Effect of injection-current fluctuations on the spectral linewidth of semiconductor lasers

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The effect of current fluctuations on the linewidth of semiconductor lasers is analyzed using the single-mode rate equations. Since the time scale of such fluctuations can generally be longer than the intrinsic time scale of relaxation oscillations, current fluctuations are modeled using a non-Markovian random force in the rate equation governing the carrier dynamics. In the absence of nonlinear-gain effects, the contribution of current fluctuations to the linewidth is negligible at all power levels. However, when the gain saturation resulting from spectral hole burning is included, current fluctuations are found to give rise to a power-independent contribution to the linewidth. At low operating powers, this contribution is small compared with the spontaneous-emission contribution. For InGaAsP lasers, the power-independent contribution is estimated to be ~ 1 MHz/μA and can significantly affect the intrinsic linewidth at high power levels (> 10 mW). Furthermore, the line shape is not strictly Lorentzian and tends towards Gaussian with increasing power. We have discussed the dependence of the line shape and linewidth on various device parameters.

I. INTRODUCTION

The linewidth of a single longitudinal mode is a measure of spectral purity of any laser oscillating predominantly in a single longitudinal mode. Under ideal conditions the linewidth is determined by quantum noise through spontaneous emission and is given by the modified Schawlow-Townes (MST) formula.[1–3] For most lasers, however, the linewidth is dominated by the pump noise (external fluctuations associated with the pumping mechanism) whose contribution is generally larger by several orders of magnitude compared with that of the quantum noise. An exception occurs for semiconductor lasers whose small size and a relatively low-Q cavity lead to a linewidth dominated by the spontaneous-emission noise. It is for this reason that semiconductor lasers have recently become a testing ground for the MST formula.[3–15]

One of the predictions of the MST formula is that the linewidth decreases inversely with an increase in the output power. Although such a behavior is indeed observed experimentally over some range, the linewidth is often found to saturate at high power levels. For solitary semiconductor lasers, the limiting value is generally in the range 1–10 MHz.[4,11–15] These experimental results suggest that the linewidth of a semiconductor laser has a power-independent contribution that manifests itself at high power levels when the spontaneous-emission contribution becomes relatively small. A large number of physical mechanisms have been proposed to explain the power-independent contribution; some of them are electron-number fluctuations,[4,16] occupation fluctuations,[17] longitudinal-mode interaction,[12,18] and $1/f$ noise.[19–21] The effect of pump noise (current fluctuations) on the semiconductor-laser linewidth has not attracted much attention even though one may expect it to contribute in a manner similar to the case of gas or dye lasers. Although external current fluctuations can be reduced by taking special precautions,[4] they are generally non-negligible and should be accounted for. In this paper we calculate the contribution of these fluctuations to the linewidth and show that under typical operating conditions it amounts to a few megahertz and may well explain the experimental results. Furthermore, spectral hole burning,[22,23] a phenomenon that leads to a power-dependent reduction of the gain, makes the current-noise contribution to the linewidth power independent.

The paper is organized as follows. In Sec. II we give the Langevin equations based on the single-mode rate equations for semiconductor lasers.[24] Current fluctuations are included in the rate equation for the carriers (electrons or holes) through a non-Markovian Langevin force. The Langevin equations are solved in Sec. III using a linearization procedure around the steady-state average values to obtain the phase variance analytically. The optical spectrum is then calculated by taking the Fourier transform of the autocorrelation function. The resulting line shape and the linewidth are discussed in Sec. IV. Simple analytic expressions for the linewidth are obtained in the low-power and high-power limits. Finally, the results are discussed in Sec. V, where we comment on the extension of the present calculations to external-cavity semiconductor lasers.

