Modulation Performance of a Semiconductor Laser Coupled to an External High-Q Resonator

GOVIND P. AGRAWAL, SENIOR MEMBER, IEEE, AND CHARLES H. HENRY

Abstract—The dynamic response of a semiconductor laser coupled to an external resonator is studied using the single-mode rate equations modified to account for the dispersive feedback. Both the frequency and the damping rate of relaxation oscillations are affected by the feedback. The frequency chirp that invariably accompanies amplitude modulation is significantly reduced. The feedback also reduces the phase noise and the linewidth. To investigate the usefulness of external-resonator lasers in high-speed optical communication systems, we have solved the rate equations numerically to obtain the emitted chirped pulse; the pulse is propagated through the fiber, detected, and filtered at the receiver. The simulated eye diagrams show that such lasers can be operated at high bit rates with negligible dispersion penalty owing to their reduced frequency chip. An improvement in the bit rate by about a factor of two is predicted by the use of an external high-Q resonator under typical operating conditions.

I. Introduction

THE coupling of a semiconductor laser to a passive resonator can improve the laser performance by, among other things, reducing the linewidth and the frequency chirp [1]–[16]. The reduced linewidth is useful in coherent-communication applications; indeed, the use of an external grating placed at 10–20 cm from the laser has led to linewidths ~10 kHz [1]. Low chirp is necessary for direct-detection systems in which information is transmitted along dispersive fibers at high bit rates [17]. The physical mechanism behind the chirp and linewidth reductions realized by coupling the semiconductor laser to an external passive resonator is the resonant, frequency-dependent optical feedback into the laser cavity from the high-Q resonator. While the coupling can greatly reduce the chirp, it also alters the dynamic response of the laser, especially by reducing the damping rate of relaxation oscillations. The purpose of this paper is primarily to investigate whether such coupling can improve the bit rate for transmission along dispersive fibers.

Several schemes have been used to form the high-Q resonator by using the techniques such as 1) the formation of a long external cavity by placing a mirror or grating [1], 2) the use of an external Bragg reflector butt-coupled to a laser facet [13]–[16], and 3) the use of a narrow-band resonant optical reflector [18] to form the external resonator. For ease of reference, we shall collectively call these devices external-resonator lasers. Recently, Kazarinov and Henry [12] have analyzed such lasers under CW or low-frequency operation and showed how the reductions in the adiabatic chirp and the linewidth are simply related. In this paper, we extend their analysis to study the dynamic response of an external-resonator laser under high-frequency modulation. In Section II we obtain the single-mode rate equations modified to account for the resonant optical feedback from the external resonator governed by two feedback parameters A and B. These equations are used to study the small-signal modulation response in Section III. The resonant optical feedback affects both the frequency and the damping rate of relaxation oscillations. Whereas the damping rate is always reduced, the relaxation-oscillation frequency can be increased (up to a factor of two) by an appropriate choice of the feedback parameters A and B. The frequency chirp that invariably accompanies amplitude modulation is significantly reduced and the reduction factor depends on the modulation frequency. In particular, the low-frequency or adiabatic chirp is reduced by a factor of F determined by the feedback characteristics of the external resonator [12]. The consideration of phase noise in Section IV and [12] shows that the linewidth is reduced by a factor of F^2. Furthermore, the relative amplitude of the satellite peaks occurring at multiples of the relaxation-oscillation frequency is also reduced. The linewidth reduction by the use of external resonators has been predicted in earlier work [2]–[5].

The enhancement of the relaxation-oscillation frequency and the reduction of the linewidth have been observed for an external reflector by Vahala *et al.* [9]. The relation between chirp and linewidth reductions was recently verified by Olsson *et al.* [14] using an external Bragg reflector. Olsson *et al.* [13] have demonstrated the potential of such external-resonator lasers in a 1.55 μm transmission experiment at 1.7 Gbits/s over a 82.5 km long dispersive fiber. There is some experimental evidence [13] that chirp reduction by resonant optical feedback can improve the system performance over that achieved by conventional single-frequency lasers (e.g., distributed feedback lasers).

In order to quantify the performance improvement expected by the use of resonant optical feedback, we have simulated numerically the transmission experiments and the results are given in Section V. More specifically, we solve the rate equations to obtain the chirped pulse emitted by the laser, propagate the pulse inside a dispersive fiber, and fiter the pulse at the receiver. The laser perfor-

Manuscript received July 1, 1987; revised September 14, 1987.
The authors are with AT&T Bell Laboratories, Murray Hill, NJ 07974.
IEEE Log Number 8718261.

0018-9197/88/0200-0134$01.00

mance is measured through the dispersion-induced degradation of the eye diagram. We find that the reduced chirp can increase the bit rate by about a factor of two for dispersion-limited optical communication systems.

II. Rate Equations for External-Resonator Lasers

The modulation and noise characteristics of a simple Fabry–Perot laser without external feedback are well described by a set of rate equations given by [11], [12]

$$\dot{\beta} = \tfrac{1}{2}(G - \gamma)(1 - i\alpha)\beta + F_\beta(t) \tag{1}$$

$$\dot{N} = C - S - G|\beta|^2 + F_N(t) \tag{2}$$

where β is the complex optical amplitude, α is the linewidth enhancement factor, γ^{-1} is the photon lifetime, and G is the gain or net rate of stimulated emission. In (2), N is the number of electrons inside the active region, C is the carrier-generation rate, and S represents the rate of carriers lost due to the spontaneous recombination processes. The fluctuations in β and N are accounted for by the Langevin noise sources $F_\beta(t)$ and $F_N(t)$ which are assumed to be delta-correlated in the Markoffian approximation [19].

