# Gain Nonlinearities in Semiconductor Lasers: Theory and Application to Distributed Feedback Lasers

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*Abstract*—**The gain spectrum in semiconductor lasers is affected by the intensity-dependent nonlinear effects taking place due to a finite intraband relaxation time of charge carriers. We obtain an analytic expression for the nonlinear gain in multimode semiconductor lasers using the density-matrix formalism. In general, the nonlinear gain is found to consist of the symmetric and asymmetric components. The asymmetry does not have its origin in the carrier-induced index change, but is related to details of the gain spectrum. The general expression for the nonlinear gain is used to discuss the range of single-longitudinal-mode operation of distributed feedback lasers. It is also used to obtain an analytic expression for the self-saturation coefficient and to compare the predicted value to the experimental value for both GaAs and InGaAsP lasers. The agreement between the theoretical and the experimental values supports the hypothesis that spectral hole burning is the dominant mechanism for the gain nonlinearities in semiconductor lasers.**

## I. Introduction

THE theory of semiconductor lasers has evolved somewhat differently from that of gas and solid-state lasers. Whereas in the semi-classical laser theory [1], [2] developed originally for gas lasers, the nonlinear contributions to the induced polarization (and hence to the optical gain) were included using the density-matrix formalism, such nonlinear effects, with few exceptions [3]–[6], have generally been ignored for semiconductor lasers. It has, however, been realized in recent years that gain nonlinearities play an important role in determining the dynamic response and modulation performance of semiconductor lasers [7]–[13]. These nonlinear-gain effects have been phenomenologically included through a power-dependent gain-suppression term in the single-mode rate equations [11]–[13]. Recently, attention has been paid to include the effects of nonlinear gain in multimode rate equations [10], [14], [15] so that cross saturation of the gain is also accounted for. Here it has been pointed out that the cross-saturation term is generally asymmetric [16], [17]. Indeed, the asymmetric component of the nonlinear gain was invoked to explain the asymmetric mode spectra and the shift of the dominant mode towards longer wavelengths occurring with an increase in the output power [10]. The form of the nonlinear gain used in previous work [10], [14], [15] is, however, based on an intuitive generalization [17] of the model used by Bogatov *et al.* [16] and does not appear to have a firm theoretical basis. As a result, the resulting expression for the nonlinear gain contains parameters whose value and physical interpretation are uncertain.

The objective of this paper is to obtain an analytic expression for the nonlinear gain in multimode semiconductor lasers using the density-matrix formalism. Although the density-matrix formalism has previously been used to obtain the nonlinear gain [4]–[6], the asymmetric nature of mode interaction attracted little attention. In their three-mode analysis, Kazarinov *et al.* [5] found that the asymmetric component of the nonlinear gain was essential to explain the experimentally observed gain spectra. We generalize their analysis to the multimode case and discuss the conditions under which the expression for the nonlinear gain can be reduced to the phenomenological form used in earlier work [10], [14]. We find that, in contrast to the model of Bogatov *et al.* [16], the nonlinear-gain asymmetry does not have its origin in the carrier-induced index reduction (governed by the linewidth enhancement factor), but is related to the slope of the gain profile at the frequencies of various longitudinal modes. As an application of the general expression for the nonlinear gain, we apply it to discuss the range of single-longitudinal-mode operation of distributed feedback (DFB) semiconductor lasers using a two-mode model. The results show that both main and side modes can oscillate simultaneously when the Bragg wavelength deviates considerably from the gain peak. We also use our theory to obtain an analytic expression for the self-saturation coefficient; the predicted values are in agreement with the experimental values for both GaAs and InGaAsP lasers.