II. LANGEVIN EQUATIONS

The linewidth analysis of semiconductor lasers is based on the rate equations modified by adding Langevin noise sources to account for the various noise mechanisms.[5–8] We follow the same procedure except for considering an additional noise mechanism arising from the pump noise or current fluctuations. The resulting Langevin equations are[10,24]

$$\dot{P}=(G-\gamma)P+R+F_P(t) , \quad (1)$$

$$\dot{N}=\tilde{C}-S-GP+F_N(t) , \quad (2)$$

$$\dot{\Phi}=\frac{\alpha}{2}(G-\gamma)+F_\Phi(t) , \quad (3)$$

where P and N represent the number of photons and electrons inside the cavity, Φ is the optical phase, G is the net gain, γ is the cavity-decay rate, R is the spontaneous-emission rate, $\tilde{C}$ is the rate of carrier generation, S is the rate of carrier recombination (through all processes except for stimulated emission), and α is the linewidth enhancement factor.[3] The Gaussian Langevin noise sources have zero mean and are δ-function correlated (in the Markovian approximation), i.e.,

$$\langle F_i(t)F_j(t')\rangle=2D_{ij}\delta(t-t') \quad (4)$$

for $i,j=P$, N, and Φ. The nonzero diffusion coefficients are given by[2,10]

$$D_{PP}=RP, \quad D_{NN}=RP+S , \quad (5a)$$

$$D_{\Phi\Phi}=R/2P, \quad D_{PN}=-RP . \quad (5b)$$

In the presence of current fluctuations, the rate of carrier generation $\tilde{C}$ in Eq. (2) fluctuates ($\tilde{C}=I/q$). We therefore write

$$\tilde{C}=C+F_C(t) , \quad (6)$$

where C is the average value and the random force $F_C(t)$ accounts for current fluctuations. It is important to note that $F_C(t)$ is not δ-function correlated [i.e., $F_C(t)$ is non-Markovian] since the time scale of current fluctuations (~ 1 μs) is generally larger than the photon or carrier lifetime. Similar to the case of dye lasers,[25] we assume an Ornstein-Uhlenbeck model for $F_C(t)$ with

$$\langle F_C(t)F_C(t')\rangle=D_C\Gamma_C \exp(-\Gamma_C\,|\,t-t'\,|\,) , \quad (7)$$

where D_C is the diffusion coefficient and Γ_C is the decay rate of current fluctuations. An exponential correlation amounts to assuming a Lorentzian spectral density of current fluctuations (FWHM is Γ_C/π). A noise such as $F_C(t)$ with a finite correlation time is generally referred to as colored noise in contrast to the δ-function correlated noise that has a white spectral density.

Before proceeding further we need an order-of-magnitude estimate of the parameters Γ_C and D_C. Assuming a 1-μs time scale of current fluctuations, $\Gamma_C\simeq 10^6$ s^{-1}. The parameter D_C can be related to the standard deviation σ_I of current fluctuations by using Eq. (7) and noting that

$$\langle F_C^2(t)\rangle=\sigma_I^2/q^2=D_C\Gamma_C$$

or

$$D_C=\sigma_I^2/(q^2\Gamma_C) . \quad (8)$$

If we take a typical value $\sigma_I\simeq 1$ μA, $D_C\simeq 4\times 10^{19}$ s^{-1}. This is about two orders of magnitude larger than the shot-noise term D_{NN}.

III. OPTICAL SPECTRUM

The optical spectrum of an electromagnetic field is calculated by using

$$G(\omega)=\int_{-\infty}^{\infty}\langle E^*(t)E(t+\tau)\rangle \exp(i\omega\tau)d\tau , \quad (9)$$

where $E=\sqrt{P}\exp(-i\Phi)$ is the electric field. Assuming that P is relatively constant and that phase fluctuations obey Gaussian statistics,

$$G(\omega)\simeq\bar{P}\int_{-\infty}^{\infty}\exp[-\tfrac{1}{2}\langle\Delta\phi^2(\tau)\rangle+i\omega\tau]d\tau , \quad (10)$$

where $\bar{P}$ is the average photon number and

$$\langle\Delta\phi^2(\tau)\rangle=\langle[\Phi(t+\tau)-\Phi(t)]^2\rangle$$
$$=\mathrm{Re}\left[\frac{1}{\pi}\int_{-\infty}^{\infty}\langle\,|\tilde{\Phi}(\omega)|^2\rangle(1-e^{i\omega\tau})d\omega\right] . \quad (11)$$