For an external-resonator laser, the facet reflectivity becomes frequency dependent and (1) must be modified. The modification can be carried out using the Green's function approach [10]. The modification consists of multiplying $\dot{\beta}$ by a complex factor determined by the resonant nature of the external feedback. More specifically, the modified form of (1) is [2]–[4], [12]

$$\left(1 + A - i\frac{B}{\alpha}\right)\dot{\beta} = \frac{(G - \gamma)}{2}(1 - i\alpha)\beta + F_\beta(t) \tag{3}$$

where A and B are frequency-dependent feedback parameters defined by [12]

$$A(\omega) = \frac{1}{\tau_0}\frac{d\phi}{d\omega} \quad \text{and} \quad B(\omega) = \frac{\alpha}{\tau_0}\left|\frac{1}{r}\frac{dr}{d\omega}\right|. \tag{4}$$

In (4) $r = |r(\omega)| \exp[i\phi(\omega)]$ is the effective reflection coefficient of the facet facing the external reflector, and τ_0 is the roundtrip time of the laser cavity. Dividing (3) by $(1 + A - iB/\alpha)$, we see that the external feedback affects not only the gain change but also the spontaneous-emission noise governed by $F_\beta(t)$.

We can separate (3) into two real equations using

$$\beta = \sqrt{I}\exp(-i\Phi), \tag{5}$$

where I has been normalized such that I represents the number of photons in the active cavity. Before separating (3) into its real and imaginary parts, an important point should be considered. The gain change $\Delta G = G - \overline{G}$, where $\overline{G}$ is the steady-state value. However,

$$\Delta G = \Delta G_N + \Delta G_I$$

consists of two contributions ΔG_N and ΔG_I arising from changes in the electron and photon populations, respectively. Only ΔG_N leads to an index change; the index change associated with ΔG_I is negligible. Thus, $\alpha \Delta G$ must be replaced by $\alpha \Delta G_N$ in (3). To obtain the intensity and phase equations, we follow the method of Lax [19]. Details are given in the Appendix. The result is

$$\dot{I} = \frac{[(1 + A + B)\Delta G_N + (1 + A)\Delta G_I]I + R}{(1 + A)^2 + (B/\alpha)^2} + \frac{F_I(t)}{[(1 + A)^2 + (B/\alpha)^2]^{1/2}} \tag{6}$$

$$\dot{\Phi} = \frac{1}{2}\frac{\alpha(1 + A)\Delta G_N - (B/\alpha)(\Delta G_N + \Delta G_I)}{(1 + A)^2 + (B/\alpha)^2} + \frac{F_\phi(t)}{[(1 + A)^2 + (B/\alpha)^2]^{1/2}} \tag{7}$$

where R is the spontaneous emission rate. These equations reduce to those describing a laser without feedback when A and B are set to zero.

As discussed in [12], the parameters A and B have a simple physical interpretation in the gain–frequency diagram showing how the gain and the optical frequency change with the external feedback. Such a gain–frequency diagram is shown in Fig. 1 for the specific case of a high-Q resonator formed by using a resonant optical reflector [18]. The "loss curve" with a central dip shows the frequency dependence of the laser-cavity loss resulting from the resonant optical feedback; the minimum of the loss curve corresponds to the frequency at which the feedback is strongest. The laser, however, does not necessarily operate at the minimum-loss wavelength since is has to satisfy the phase condition that the roundtrip phase change of the field should equal $2\pi N$, where N is an integer. This condition is expressed graphically by the phase curves shown dashed in Fig. 1. The gain and the frequency of the lasing mode at threshold are determined by the crossing of the loss and phase curves.

The frequency dependence of the parameters A and B obtained by using (4) is also shown in Fig. 1. A is related to the slope of the phase curve while B is related to the slope of the loss curve. Clearly, B is large only when the laser operates away from the loss minimum; this situation is sometimes referred to as the detuned-loading configuration [5], [9]. By suitable design optimization, it is possible to choose the operating point corresponding to the frequency at which B is maximum. Fig. 1 also shows $F = 1 + A + B$ since, as shown below, the factor F plays an important role in determining the effect of resonant optical feedback on the laser characteristics.

III. Small-Signal Modulation Response

In this section we consider the small-signal response of an external-resonator laser to a weak modulation of the applied current. The purpose is to study how the relaxa-

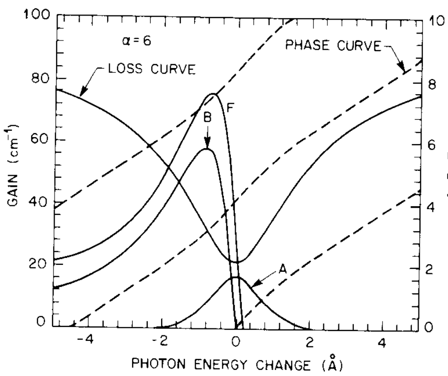

Fig. 1. Frequency dependence of the feedback parameters A and B for a specific external-resonator laser for which the high-Q resonator is formed by using a resonant optical reflector. A is related to the slope of the phase curve while B is related to the slope of the loss curve. The intersection of the loss and phase curves determined the operating point and the values of A and B. The chirp reduction factor $F = 1 + A + B$ is also shown.

tion oscillation characteristics are affected by the resonant optical feedback governed by the parameters A and B. Let $p(t)$, $n(t)$, and $\phi(t)$ represent the deviations in I, N, and Φ from their steady-state values (corresponding to the bias level). We linearize (2), (6), and (7) in p, n, and ϕ. For simplicity, we neglect the effect of the Langevin noise sources on the modulation response. The gain changes ΔG_N and ΔG_I in (6) and (7) are taken to be

$$\Delta G_N = G_N n, \ \Delta G_I = -G_I p \tag{8}$$

where $G_N = \partial G/\partial N$ is the gain coefficient, and ΔG_I governs the nonlinear gain reduction occurring due to spectral hole burning. The linearized rate equations can be written in a form similar to that for solitary lasers [11] and become

$$\dot{n} = -\Gamma_N n - Gp + C_m \tag{9}$$

$$\dot{p} = -\Gamma_I p + \overline{G}_N I n \tag{10}$$

$$\dot{\phi} = \tfrac{1}{2}\overline{\alpha}\,\overline{G}_N n + \tfrac{1}{2}\overline{\beta}\,\overline{G}_I p \tag{11}$$

where C_m is the modulation current and

$$\Gamma_N = dS/dN + G_N I \tag{12}$$

$$\Gamma_I = \overline{R}/I + \overline{G}_I I \tag{13}$$

are the damping coefficients for the electron and photon populations. The parameters affected by the resonant optical feedback are R, α, G_N, and G_I, and the effective values are given by