## II. Density-Matrix Formalism

We use the density-matrix formalism [1], [2] to obtain the linear and nonlinear contributions to the optical gain. This formalism has been used previously [3]–[6] for semiconductor lasers. However, except for the work of Kazarinov *et al.* [5], the asymmetric nature of nonlinear gain attracted little attention. We generalize the analysis of [5] to include an arbitrary number of longitudinal modes and pay particular attention to the asymmetric nature of the nonlinear interaction among longitudinal modes. Within the framework of the density-matrix formalism, the nonlinear gain has its origin in spectral hole burning, a phenomenon that leads to inhomogeneous saturation of the gain spectrum in semiconductor lasers [18], [19], similar

Manuscript received October 15, 1986; revised January 18, 1987.
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IEEE Log Number 8714253.

to the case of gas lasers [2]. Physically, the gain at the frequency of an oscillating mode is slightly reduced because of the finite intraband relaxation time of charge carriers. The amount of reduction depends not only on the power of that mode (self saturation), but also on the power of neighboring modes (cross saturation).

In the density-matrix approach for semiconductor lasers [3]–[6], each conduction-band state $|1>$ and the corresponding valence-band state $|2>$ participating in the band-to-band transitions are modeled as a two-level system with analogous density-matrix equations of the form [2]

$$\frac{d}{dt}\rho_{11} + \gamma_c(\rho_{11} - \bar{\rho}_{11}) = \frac{\mu}{i\hbar}(\rho_{12} - \rho_{21})E(r, t) \qquad (1)$$

$$\frac{d}{dt}\rho_{22} + \gamma_v(\rho_{22} - \bar{\rho}_{22}) = -\frac{\mu}{i\hbar}(\rho_{12} - \rho_{21})E(r, t) \qquad (2)$$

$$\frac{d}{dt}\rho_{12} + (\gamma + i\omega_T)\rho_{12} = \frac{\mu}{i\hbar}(\rho_{11} - \rho_{22})E(r, t) \qquad (3)$$

where $\gamma_c$ and $\gamma_v$ are the intraband energy relaxation rates for the conduction and valence bands, respectively, $\gamma$ is the polarization relaxation rate, $\omega_T$ is the transition frequency, and $\mu$ is the transition dipole moment. The dipole-moment operator is assumed to have only off-diagonal matrix elements which are further taken to be real without any loss of generality ($\mu_{12} = \mu_{21} = \mu$). $\bar{\rho}_{11}$ and $\bar{\rho}_{22}$ are the occupation probabilities of electrons and holes in thermal equilibrium and are determined by the quasi-Fermi levels of the conduction and valence bands, respectively. Finally, $E(r, t)$ is the optical field given by

$$E(r, t) = \sum_j E(\omega_j)\exp(-i\omega_j t)\,F_j(r) + \text{c.c.} \qquad (4)$$

To separate the linear and nonlinear contributions in the susceptibility, the solution of (1)–(3) is taken to be of the form

$$\rho_{11}(t) = \bar{\rho}_{11} + \Delta\rho_{11}(t) \qquad (7)$$

$$\rho_{22}(t) = \bar{\rho}_{22} + \Delta\rho_{22}(t) \qquad (8)$$

$$\rho_{12}(t) = \sum_j [\bar{\rho}_{12}(\omega_j) + \Delta\rho_{12}(\omega_j)]\exp(-i\omega_j t). \qquad (9)$$

By using (7)–(9) in (3), we obtain the formal solution

$$\bar{\rho}_{12}(\omega_j) = \frac{\mu(\bar{\rho}_{11} - \bar{\rho}_{22})\,E(\omega_j)\,F_j(r)}{\hbar(\omega_j - \omega_T + i\gamma)} \qquad (10)$$

$$\Delta\rho_{12}(\omega_j) = \frac{\mu[(\Delta\rho_{11} - \Delta\rho_{22})\,E(t)\exp(i\omega_j t)]'}{\hbar(\omega_j - \omega_T + i\gamma)} \qquad (11)$$

where the prime denotes the time-independent part of the bracketed term. To obtain $\Delta\rho_{11}(t)$ and $\Delta\rho_{22}(t)$, we use third-order perturbation theory [2] and assume that $\rho_{12}$ in (1) can be replaced by $\bar{\rho}_{12}$. Equation (1) has a general solution of the form

$$\Delta\rho_{11}(t) = \Delta\rho_{11}(0) + \sum_{m\neq n}\sum \Delta\rho_{11}(\omega_m - \omega_n) \cdot \exp[-i(\omega_m - \omega_n)t]. \qquad (12)$$

