of spontaneous emission into the laser mode and N_{ph} is the number of photons inside the cavity. When α is treated as a material constant, $\Delta\nu$ is expected to decrease with the output power as $1/P$. This is evident from (15) if we replace α by α_0 and note that $P = CN_{\text{ph}}$, where C is constant. However, if α from (12) is used in (15), $\Delta\nu$ deviates from the $1/P$ dependence for large output powers. This behavior is shown in Fig. 1 for three values of β by plotting the relation.

$$\frac{\Delta\nu}{\Delta\nu_s} = \frac{1+\alpha^2}{1+\alpha_0^2}\frac{1}{I} \tag{16}$$

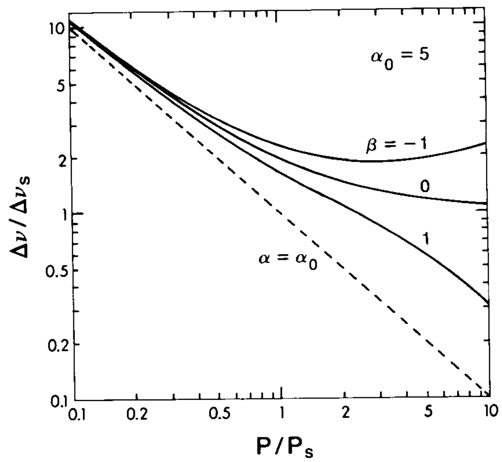

Fig. 1. Variation of the laser linewidth $\Delta\nu$ with the output power P for three values of β. Dashed line shows the expected behavior when the intensity dependence of the linewidth enhancement factor α is ignored. $\Delta\nu_s$ is the linewidth at the saturation output power P_s; numerical estimates are given in the text.

where

$$\Delta\nu_s = CR_{\text{sp}}(1+\alpha_0^2)/4\pi P_s, \tag{17}$$

$I = P/P_s$, and P_s is the saturation output power. The dashed line shows the conventional theory where $\alpha = \alpha_0$ at all power levels. We take $\alpha_0 = 5$ in Fig. 1.

Fig. 1 shows that the intensity dependence of the LEF leads to saturation of the laser linewidth. Consider first the $\beta = 0$ case. Equations (12) and (15) predict a power-independent contribution to the laser linewidth resulting from spectral hole burning in semiconductor lasers. The linewidth is thus expected to saturate to a level $\Delta\nu \simeq \Delta\nu_s$ for $P \gg P_s$. For nonzero values of β the linewidth can increase or decrease from $\Delta\nu_s$ depending on the sign of β. The case of negative β is interesting as it corresponds to rebroadening of the linewidth, a phenomenon also observed experimentally [8]. In our model, rebroadening can occur for DFB lasers oscillating on the low-frequency side of the gain peak.

For a comparison of theory and experiment, it is important to estimate the saturation output power P_s and the saturated linewidth $\Delta\nu_s$. Consider an index-guided InGaAsP laser with a mode cross section $\sim 1\ \mu\text{m}^2$. By using (7) together with the typical parameter values for such a laser [7], we estimate that $I_s \sim 10$ MW/cm^2 and the intracavity saturation power ~ 100 mW. The output saturation power P_s is lower by a factor that depends on the facet reflectivities among other things. As a rough estimate, $P_s \sim 10$ mW and $\Delta\nu_s \sim 1$ MHz. These order-of-magnitude estimates are consistent with the experimental values, [8], [11], indicating that the nonlinear intraband effects may be responsible for linewidth saturation. It should be noted that several other mechanisms [9]–[11] have been invoked to explain the linewidth saturation and rebroadening. The results presented here suggest that the intensity dependence of the LEF should also be included when interpreting the experimental data.

In conclusion, the inclusion of the intraband relaxation effects shows that the LEF is generally intensity dependent and may increase significantly when the laser output power exceeds a saturation level (~ 10 mW). The intensity dependence of the LEF is expected to influence many laser characteristics such as the frequency chirp, the modulation response, the injection-locking range, and the phase noise. As a specific example, we have investigated its effect on the laser linewidth and found that the intraband effects lead to a power-independent contribution to the linewidth. Furthermore, for semiconductor lasers detuned to operate away from the gain peak, the nonlinear index changes can even lead to a rebroadening of the laser linewidth at high-output powers.

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