Here Re stands for the real part and $\tilde{\Phi}(\omega)$ is the Fourier transform of $\Phi(t)$. To evaluate $\tilde{\Phi}(\omega)$, one needs to solve the set of three stochastic nonlinear equations (1)–(3). In a commonly employed method,[6–8] these equations are linearized around the steady-state average values $\bar{P}$, $\bar{\Phi}$, and $\bar{N}$ to obtain

$$\dot{p}=G_N\bar{P}n-\Gamma_P p+F_P(t) , \quad (12)$$

$$\dot{n}=-Gp-\Gamma_N n+F_N(t)+F_C(t) , \quad (13)$$

$$\dot{\phi}=\frac{\alpha}{2}G_N n+F_\Phi(t) , \quad (14)$$

where $p=P-\bar{P}$, $\phi=\Phi-\bar{\Phi}$, and $n=N-\bar{N}$ are small deviations from the steady state and the decay rates

$$\Gamma_P=R/\bar{P}+G_P\bar{P}, \quad \Gamma_N=dS/dN+G_N\bar{P} . \quad (15)$$

In obtaining Eqs. (12)–(14), the gain G in Eq. (1) is approximated by

$$G(N,P)\simeq G(\bar{N},\bar{P})+G_N n-G_P p , \quad (16)$$

where G_P accounts for the nonlinear reduction in gain resulting from phenomena such as spectral hole burning.[22,23] Its main effect is to enhance Γ_P in direct proportion with the laser power [see Eq. (15)]. It will be seen later that it is this effect that makes the pump-noise contribution to the linewidth power independent.

Because of their linear nature, Eqs. (12)–(14) are readily solved in the Fourier domain to obtain $\tilde{\phi}(\omega)$:

$$\tilde{\phi}(\omega)=\frac{1}{i\omega}\tilde{F}_\Phi+\frac{\alpha G_N}{2i\omega\Delta}[(\Gamma_P+i\omega)(\tilde{F}_N+\tilde{F}_C)-G\tilde{F}_P] , \quad (17)$$

where the tilde denotes the Fourier-transform operation and

$$\Delta=(\omega_1-\omega)(\omega-\omega_2) , \quad (18)$$

$$\omega_1=\Omega+i\Gamma, \quad \omega_2=-\Omega+i\Gamma . \quad (19)$$

The frequency Ω and the damping rate Γ of relaxation oscillations are given by

$$\Omega \simeq (GG_N\bar{P})^{1/2}, \quad \Gamma = \tfrac{1}{2}(\Gamma_N + \Gamma_P) . \tag{20}$$

Equation (17) shows how various noise sources lead to phase fluctuations. The first term is due to instantaneous phase fluctuations as a result of spontaneous emission. The second term proportional to α results from carrier fluctuations and has three distinct contributions arising from $\tilde{F}_N$ (shot noise), $\tilde{F}_C$ (pump noise), and $\tilde{F}_P$ (intensity noise). The term proportional to $\tilde{F}_P$ generally provides the dominant contribution. However, the contributions from $\tilde{F}_N$ and $\tilde{F}_C$ should be included since they become important at high power levels.

Equations (11) and (17) can be used to calculate the phase variance. The use of Eqs. (4)–(7) in (17) leads to the following expression:

$$\langle |\tilde{\phi}(\omega)|^2 \rangle = \frac{R}{2P\omega^2}\left[1 + \frac{\alpha^2}{|\Delta|^2}\{\Omega^2 + G_N^2 P[(\omega^2+\Gamma_P^2)(P+S/R) + 2G\Gamma_P P]\} + \frac{D_C\alpha^2 G_N^2 P\Gamma_C^2(\omega^2+\Gamma_P^2)}{R(\omega^2+\Gamma_C^2)}\right] . \tag{21}$$

For brevity, the bar over $\bar{P}$ and $\bar{N}$ is omitted from now onward. The last term, proportional to D_C, is due to the pump noise. We substitute Eq. (21) in Eq. (11) and carry out the integration using the method of contour integration. The integrand has four poles located at $\omega=0$, $\omega=i\Gamma_C$, $\omega=\omega_1$, and $\omega=\omega_2$, where ω_1 and ω_2 are given by Eq. (19). The contribution of poles at ω_1 and ω_2 leads to weak satellite peaks at multiples of the relaxation frequency Ω (Refs. 7 and 8). Since $\Omega \gg \Gamma_C$, these satellite peaks are not significantly affected by the pump noise. In the following discussion we ignore them and consider only the poles at $\omega=0$ and $\omega=i\Gamma_C$. Both poles contribute to the linewidth of the dominant central peak in the optical spectrum. Since the procedure is straightforward, we write the result