The first term represents the static change in the electron occupation probability and causes static hole burning. The second term arises from the beating of two longitudinal modes that modulates the occupation probability at the beat frequency $\Omega_{mn} = \omega_m - \omega_n$. In the context of semiclassical laser theory, these dynamic variations are often referred to as population pulsations [2]. By substituting (7) and (12) in (1), we obtain

$$\Delta\rho_{11}(0) = \frac{\mu}{i\hbar\gamma_c}\sum_j [\bar{\rho}_{12}(\omega_j)\,E^*(\omega_j)\,F_j^*(r) - \text{c.c.}] \qquad (13)$$

$$\Delta\rho_{11}(\omega_m - \omega_n) = \frac{\mu[\bar{\rho}_{12}(\omega_m)\,E^*(\omega_n)\,F_n^*(r) - \bar{\rho}_{12}^*(\omega_n)\,E(\omega_m)\,F_m(r)]}{\hbar[\omega_m - \omega_n + i\gamma_c]}. \qquad (14)$$

where sum extends over all longitudinal modes and $\omega_j$ are the corresponding frequencies. The spatial structure of the mode is accounted for through $F_j(r)$. The induced polarization is calculated by summing over all possible band-to-band transitions, i.e.,

$$P(r, t) = \sum_{\omega_T}\mu(\rho_{12} + \rho_{21}) = \int \mu D(\omega_T)(\rho_{12} + \rho_{21})\,d\omega_T \qquad (5)$$

where $D(\omega_T)$ is the density of states per unit volume. The susceptibility $\chi(\omega_j)$ of the medium for various modes is then obtained by writing

$$P(r, t) = \epsilon_0 \sum_j \chi(\omega_j)\,E(\omega_j) \cdot \exp(-i\omega_j t)\,F_j(r) + \text{c.c.} \qquad (6)$$

where $\epsilon_0$ is the vacuum permittivity.

The expression for $\Delta\rho_{22}(t)$ can be obtained from (12)–(14) by changing $\mu$ to $-\mu$ and $\gamma_c$ to $\gamma_v$.

Equations (7)–(14) complete the formal solution of the density-matrix equations. The susceptibility is obtained by using (9) in (5) and then expressing the result in the form of (6). We find that the susceptibility consists of several contributions which can be identified as [5]

$$\chi(\omega_j) = \chi_L(\omega_j) + \chi_1(\omega_j) + \chi_2(\omega_j) + \chi_{\text{FWM}}(\omega_j) \qquad (15)$$

where

$$\chi_L(\omega_j) = C_j \int \frac{f(\omega_T)\,d\omega_T}{\omega_j - \omega_T + i\gamma} \qquad (16)$$

is the linear contribution with $C_j = \langle |F_j(r)|^2 \rangle$ and $f(\omega_T)$ defined as

$$f(\omega_T) = \mu^2(\bar{\rho}_{11} - \bar{\rho}_{22})\,D(\omega_T)/(\epsilon_0\hbar).$$

The other three terms represent the nonlinear contributions to the susceptibility. Their explicit expressions are

$$\chi_1(\omega_j) = \sum_k C_{jk} \int \frac{d\omega_T f(\omega_T)\gamma}{\omega_j - \omega_T + i\gamma} \left( \frac{1}{\omega_k - \omega_T + i\gamma} - \frac{1}{\omega_k - \omega_T - i\gamma} \right) \frac{|E(\omega_k)|^2}{iI_s} \tag{17}$$

$$\chi_2(\omega_j) = \sum_{j \neq k} C_{jk} \int \frac{d\omega_T f(\omega_T)\gamma}{\omega_j - \omega_T + i\gamma} \left( \frac{1}{\omega_j - \omega_T + i\gamma} - \frac{1}{\omega_k - \omega_T - i\gamma} \right) \times \left( \frac{\overline{\gamma}}{\omega_j - \omega_k + i\gamma_c} + \frac{\overline{\gamma}}{\omega_j - \omega_k + i\gamma_v} \right) \frac{|E(\omega_k)|^2}{I_s} \tag{18}$$