$$\overline{R} = \frac{R}{(1 + A)^2 + (B/\alpha)^2} \tag{14}$$

$$\overline{\alpha} = \frac{\alpha(1 + A) - B/\alpha}{1 + A + B} \tag{15}$$

$$\overline{G}_N = \frac{1 + A + B}{(1 + A)^2 + (B/\alpha)^2} G_N \tag{16}$$

$$\overline{G}_I = \frac{1 + A}{(1 + A)^2 + (B/\alpha)^2} G_I. \tag{17}$$

The parameter $\overline{\beta}$ in (11) is given by

$$\overline{\beta} = \frac{B}{\alpha(1 + A)}. \tag{18}$$

Since (9) and (10) are formally identical to the case of a solitary laser, one can write the expressions for the frequency and the damping rate of relaxation oscillations directly as [11]

$$\Omega \simeq (G\overline{G}_N I)^{1/2}, \ \Gamma = \tfrac{1}{2}(\Gamma_I + \Gamma_N). \tag{19}$$

Using (12)–(17) and noting that the dominant contribution to Γ comes from the spectral hole-burning term in (13), we obtain

$$\frac{\Omega^2}{\Omega_0^2} = \frac{1 + A + B}{(1 + A)^2 + (B/\alpha)^2} \frac{G}{G_0} \tag{20}$$

$$\frac{\Gamma}{\Gamma_0} \simeq \frac{1 + A}{(1 + A)^2 + (B/\alpha)^2} \tag{21}$$

where Ω_0 and Γ_0 are the frequency and the damping rate of relaxation oscillations for the solitary laser. An expression identical to (20) was obtained by Vahala and Yariv [5]. For simplicity, we assume that $G \simeq G_0$, i.e., the threshold gain or the photon lifetime is not significantly affected by the resonant feedback. Fig. 2 shows the variation of Ω/Ω_0 and Γ/Γ_0 with B for several values of A after choosing $\alpha = 6$. In general, low values of A are desirable for enhancing the relaxation-oscillation frequency; for $A = 0$, an enhancement by nearly a factor of 2 can occur when $B \simeq \alpha$. The damping rate Γ is always reduced, an undesirable feature from the viewpoint of the system applications. Again, low values of A are required to minimize the decrease in Γ. As seen in Fig. 1, low values of A are realized when the laser operates away from the minimum-loss wavelength.

In the absence of parasitics, the small-signal modulation bandwidth ν_m is governed by the relaxation-oscillation frequency ($\nu_m \simeq \Omega/2\pi$). Fig. 2 shows that ν_m can be nearly doubled for external-resonator lasers by an appropriate choice of the parameters A and B [5], [9]. In practice, however, A and B cannot be chosen independently. As seen in Fig. 1, when B takes its maximum value of α, $A \simeq 1$. If we assume that $B = \alpha = 6$ and $A = 1$, Fig. 2 shows an enhancement of about 25 percent for the relaxation-oscillation frequency and a decrease of 60 percent for the damping rate of relaxation oscillations. If B is reduced to about 4 so that $A \simeq 0.5$ in Fig. 1, it is possible to enhance Ω by 50 percent, as also observed experimentally for a specific scheme [9]. However, as shown below, from the point of view of the chirp and linewidth reduction, it is important to maximize the factor $F = 1 + A + B$. For the case of a resonant optical reflector [18] when F is maximum, the ratio $B/A = \alpha$.

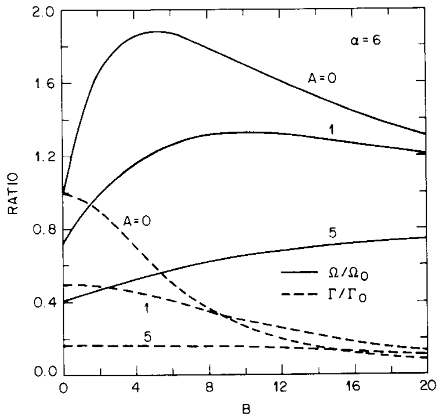

Fig. 2. Variation of the frequency Ω and the damping rate Γ of relaxation oscillations with the feedback parameter B for three values of A. Ω_0 and Γ_0 are the frequency and the damping rate of a solitary laser ($A = 0$, $B = 0$).

The frequency chirp $\delta\nu = \dot{\phi}/2\pi$ under small-signal modulation can be obtained from (11). A relevant measure of chirp [19] is the chirp-to-power ratio (CPR). For the case of sinusoidal modulation at the frequency ω_m, we obtain the following expression of CPR from (9)–(11):

$$\text{CPR} = \frac{\dot{\phi}}{2\pi p} = \frac{1}{2}\left|(\Gamma_I + i\omega_m)\overline{\alpha}/I + \overline{\beta}\,\overline{G}_I\right|. \tag{22}$$

For modulation frequencies such that $\omega_m << \Gamma_I$, the use of (15)–(17) in (22) yields the following result for the low-frequency or adiabatic CPR.

$$\text{CPR} = \frac{1}{2}\frac{\alpha G_I}{1 + A + B}. \tag{23}$$

Thus, the static or low-frequency chirp is reduced by a factor of

$$F = 1 + A + B \tag{24}$$

for external-resonator lasers. As discussed in [12] and in the following section, the same factor F governs the linewidth reduction; more specifically, the linewidth reduces by a factor of F^2.