$$\langle \Delta\phi^2(\tau) \rangle = \frac{R}{2P}[(1+\alpha^2 A)\tau] - \frac{\alpha^2 D_C \Gamma_P^2}{2G^2P^2\Gamma_C}[1-\exp(-\Gamma_C\tau)] , \tag{22}$$

where

$$A = \frac{\Omega^4}{(\Omega^2+\Gamma^2)^2}\left[\left(1+\frac{\Gamma_P}{G}\right)^2 + \frac{\Gamma_P^2}{RG^2P}(D_C+S)\right] \simeq 1 + \frac{\Gamma_P^2(D_C+S)}{RG^2P} . \tag{23}$$

In obtaining Eq. (23) we used the approximations $\Gamma \ll \Omega$ and $\Gamma_P \ll G$ which generally hold under practical operating conditions. Using Eq. (23) in Eq. (22) we obtain

$$\langle \Delta\phi^2(\tau) \rangle = \frac{R(1+\alpha^2)}{2P}\tau + \frac{\alpha^2\Gamma_P^2 D_C}{2G^2P^2}\left[\left(1+\frac{S}{D_C}\right)\tau - \frac{1}{\Gamma_C}[1-\exp(-\Gamma_C\tau)]\right] . \tag{24}$$

The first term (proportional to R) is due to spontaneous emission while the second term results from the pump noise and shot noise. In the absence of the pump noise ($D_C=0$), the shot-noise contribution (proportional to S) is generally negligible compared to the spontaneous-emission contribution. For this reason, the linewidth analysis of semiconductor lasers is generally based on the first term alone in Eq. (24). However, in the presence of pump noise ($D_C \neq 0$), the second term contribution may not be negligible, particularly at high power levels. This is discussed in Sec. IV.

IV. LASER LINEWIDTH

The line shape is obtained by taking the inverse Fourier transform of $\langle \Delta\phi^2(\tau) \rangle$ given by Eq. (24), as seen from Eq. (10). We have evaluated the line shape numerically using typical parameter values for a 1.55-μm InGaAsP laser.[24] In particular, $\alpha=6$, $G=6\times 10^{11}$ s^{-1}, $R=n_{sp}G$ with the spontaneous emission factor $n_{sp}=1.7$, and $P=3\times 10^4 P_{out}$ where P_{out} is the output power in milliwatts. The gain G was estimated using

$$G = \tau_p^{-1} = (c/n_g)g_{th} , \tag{25}$$

with the threshold gain $g_{th}=80$ cm^{-1} and the group index $n_g=4$. This corresponds to a photon lifetime of $\tau_p=1.66$ ps. The decay rate Γ_P from Eq. (15) is given by

$$\Gamma_P = R/P + G_P P . \tag{26}$$

The hole-burning parameter G_P is estimated as $G_P = 1\times 10^5$ s^{-1}. This value corresponds to a gain compression of 0.5% at 1 mW of the output power as a result of spectral hole burning.[23] For the decay rate of current fluctuations, we vary Γ_C from 10^4 to 10^8 s^{-1}. The parameter D_C is calculated using Eq. (8) for a given noise current σ_I. The effect of shot noise is generally negligible since $S \ll D_C$.

Figure 1 shows the line shapes at 10 and 50 mW of output powers for $\sigma_I = 5$ μA and $\Gamma_C = 10^6$ s^{-1}. For comparison, dashed curves show the Lorentzian line shape in the absence of current fluctuations ($\sigma_I=0$). The effect of pump noise is to increase the linewidth, and the increase is most evident at high operating powers. For $P_{out} < 10$ mW, the increase is modest (a few percent) and the line shape remains approximately Lorentzian because the spontaneous-emission contribution [first term in Eq. (24)]