$$\chi_{\mathrm{FWM}}(\omega_j) = \sum_{j \neq k} C'_{jk} \int \frac{d\omega_T f(\omega_T)\gamma}{\omega_j - \omega_T + i\gamma} \left( \frac{1}{\omega_k - \omega_T + i\gamma} - \frac{1}{\omega_m - \omega_T - i\gamma} \right) \times \left( \frac{\overline{\gamma}}{\omega_k - \omega_m + i\gamma_c} + \frac{\overline{\gamma}}{\omega_k - \omega_m + i\gamma_v} \right) \frac{E^*(\omega_m)}{E(\omega_j)} \frac{E^2(\omega_k)}{I_s} \tag{19}$$

where $\overline{\gamma} = \gamma_c \gamma_v / (\gamma_c + \gamma_v)$. The saturation intensity

$$I_s = \hbar^2 \overline{\gamma}_c \gamma / \mu^2 \tag{20}$$

is generally a function of $\omega_T$ because of its dependence on $\mu$. The overlap factor $C_{jk} = \langle \, | F_j(r) \, F_k(r) |^2 \rangle$ arises from the spatial structure of the optical mode: angular brackets denote averaging over the active volume. In (19), the index $m$ is chosen such that $2\omega_k - \omega_m = \omega_j$. Further, the overlap factor $C'_{jk} = \langle F_j^*(r) \, F_m^*(r) \, F_k^2(r) \rangle$. Expressions (17)–(19) are generalizations of the corresponding results of [5] to the multimode case. In physical terms, $\chi_1$ is due to static hole burning, $\chi_2$ is due to population pulsations, and $\chi_{\mathrm{FWM}}$ is due to four-wave mixing occurring in the presence of population pulsations. Note that the $\chi_{\mathrm{FWM}}$ term can lead to efficient nondegenerate four-wave mixing [20] even when pump and probe frequencies differ by more than 100 GHz.

## III. Nonlinear Gain

The susceptibility $\chi(\omega_j)$ given by (15) can be used to obtain the linear and nonlinear contributions to the optical gain at the frequency $\omega_j$. However, the contribution of the four-wave mixing term considerably complicates the nonlinear-gain analysis. The reason is that $\chi_{\mathrm{FWM}}$ couples the amplitudes of the neighboring modes. As a result, it is not possible to obtain the effective nonlinear gain for a particular mode without solving the coupled set of amplitude equations. Kazarinov *et al.* [5] have solved the coupled-mode problem for the specific case of a three-mode laser. They found that the contributions of $\chi_2$ and $\chi_{\mathrm{FWM}}$ to the nonlinear gain cancel at the line center while the cancellation is incomplete for modes away from the gain peak. A further consideration occurs for single-frequency lasers such as DFB lasers where the main mode is not located at the gain peak. Since the modes of such a laser are not necessarily equispaced, the four-wave mixing contribution may vanish completely due to phase mismatch. To account for these features in a simple manner, we define the effective nonlinear gain as

$$g_{\mathrm{NL}}(\omega) = -\frac{\omega}{nc} \operatorname{Im} \left[ \chi_1(\omega) + b(\omega) \, \chi_2(\omega) \right] \tag{21}$$

where $n$ is the refractive index and Im stands for the imaginary part of the bracketed expression. The phenomenological parameter $b$ controls the net contribution to the nonlinear gain resulting from intermode beating (population pulsation). In general, $0 \leq b \leq 1$. We expect $b$ to be small for Fabry–Perot lasers, while it approaches 1 for DFB lasers. If we write $g_{\mathrm{NL}}(\omega_j) = \Sigma_k \, \alpha^{(3)}(\omega_j, \omega_k) \, |E_k|^2$, the resulting expression for $\alpha^{(3)}(\omega_j, \omega_k)$ reduces to that obtained in [4] if we set $b = 1$ (no four-wave mixing). The nonlinear gain $g_{\mathrm{NL}}(\omega_j)$ can be evaluated numerically [4], [6] and depends, in general, on the band-structure details as both the density of states $D$ and the dipole moment $\mu$ vary with the transition frequency $\omega_T$. Note also that the integrations in (17)–(19) should be made in the momentum space in order to take into account the nonparabolicity of the conduction band [6]. Since our purpose is to study the qualitative dependence of the nonlinear gain on various physical parameters, we evaluate the integrals approximately using the method of [5].