IV. Phase Noise and Linewidth

In this section we consider the effect of the resonant optical feedback on the phase noise and the linewidth of semiconductor lasers. Both the intensity and phase noises can be studied using the rate equations (2), (6), and (7). In terms of small fluctuations $p(t)$, $n(t)$, and $\phi(t)$ from the steady-state values, these equations become

$$\dot{n} = -\Gamma_N n - Gp + F_N(t) \tag{25}$$

$$\dot{p} = -\Gamma_I p + \overline{G}_N I n + \frac{F_I(t)}{\left[(1 + A)^2 + (B/\alpha)^2\right]^{1/2}} \tag{26}$$

$$\dot{\phi} = \frac{1}{2}\left(\overline{\alpha}\,\overline{G}_N n + \overline{\beta}\,\overline{G}_I p\right) + \frac{F_\phi(t)}{\left[(1 + A)^2 + (B/\alpha)^2\right]^{1/2}} \tag{27}$$

where various quantities are defined by (12)–(18). We neglect the carrier noise $F_N(t)$ in the following discussion since it does not affect the phase noise significantly in the above-threshold regime. The Langevin noise sources are assumed to be delta-correlated, i.e.,

$$\langle F_i(t)\, F_j(t) \rangle = 2D_{ij}\delta(t - t') \tag{28}$$

with $D_{II} = RI$ and $D_{\phi\phi} = R/4I$. The phase noise is governed by the variance [11]

$$\langle \Delta\phi^2(t) \rangle = \frac{1}{\pi}\,\text{Re}\int_{-\infty}^{\infty} \langle |\tilde{\phi}(\omega)|^2 \rangle \left[1 - \exp(i\omega t)\right] d\omega \tag{29}$$

where $\tilde{\phi}(\omega)$ is the Fourier transform of $\phi(t)$. The solution of (25)–(27) in the Fourier domain leads to the following expression.

$$\langle |\tilde{\phi}(\omega)|^2 \rangle = \frac{\overline{R}}{2I}\left(\frac{1}{\omega^2} + \frac{\left|\overline{\alpha}\,\overline{G}_N G - \overline{\beta}\,\overline{G}_I(\Gamma_N + i\omega)\right|^2 I^2}{\omega^2 |\omega - \Omega - i\Gamma|^2\, |\omega + \Omega - i\Gamma|^2}\right). \tag{30}$$

The integration in (29) is performed using the method of contour integration, and the result is

$$\langle \Delta\phi^2(t) \rangle = \frac{\overline{R}}{2I}\left[Ct + \frac{D}{2\Gamma}\,\text{Re}\left(\frac{1 - \exp\left[i(\Omega + i\Gamma)t\right]}{(\Omega + i\Gamma)^3}\right)\right], \tag{31}$$

where

$$C = 1 + \frac{\left(\overline{\alpha}\,\overline{G}_N G - \overline{\beta}\,\overline{G}_I \Gamma_N\right)^2 I^2}{\left(\Omega^2 + \Gamma^2\right)^2} \tag{32}$$

$$D = \left|\overline{\alpha}\,\overline{G}_N G - i\overline{\beta}\,\overline{G}_I\left[\Omega + i(\Gamma - \Gamma_N)\right]\right|^2. \tag{33}$$

Similar to the case of a solitary laser [11], the phase variance is the sum of a linear term and a damped oscillatory term that has its origin in relaxation oscillations. The spectral line shape is obtained by taking the Fourier transform of $\exp[-\langle \Delta\phi^2(t) \rangle]$. In the absence of the oscillatory term, the line shape is Lorentzian with the linewidth (FWHM)

$$\Delta\nu = \frac{\overline{R}C}{4\pi I}. \tag{34}$$

The oscillatory term gives rise to satellites separated from the main peak by multiples of the relaxtion-oscillation frequency Ω. Resonant optical feedback in semiconductor lasers changes the linewidth as well as the location and the relative height of the satellites. In general, the satellites are almost completely suppressed for external-resonator lasers with $B/A = \alpha$.

A simple expression for the linewidth $\Delta\nu$ is obtained if we note that typically $\Omega >> \Gamma$ and $\overline{\beta}\,\overline{G}_I \Gamma_N << \overline{\alpha}\,\overline{G}_N G$ in (32). Using $\Omega^2 = G\overline{G}_N I$, $C \simeq 1 + \overline{\alpha}^2$. If we now use

(14) and (15), for $\bar{R}$ and $\bar{\alpha}$, the linewidth is given by

$$\Delta\nu = \frac{R}{4\pi I}\frac{1+\alpha^2}{(1+A+B)^2} = \frac{\Delta\nu_0}{F^2} \tag{35}$$

where $\Delta\nu_0$ is the linewidth of the solitary laser and $F = 1 + A + B$. Thus, the resonant optical feedback reduces the linewidth by F^2. Since $F \simeq 30$ should be achievable, a reduction by up to three orders of magnitude can be realized ($\Delta\nu \sim 100$ kHz at 1 mW).

For coherent communication systems, phase fluctuations occurring during a single bit generally limit the system performance. Thus, it is not the linewidth but the phase variance $\langle \Delta\phi^2(t) \rangle$, where $t \sim B^{-1}$, that needs to be minimized (the linewidth is a measure of only low-frequency phase fluctuations). Fig. 3 shows the variation of $\langle \Delta\phi^2(t) \rangle$ with time obtained from (31) using typical parameter values. The dashed line shows the linear increase resulting from the first term in (31). The second term in (31) arising from relaxation oscillations enhances the phase noise; its contribution is oscillatory and depends on the specific values of parameters A and B. The two curves in Fig. 3 correspond to $F = 30$ with $B = 0$ and $B/A = \alpha = 6$. Even though the linewidth is reduced by the same amount in both cases, high values of B are desirable to reduce the overall phase noise.

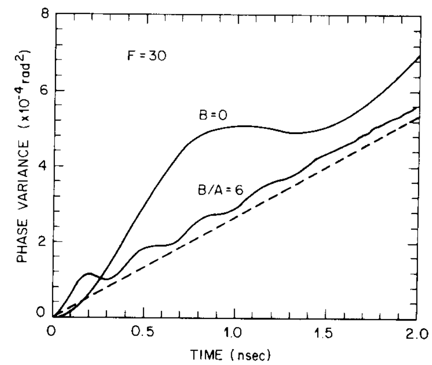

Fig. 3. Time dependence of the phase variance $\langle \Delta\phi^2(t) \rangle$ for two values of B. In each case, the chirp reduction $F = 1 + A + B = 30$. Dashed line shows the linear increase expected from the linewidth consideration alone.