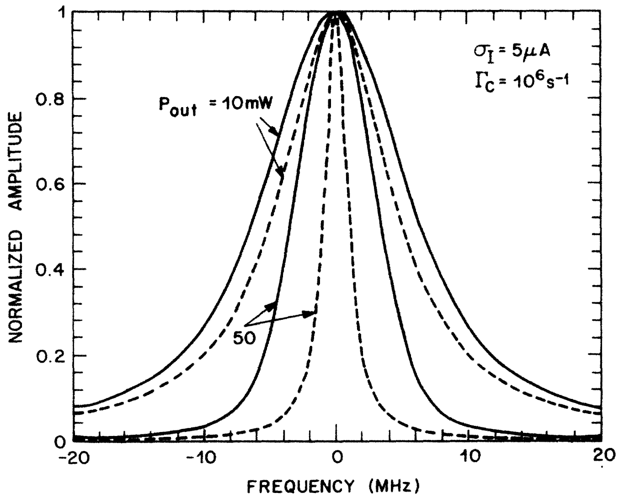

FIG. 1. Line shapes of a semiconductor laser at 10 and 50 mW output powers showing the effect of current fluctuations for $\sigma_I = 5$ μA and $\Gamma_C = 10^6$ s^{-1}. Dashed curves show the line shapes in the absence of current fluctuations ($\sigma_I = 0$).

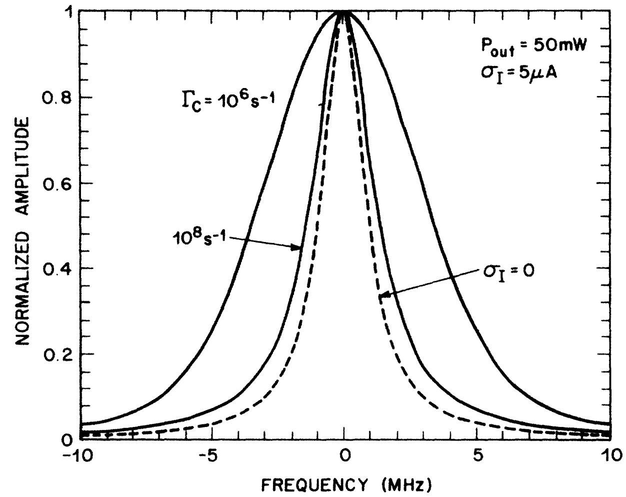

FIG. 3. The effect of varying the time scale Γ_C^{-1} of current fluctuations on the line shape of a semiconductor laser operating at 50 mW with $\sigma_I = 5$ μA. For comparison, the dashed curve shows the Lorentzian line shape in the absence of current fluctuations ($\sigma_I = 0$). For $\Gamma_C < 10^6$ s^{-1}, the line shape cannot be distinguished with that obtained for $\Gamma_C = 10^6$ s^{-1} on the scale used in the figure. For $\Gamma_C > 10^9$ s^{-1}, the line shape nearly coincides with the dashed curve.

dominates over the pump-noise contribution. The two contributions become comparable at 50 mW in Fig. 1. The line shape is then no longer Lorentzian and the linewidth is considerably enhanced. Of course, the enhancement factor depends on the pump-noise level. This is seen clearly in Fig. 2 where σ_I is varied in the range 0–10 μA for $P_{out} = 50$ mW and $\Gamma_C = 10^6$ s^{-1}. Finally, Fig. 3 shows the effect of Γ_C variation on the linewidth for constant values of $\sigma_I = 5$ μA and $P_{out} = 50$ mW. As Γ_C is decreased from 10^8 to 10^4 s^{-1}, the pump-noise contribution to the linewidth initially increases and then saturates to a constant value for $\Gamma_C \lesssim 10^6$ s^{-1}. The curve for $\Gamma_C = 10^4$ s^{-1} is indistinguishable from that for $\Gamma_C = 10^6$ s^{-1} on the scale shown in Fig. 3.