V. Large-Signal Modulation

In Section III we considered the small-signal modulation response and found that the resonant optical feedback reduces the frequency chirp, but at the same time it decreases the damping rate of relaxation oscillations. Although low chirp is certainly beneficial, the reduced damping of relaxation oscillations can lead to degradation of the system performance [21]. In this section, we study the performance of external-resonator lasers under large-signal modulation conditions such as those encountered in a realistic 1.55 μm optical communication system operating at high bit rates ($B_R \sim$ a few gigabits). More specifically, we assume that the laser current is modulated by a random sequence of on–off pulses of duration $T = B_R^{-1}$ in the nonreturn-to-zero (NRZ) format. The resulting sequence of optical pulses is transmitted through a dispersive fiber and filtered at the receiver. To simulate a realistic receiver, we assume that the receiver incorporates a low-pass second-order Butterworth filter with a frequency response [22]

$$H(f) = \frac{f_c^2}{f_c^2 - f^2 + i\sqrt{2}ff_c} \tag{36}$$

where f_c is the cutoff frequency. In our numerical simulations, we choose $f_c = 0.75B_R$. The system performance is measured through the degradation of the "eye diagram" formed by superposition of a large number of received pulses. Our procedure is similar to that employed by Corvini and Koch [23]. However, rather than using a pseudorandom bit sequence, we have considered isolated pulses up to 3 bits long and formed the eye diagram by a superposition of many such pulses. Although this procedure may have excluded some pattern effects, it suffices for the present purpose. The use of a fourth-order Butterworth filter [22] did not alter the conclusions reached here.

The first step in our simulations consists of solving the rate equations (2), (6), and (7) numerically to obtain the pulse shape and the associated transient chirp for a specific sequence of bits. For a current pulse of m bits long, the carrier-generation rate C in (2) is taken to be

$$C = [I_b + I_m f(t)]/q \tag{37}$$

where I_b is the bias current, I_m is the modulation current, and $f(t)$ is given by

$$f(t) = \begin{cases} 1 - \exp(-t/\tau_r) & \text{for } 0 \le t \le \tau \\ \exp(-(t-\tau)/\tau_r] & \text{for } t > \tau \end{cases} \tag{38}$$

where $\tau = mT$ is the duration of the m bit long current pulse and τ_r is the rise time.

Fig. 4 shows the pulse shape and the transient chirp for an external-resonator laser after choosing $B_R = 4$ Gbits/s, $I_b = 1.1I_{th}$, $I_m = I_{th}$, $\tau_r = 0.2T$, and $\tau = T$ (single isolated bit) in (37) and (38). The feedback parameters are chosen to be $A = 1$ and $B = 6$ (see Fig. 1). To clarify the changes associated with the resonant optical feedback, the bottom part of Fig. 4 shows the large-signal modulation behavior for $A = 0$ and $B = 0$, the values corresponding to a solitary single-frequency laser. All other parameters are identical in both cases and were taken corresponding to a typical buried-heterostructure laser with active-region dimensions $0.2 \times 2 \times 250$ μm^3. A parameter that needs special attention is the nonlinear gain parameter ϵ that leads to the power-dependent gain change ΔG_l resulting from phenomena such as spectral hole buring [24]–[27]. It is defined using

$$\Delta G_l = -G\epsilon S \tag{39}$$

where S is the photon density. The parameter ϵ is estimated to be in the range $1\text{-}3 \times 10^{-17}$ cm^3 for 1.55 μm InGaAsP lasers; we used $\epsilon = 2 \times 10^{-17}$ cm^3 in our simulations.

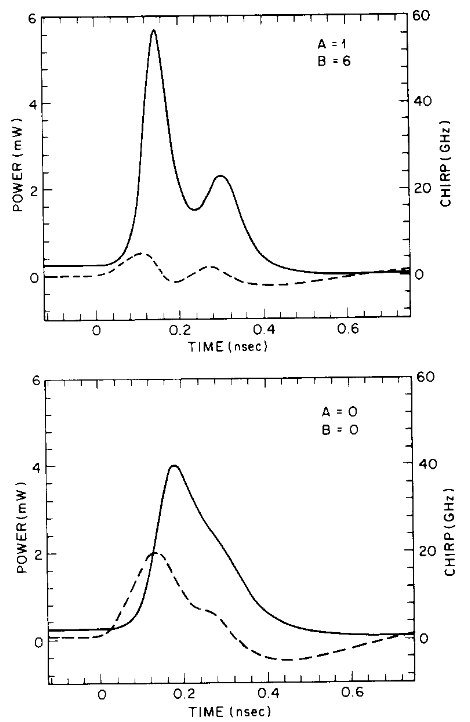

Fig. 4. Pulse shape (solid line) and transient chirp (dashed line) for external resonator (upper figure) and conventional (lower figure) lasers for a 250 ps wide current pulse (B = 4 Gbits/s) with $I_b = 1.1 I_{th}$ and $I_m = I_{th}$.

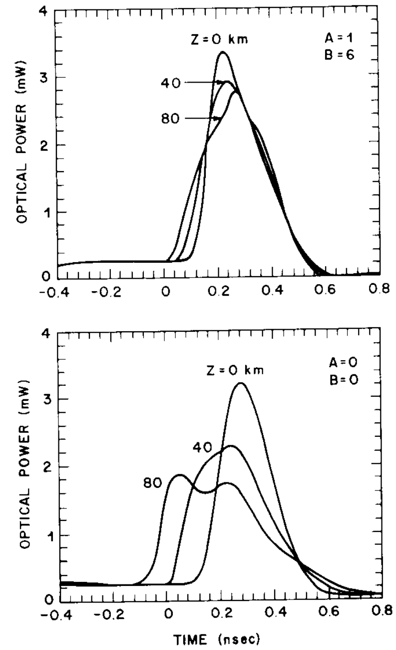

Fig. 5. Comparison of received pulses for three different fiber lengths. The parameter values are identical to those used for Fig. 5.

A comparison of the two cases shown in Fig. 4 reveals that the transient chirp is considerably reduced by the resonant optical feedback but at the same time the pulse shape is distorted because of a decrease in the damping rate of relaxation oscillations. Thus, the qualitative features of the small-signal analysis considered in Section III remain valid even in the large-signal regime. The chirp reduction factor is, however, different than $F = 1 + A + B$ that occurs at low frequencies. In Fig. 4 the chirp is reduced by nearly a factor of 5 if we define the chirp as the maximum frequency shift occurring during the pulse duration. The reduced chirp is beneficial for optical communications systems since it reduces the dispersion-induced broadening of the optical pulse. However, one has to consider simultaneously the effect of pulse distortion resulting from the enhanced relaxation oscillations.

To quantify these effects, the emitted pulse is propagated down the fiber assuming a chromatic dispersion of 16 ps/km/nm. We used the Fourier transform method to propagate the pulse inside the dispersive fiber. The detected pulse was filtered using (36) to simulate a realistic receiver. Fig. 5 shows the received pulse shape for three different fiber lengths corresponding to the emitted laser pulse shown in Fig. 4. As in Fig. 4, an external-resonator laser with $A = 1$ and $B = 6$ is compared to a conventional single-frequency laser for which $A = B = 0$. As expected, the pulse broadens considerably for a conventional laser because of the relatively large chirp (~ 20 GHz), and the broadening is proportional to the fiber length. For an external-resonator laser, by contrast, little broadening occurs because of the reduced chirp. Fig. 5 also shows that the broadening is mainly due to a chirp-induced shift of the leading edge of the pulse. In fact, if we use the simple relation that the shift $\Delta\tau = Dz\Delta\lambda$ with D = 16 ps/km/nm, z = 80 km, and $\Delta\lambda$ = 0.16 nm (corresponding to a frequency chirp of 20 GHz), we obtain $\Delta\tau \simeq 0.2$ ns in agreement with the numerical result shown in the lower part of Fig. 6. We may note that relaxation oscillations occurring in the emitted pulse (Fig. 4) do not appear in the received pulse because of the use of the low-pass Butterworth filter with a bandwidth less than the relaxation-oscillation frequency. The filter, however, distorts the pulse shape to some extent because of the nonlinear phase response associated with (36).

The pulse shapes shown in Fig. 5 correspond to a single isolated bit. To simulate the performance of an actual communication system, all possible bit patterns should be considered. For this purpose, we have numerically generated the "eye diagram" which is commonly used to monitor the system performance. Fig. 6 compares the eye diagrams for an external-resonator laser and a conventional single-frequency laser when a 4 Gbits/s signal is transmitted over 80 km long fiber. All parameters are identical to those used for Figs. 4 and 5. Because of the chirp-induced broadening of the pulse seen in Fig. 5, the eye is almost closed for the conventional laser (indicative of the error floor in the bit-error-rate curves). By contrast, the external-resonator laser has an open eye because of the reduced chirp. It is important to stress that the performance of the conventional laser cannot be improved simply by increasing the laser power.

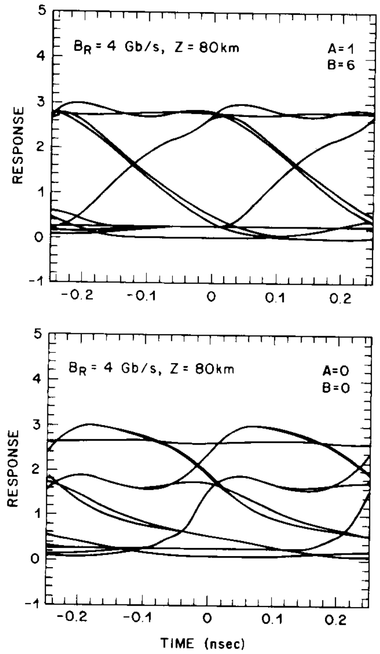

Fig. 6. Comparison of eye diagrams for an external-resonator laser and a conventional laser for data transmission over 80 km of fiber at the bit rate of 4 Gbits/s. The bias level $I_b = 1.1 I_{th}$ and the modulation current $I_m = I_{th}$.

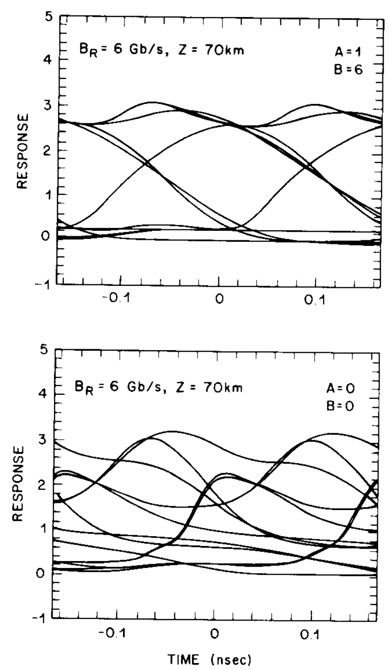

Fig. 7. Same as Fig. 6 except that the bit rate B = 6 Gbits/s and the fiber length z = 70 km.

It is possible to operate external-resonator lasers at even higher bit rates. As an example, Fig. 7 compares the eye diagram for the external-resonator and conventional lasers for B_R = 6 Gbits/s and z = 70 km. The other parameters

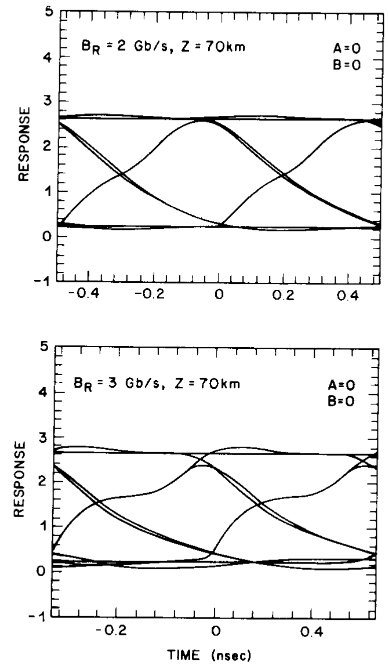

Fig. 8. Eye diagrams for a conventional laser under conditions identical to those used in Fig. 7 but at lower bit rates of 2 and 3 Gbits/s.

are identical to those used for Fig. 6. Again, the eye is nearly open for the external-resonator laser while it is completely closed for the conventional laser. We can estimate the improvement in the system performance realized by the use of resonant optical feedback by reducing the bit rate for the conventional laser and noting the bit rate at which the eye is open enough to be acceptable in the system experiments. Fig. 8 shows the eye diagrams for the conventional laser under identical operating conditions for a bit rate of 2 and 3 Gbits/s. The eye is wide open at 2 Gbits/s but starts to deteriorate at higher bit rates. Although there is a partial closing of the eye (indicative of the chirp-induced power penalty), the laser can be used at 3 Gbits/s. We have performed similar simulations for different operating conditions. In all cases we have found that the lower chirp for an external resonator can increase the bit-rate-distance product by about a factor of two.