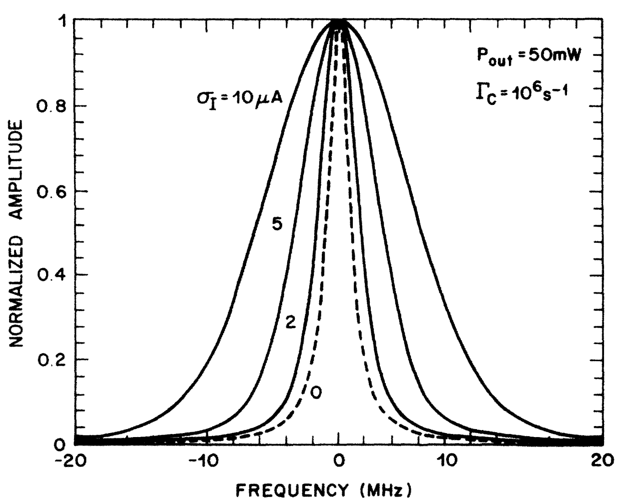

FIG. 2. Line shapes at 50 mW output power showing the broadening with an increase in the current noise σ_I. The dashed curve for $\sigma_I = 0$ corresponds to the Lorentzian line shape expected in the absence of current fluctuations.

In order to obtain further physical insight we evaluate $G(\omega)$ approximately using Eqs. (10) and (24) with the assumption that the dominant contribution to the integral comes from values of $|\tau| \ll \Gamma_C^{-1}$. Expanding $\exp(-\Gamma_C\tau)$ in a power series and neglecting the cubic and higher-order terms, we obtain

$$G(\omega) = P\int_{-\infty}^{\infty} \exp(-\tfrac{1}{2}\beta^2\tau^2 - \gamma|\tau| + i\omega\tau)d\tau\ , \tag{27}$$

where

$$\beta = \frac{\alpha\Gamma_P\sqrt{D_C\Gamma_C}}{2GP}\ , \tag{28}$$

$$\gamma = \frac{R(1+\alpha^2)}{4P} + \frac{\alpha^2\Gamma_P^2 S}{4G^2P^2}\ . \tag{29}$$

The integral in Eq. (27) can be evaluated in terms of the complimentary error function[26] with the result

$$G(\omega) = P\,\mathrm{Re}\left[\left(\frac{\pi}{2\beta}\right)^{1/2} \exp\left(\frac{(\gamma - i\omega)^2}{2\beta^2}\right) \times \mathrm{erfc}\left(\frac{\gamma - i\omega}{\sqrt{2}\beta}\right)\right]\ . \tag{30}$$

It is difficult to obtain an expression for the linewidth for arbitrary values of β and γ. We therefore consider the two limiting cases of low and high operating powers.

A. Low operating powers

At low operating power ($\lesssim 10$ mW), $\gamma \gg \beta$, and the spontaneous-emission contribution dominates. It is then possible to use the asymptotic expansion[26]

$$\text{erfc}(z) \simeq \frac{1}{\sqrt{\pi}} \exp(-z^2) \left[\frac{1}{2} - \frac{1}{2z^3} \right] . \tag{31}$$

Using Eqs. (30) and (31) we obtain

$$G(\omega) = \frac{\gamma P}{\gamma^2 + \omega^2} \left[1 - \frac{\beta^2(\gamma^2 - 3\omega^2)}{(\gamma^2 + \omega^2)^2} \right] . \tag{32}$$

As expected, the line shape is Lorentzian in the absence of pump noise ($\beta = 0$). The spectral width (FWHM) of this Lorentzian is given by

$$\Delta\nu_0 = \frac{\gamma}{\pi} = \frac{R(1+\alpha^2)}{4\pi P} + \frac{\alpha^2 \Gamma_P^2 S}{4\pi G^2 P^2} . \tag{33}$$

This is the well-known expression for the semiconductor laser linewidth.[3-8] The shot-noise contribution (proportional to S) is often neglected in comparison to the spontaneous-emission contribution. Noting from Eq. (26) that $\Gamma_P \simeq G_P P$, the shot-noise contribution can be seen to be power independent. Its typical value estimated using the parameter values used for Figs. 1–3 is 15 kHz. We shall ignore this contribution in the following discussion.