VI. Conclusion

The coupling of a multimode semiconductor laser to a passive high-Q resonator not only provides mode selectivity but also leads to a reduction in the frequency chirp and the linewidth. Such low-chirp, narrow-linewidth, single-frequency lasers may prove useful for high-speed 1.55 μm optical communication systems whether a direct-detection or coherent-detection scheme is employed. In the case of coherent detection, the linewidth requirements are easily met since the linewidth can be reduced by up to three orders of magnitude by suitably optimizing the feedback parameters A and B. The phase-noise analysis presented here shows that relaxation oscillations do not significantly increase the phase variance deduced from the consideration of the linewidth alone. In the case of direct detection, it is the low chirp that helps to improve the system performance. Our numerical simulations indicate that the bit rate can be increased by about a factor of two by the use of external-resonator lasers for nearly error-free transmission along dispersive fibers.

Appendix
Derivation of (6) and (7)

The complex field β satisfies (3) or equivalently

$$\frac{d\beta}{dt} = \frac{\Delta G(1 - i\alpha)}{2D}\beta + \frac{F_\beta(t)}{D} \tag{A1}$$

where $\Delta G = G - \gamma$ and

$$D = 1 + A - i\beta/\alpha. \tag{A2}$$

The Langevin noise source is taken to be delta-correlated in the Markoffian approximation, i.e.,

$$\langle F_\beta(t)\, F_\beta^*(s) = R\delta(t - s) \tag{A3}$$

where R is the rate of spontaneous emission.

Consider the Langevin equation for the intensity $I = |\beta|^2$. Using (A1), one readily obtains

$$\frac{dI}{dt} = \beta^* \frac{d\beta}{dt} + \beta \frac{d\beta^*}{dt} = \frac{QI}{|D|^2} + \left(\frac{\beta^* F_\beta}{D} + \frac{\beta F_\beta^*}{D^*}\right) \tag{A4}$$

where

$$Q = \text{Re}\left[\Delta G(1 - i\alpha) D^*\right] = (1 + A + B)\Delta G_N + (1 + A)\Delta G_I. \tag{A5}$$

In obtaining (A5), we used $\Delta G = \Delta G_N + \Delta G_I$ and replaced $\alpha \Delta G$ by $\alpha \Delta G_N$ as discussed in Section II. Since $\langle \beta F_\beta^* \rangle \neq 0$, it is necessary to evaluate this average. Using the method of Lax [19], we write $\beta(t) = \beta(t_c) + \int_{t_c}^{t} \dot\beta(s)\, ds$ so that

$$\langle \beta(t)\, F_\beta^*(t) \rangle = \int_{t_c}^{t} \langle \dot\beta(s)\, F_\beta^*(t) \rangle\, ds, \tag{A6}$$

where $\dot\beta = d\beta/dt$ and t_c is slightly smaller than t. We substitute (A1) in (A6) and use (A3) to perform the average. The result is

$$\langle \beta(t)\, F_\beta^*(t) \rangle = \frac{R}{2D} \tag{A7}$$

where the factor of 1/2 results from integration over only half of the area covered by the delta function. Using (A7) in (A4), we obtain

$$\frac{dI}{dt} = \frac{QI + R}{|D|^2} + \frac{F_I(t)}{|D|}, \tag{A8}$$

where $F_I(t)$ has zero mean and is delta-correlated according to

$$\langle F_I(t)\, F_I(s) \rangle = 2D_{II}\delta(t - s) \tag{A9}$$

with $D_{II} = RI$.

Equation (6) of the text is obtained from (A8) by substituting D and Q from (A2) and (A5). An essentially identical procedure can be used to obtain (7) for the optical phase. The Langevin noise $F_\phi(t)$ is again found to be delta-correlated with the diffusion coefficient $D_{\phi\phi} = R/4I$.

Acknowledgment

The authors are thankful to N. A. Olsson, R. F. Kazarinov, T. L. Koch, and P. J. Corvini for stimulating discussions.

References

[1] R. Wyatt and W. J. Devlin, "10 kHz linewidth 1.5 μm InGaAsP external cavity laser with 55 nm tuning range," *Electron. Lett.*, vol. 19, pp. 110–112, 1983.

[2] E. Patzak, A. Sugimura, S. Saito, T. Mukai, and H. Olesen, "Semiconductor laser linewidth in optical feedback configurations," *Electron. Lett.*, vol. 19, pp. 1026–1027, 1983.

[3] G. P. Agrawal, "Line narrowing in a single-mode injection laser due to external optical feedback," *IEEE J. Quantum Electron.* vol. QE-20, pp. 468–471, May 1984.

[4] R. A. Suris and A. A. Tager, "Anomalous contraction of the emission line of a composite resonator semiconductor laser," *Sov. Tech. Phys. Lett.*, vol. 12, pp. 320–321, 1986.

[5] K. Vahala and A. Yariv, "Detuned loading in coupled-cavity semiconductor lasers—Effect on quantum noise and dynamics," *Appl. Phys. Lett.*, vol. 45, pp. 501–503, 1984.

[6] T. Fujita, J. Ohya, S. Ishizuka, K. Fujito, and H. Sato, "Oscillation frequency shift suppression of semiconductor lasers coupled to external cavity," *Electron. Lett.*, vol. 20, pp. 416–417, 1984.