In the presence of pump noise ($\beta \neq 0$), Eq. (32) shows that the line shape deviates from Lorentzian. At low operating powers, the deviations are small. The linewidth (FWHM) is found to be slightly enhanced and is given by

$$\Delta\nu = \Delta\nu_0(1 + 2\beta^2/\gamma^2) . \tag{34}$$

The enhancement increases with an increase in the operating power since γ varies as P^{-1} [see Eq. (29)]. This is consistent with the numerical results of Fig. 1. Using Eqs. (8), (28), and (29) in (34), the enhancement factor is given by

$$\frac{\Delta\nu}{\Delta\nu_0} = 1 + \left[\frac{2 G_P \sigma_I P}{qGR} \right]^2 \frac{\alpha^2}{1+\alpha^2} . \tag{35}$$

B. High operating powers

As the laser power increases, the spontaneous-emission contribution decreases as P^{-1} while the pump-noise contribution is relatively power independent. This can be seen from Eqs. (28) and (29) by noting that $\Gamma_P \simeq G_P P$. At very high power levels ($\gamma \ll \beta$), the linewidth becomes power independent.

This limiting linewidth can be obtained from Eq. (27) or Eq. (30) by setting $\gamma = 0$. The line shape in found to be Gaussian, i.e.,

$$G(\omega) = \frac{\sqrt{2\pi} P}{\beta} \exp \left[-\frac{\omega^2}{2\beta^2} \right] . \tag{36}$$

The width (FWHM) is given by

$$\Delta\nu_{\text{PI}} = (2 \ln 2)^{1/2} \beta / \pi , \tag{37}$$

where the subscript PI is a reminder that $\Delta\nu_{\text{PI}}$ is the power-independent linewidth. Substituting β from Eq. (28) and replacing Γ_P by $G_P P$, we obtain

$$\Delta\nu_{\text{PI}} = \frac{1}{2\pi} (2 \ln 2)^{1/2} (\alpha G_P / G)(D_C \Gamma_C)^{1/2} . \tag{38}$$

We can eliminate D_C using Eq. (8), and the result is

$$\Delta\nu_{\text{PI}} = 1.18 \alpha G_P \tau_p \sigma_I / (2\pi q) , \tag{39}$$

where we have replaced G by τ_p^{-1}, τ_p being the photon lifetime. Equation (39) shows that $\Delta\nu_{\text{PI}}$ varies linearly with the noise current σ_I, the linewidth enhancement factor α, the photon lifetime τ_p, and the hole-burning parameter G_P. Using typical parameter values for InGaAsP lasers $\alpha = 6$, $G_P = 1 \times 10^5 \text{ s}^{-1}$, $\tau_p = 1.66$ ps, we estimate that $\Delta\nu_{\text{PI}} \simeq 1$ MHz for $\sigma_I = 1\ \mu$A. This value is expected to be smaller for GaAs lasers since both α and G_P are generally smaller. It is important to emphasize that the power-independent nature of $\Delta\nu_{\text{PI}}$ is a consequence of spectral hole burning, a phenomenon that enhances damping of relaxation oscillations through Γ_P given by Eq. (26). In the absence of spectral hole burning ($G_P = 0$), Γ_P in Eq. (28) should be replaced by R/P; $\Delta\nu_{\text{PI}}$ then decreases as P^{-2} and would remain negligible at all power levels.

V. DISCUSSION AND CONCLUSIONS

In this paper we have studied the dependence of the linewidth of a single-longitudinal-mode semiconductor laser on injection current fluctuations. Within the rate-equation formalism, current fluctuations are included through a non-Markovian Langevin force that is added to the usual Markovian Langevin force responsible for the carrier shot noise. Taking an Ornstein-Uhlenbeck model for the non-Markovian process, we assign a strength parameter D_C and a time scale Γ_C^{-1} to these current fluctuations. The theory then leads to a very interesting conclusion: current fluctuations are responsible for a power-independent contribution to the linewidth. The estimated value of this contribution for InGaAsP lasers in ~ 1 MHz/μA. Since fluctuations in the range of a few μA can occur at typical operating current ~ 100 mA unless special precautions are taken to eliminate them, a limiting value of the laser linewidth at high operating powers in the range of a few MHz is expected. The existence of a power-independent contribution to the linewidth has been noted in many recent experiments.[4,11-15] Although other mechanisms[16-20] may have also contributed, we believe that current fluctuations are at least in part responsible for the observed data, particularly for InGaAsP lasers.

A remarkable result of our theory is that the power-independent nature of the current-fluctuations contribution to the linewidth is a consequence of spectral hole burning.[22,23] In the absence of spectral hole burning, this contribution decreases as P^{-2} and would be negligible at all power levels compared with the spontaneous-emission contribution. The role of spectral hole burning is to partially unclamp the carrier population which has to increase to compensate for the gain compression at high powers. As a result, the damping rate of relaxation oscillations increases with the laser power ($\Gamma_P \simeq G_P P$). Since the hole-burning parameter G_P is larger for InGaAsP lasers than for GaAs lasers,[23] current fluctuations affect the linewidth of InGaAsP lasers more than that of GaAs lasers. In fact, larger values of both α and G_P for In-

GaAsP lasers suggest that $\Delta\nu_{PI}$ [Eq. (39)] may be larger as much as by one order of magnitude for InGaAsP lasers.

In general the effect of current fluctuations on the laser linewidth depends on the standard deviation σ_I as well as on the time scale Γ_C^{-1} of fluctuations. For $\Gamma_C > \Gamma$ current fluctuations occur so rapidly that carriers cannot respond, and as a result, the linewidth is not affected by the current noise. However, for a time scale of current fluctuations ~ 1 μs, the linewidth is nearly independent of Γ_C. The parameter σ_I will generally vary depending on the electrical power supply used for the current injection. Clearly σ_I can be reduced by a proper control of current fluctuations. Johnson noise sets a fundamental limit on σ_I. With a 50-Ω series resistor and a 1-MHz bandwidth for the power supply, we estimate the limiting value $\sigma_I \simeq 20$ nA. The Johnson-noise contribution to the linewidth from Eq. (39) is then estimated to be ~ 10 kHz and is similar to the shot-noise contribution.

Mooradian and co-workers[4,16] have used carrier-number fluctuations to explain their data on the power-independent contribution to the linewidth of GaAs lasers. Their phenomenological model assumes that $\Delta N = \sqrt{N}$ where the carrier number $N = \eta_i \tau I_{th}/q$ is taken to be clamped at is threshold value, η_i is the internal quantum efficiency, τ is the spontaneous carrier lifetime ($\sim 2-3$ ns), and I_{th} is the threshold current. If we assume that carrier fluctuations are due to current fluctuations, we estimate that $\sigma_I = (qI_{th}/\eta_i\tau)^{1/2} \simeq 3$ μA for a typical room-temperature value of $I_{th} = 30$ mA. This corresponds well with the σ_I values used in Sec. IV and suggests that current fluctuations can be a possible source of carrier-number fluctuations. There is however a major difference between the phenomenological model[4,16] and the rate-equation approach adopted here. Whereas the phenomenological model assumes that phase fluctuations follow carrier-number fluctuations instantaneously, in the rate-equation approach the inclusion of carrier dynamics imposes a delay governed by relaxation oscillations. As a result, the two methods predict different linewidth behavior. For example, $\Delta\nu_{PI}$ from Eq. (38) would decrease (because of lower values of α) rather than increase at low temperatures if we assume that G_P is nearly temperature independent. It appears that the observed increase of $\Delta\nu_{PI}$ at low temperatures[16] is probably due to some other mechanism.

It is well known that the linewidth of a semiconductor laser can be considerably reduced by optical feedback.[27,28] Indeed, the coupling of a semiconductor laser to an external grating has resulted in linewidths as narrow as 10 kHz.[29] The question then arises as to what extent current fluctuations limit the linewidth of such external-cavity semiconductor lasers. For the case of weak optical feedback, we have followed the analysis of Ref. 28 to study the effect of current fluctuations. We find that a combination of current fluctuations and spectral hole burning still leads to a power-independent linewidth $\Delta\nu_{PI}$. However, $\Delta\nu_{PI}$ is also reduced by exactly the same factor as the spontaneous-emission contribution. A similar analysis can be carried out for the case of strong feedback.[28,30] The result is the same. For external-cavity semiconductor lasers, $\Delta\nu_{PI}$ is reduced by a factor of $(1+n_1L_1/n_0L_0)^2$ where n_0L_0 and n_1L_1 are the optical lengths of the laser and external cavities. A similar conclusion holds for more complicated feedback configurations such as those using external Bragg reflectors.[30] The general conclusion is that current fluctuations in semiconductor lasers can limit the achievable linewidth by providing a power-independent contribution to the linewidth.

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