[7] G. P. Agrawal, "Generalized rate equations and modulation characteristics of external-cavity semiconductor lasers," *J. Appl. Phys.*, vol. 56, pp. 3110–3116, 1984.

[8] E. Patzak, P. Meissner, and D. Yevick, "An analysis of the linewidth and spectral behavior of DBR lasers," *IEEE J. Quantum Electron.*, vol. QE-21, pp. 1318–1325, Sept. 1985.

[9] K. Vahala, J. Paslaski, and A. Yariv, "Observation of modulation speed enhancement, frequency modulation suppression, and phase noise reduction by detuned loading in a coupled-cavity semiconductor laser," *Appl. Phys. Lett.*, vol. 46, pp. 1025–1027, 1985.

[10] C. H. Henry, "Theory of spontaneous emission noise in open resonators and its applications to lasers and optical amplifiers," *J. Lightwave Technol.*, vol. LT-4, pp. 288–297, Mar. 1986.

[11] ——, "Phase noise in semiconductor lasers," *J. Lightwave Technol.*, vol. LT-4, pp. 298–311, Mar. 1986.

[12] R. F. Kazarinov and C. H. Henry, "The relation of line narrowing and chirp reduction resulting from the coupling of semiconductor laser to a passive resonator," *IEEE J. Quantum Electron.*, vol. QE-23, pp. 1401–1409, Sept. 1987.

[13] N. A. Olsson, C. H. Henry, R. F. Kazarinov, H. J. Lee, and K. J. Orlowsky, "Performance characteristics of a 1.5 μm single-frequency semiconductor laser with an external waveguide Bragg reflector," *IEEE J. Quantum Electron.*, this issue, pp. 143–147.

[14] N. A. Olsson, C. H. Henry, R. F. Kazarinov, H. J. Lee, and B. H. Johnson, "Relation between chirp and linewidth reduction in external Bragg reflector semiconductor lasers," *Appl. Phys. Lett.*, vol. 51, pp. 92–93, 1987.

[15] J. M. Hammer, C. C. Neil, N. W. Carlson, M. T. Duffy, and J. M. Shaw, "Single-wavelength operation of the hybrid-external Bragg-reflector waveguide laser under dynamic conditions," *Appl. Phys. Lett.*, vol. 47, pp. 183–185, 1985.

[16] E. Brinkmeyer, W. Brennecke, M. Zürn, and R. Ulrich, "Fiber Bragg reflector for mode selection and line narrowing of injection lasers," *Electron. Lett.*, vol. 22, pp. 134-135, 1986.

[17] R. A. Linke, "Modulation induced transient chirping in single-frequency lasers," *IEEE J. Quantum Electron.*, vol. QE-21, pp. 593-597, June 1985.

[18] R. F. Kazarinov, C. H. Henry, and N. A. Olsson, "Narrow-band resonant optical reflectors and resonant optical transformers for laser stabilization and wavelength division multiplexing," *IEEE J. Quantum Electron.*, vol. QE-23, pp. 1419-1425, Sept. 1987.

[19] M. Lax, "Classical noise IV. Langevin methods," *Rev. Mod. Phys.*, vol. 38, pp. 541-566, 1966.

[20] T. L. Koch and J. E. Bowers, "Nature of wavelength chirping in directly modulated semiconductor lasers," *Electron. Lett.*, vol. 20, pp. 1038-1039, 1984.

[21] G. P. Agrawal and T. M. Shen, "Importance of rapid damping of relaxation oscillations for high-performance optical communication systems," *Electron. Lett.*, vol. 22, pp. 1087-1088, 1986.

[22] G. S. Moschytz and P. Horn, *Active Filter Design Handbook.* New York: Wiley, 1981, p. 134.

[23] P. J. Corvini and T. L. Koch, "Computer simulation of high-bit-rate optical fiber transmission using single-frequency lasers," *J. Lightwave Technol.*, vol. LT-5, pp. 1591-1595, Nov. 1987.

[24] R. S. Tucker, "High-speed modulation of semiconductor lasers," *J. Lightwave Technol.*, vol. LT-3, pp. 1180-1192, Dec. 1985.

[25] R. Olshansky, D. M. Fye, J. Manning, and C. B. Su, "Effect of nonlinear gain on the bandwidth of semiconductor lasers," *Electron. Lett.*, vol. 21, pp. 721-722, 1983.

[26] T. L. Koch and R. Linke, "Effect of nonlinear gain reduction on semiconductor laser wavelength chirping," *Appl. Phys. Lett.*, vol. 48, pp. 613-615, 1986.

[27] G. P. Agrawal, "Gain nonlinearities in semiconductor lasers: Theory and application to distributed feedback lasers," *IEEE J. Quantum Electron.*, vol. QE-23, pp. 860-868, June 1987.

Govind P. Agrawal (M'83-SM'86) was born in Kashipur, India, on July 24, 1951. He received the M.S. and Ph.D. degrees in physics from the Indian Institute of Technology, New Delhi, in 1971 and 1974, respectively.

After spending several years at the Ecole Polytechnique, France; the City University of New York, New York; and Quantel, France, he joined AT&T Bell Laboratories, Murray Hill, NJ, in 1982 as a member of the technical staff. His research interests have been in the fields of quantum electronics, nonlinear optics, and laser physics. He is an author or coauthor of more than 100 research papers and a book entitled *Long-Wavelength Semiconductor Lasers*. Currently he is engaged in the research and development of semiconductor lasers.

Dr. Agrawal is a member of the American Physical Society and a Fellow of the Optical Society of America.

Charles H. Henry was born in Chicago, IL, in 1937. He received the M.S. degree in physics from The University of Chicago, Chicago, IL, in 1959 and the Ph.D. degree in physics from the University of Illinois, Urbana, in 1965.

Since 1965 he has been a member of the staff in the Semiconductor Electronics Research Department, AT&T Bell Laboratories, Murray Hill, NJ. From 1971 to 1976 he served as head of this department. His research is primarily concerned with the physics associated with light-emitting device technology.

Dr. Henry is a Fellow of the American Physical Society and of the American Association for the Advancement of Science.