The approximation is based on the observation that the main contribution to the integral in (17) and (18) comes from a region $|\omega_T - \omega_k| \leq \gamma$. We assume that $f(\omega_T)$ varies slowly in this essential region of integration and can be approximated by

$$f(\omega_T) \simeq f(\omega_k) + f'(\omega_k)(\omega_T - \omega_k). \tag{22}$$

This allows us to evaluate the integrals using the method of contour integration. By closing the contour in the lower half complex $\omega_T$ plane, we find that a single pole at $\omega_T = \omega_k - i\gamma$ contributes to the integral. The result is

$$\chi_1(\omega_j) = i\pi \sum_k C_{jk} \frac{f(\omega_k) - i\gamma f'(\omega_k)}{1 - i\Omega_{jk}/2\gamma} \frac{|E(\omega_k)|^2}{I_s} \tag{23}$$

$$\chi_2(\omega_j) = i\pi \sum_{k \neq j} C_{jk} \frac{f(\omega_k) - i\gamma f'(\omega_k)}{1 - i\Omega_{jk}/2\gamma} \cdot \left( \frac{\overline{\gamma}}{\gamma_c - i\Omega_{jk}} + \frac{\overline{\gamma}}{\gamma_v - i\Omega_{jk}} \right) \frac{|E(\omega_k)|^2}{I_s} \tag{24}$$

where

$$\Omega_{jk} = \omega_j - \omega_k \tag{25}$$

is the beat frequency. We have assumed that the dipole moment $\mu$ is nearly constant over the width of the longitudinal-mode spectrum so that the saturation intensity $I_s$ is the same for all modes. It can be shown using (16) that $f(\omega_k)$ is approximately proportional to the linear gain $g_L(\omega_k)$, i.e.,

$$f(\omega_k) \simeq \frac{nc}{\omega\pi C_k} g_L(\omega_k). \tag{26}$$

Using (23)–(26) in (21), we finally obtain

$$\begin{aligned} g_{\mathrm{NL}}(\omega_j) \simeq & -\sum_k \frac{C_{jk}}{C_k} \frac{g_L(\omega_k)(1 + \alpha_k \Omega_{jk}\tau_{in}/2)}{[1 + (\Omega_{jk}\tau_{in}/2)^2]} \frac{|E(\omega_k)|^2}{I_s} \\ & - b_j \sum_{k \neq j} \frac{C_{jk}}{C_k} g_L(\omega_k) \operatorname{Re}\left[\frac{(1 - i\alpha_k)}{(1 - i\Omega_{jk}\tau_{in}/2)}\right. \\ & \cdot \left.\left(\frac{\tau_c}{\tau_c + \tau_v}\frac{1}{1 - i\Omega_{jk}\tau_c} + \frac{\tau_v}{\tau_c + \tau_v}\frac{1}{1 - i\Omega_{jk}\tau_v}\right)\right] \\ & \cdot \frac{|E(\omega_k)|^2}{I_s} \end{aligned} \tag{27}$$

where $b_j = b(\omega_j)$ is assumed to be real,

$$\tau_v = 1/\gamma_v, \; \tau_c = 1/\gamma_c, \; \tau_{in} = 1/\gamma \tag{28}$$

are the intraband relaxation times, and

$$\alpha_k = \frac{\gamma f'(\omega_k)}{f(\omega_k)} = \frac{1}{\tau_{in}} \frac{g'(\omega_k)}{g(\omega_k)} \tag{29}$$

is a dimensionless parameter related to the slope of the gain curve at $\omega_k$. Since $\alpha_k$ is nonzero for modes oscillating away from the gain peak, the spectrum of nonlinear gain $g_{\mathrm{NL}}$ in general has an asymmetric component. As is evident from (27), the form of nonlinear gain is more complicated than assumed previously [10], [14], [15] based on a phenomenological approach [17]. In the next section, we consider simplification of the general expression (27) and discuss the conditions under which the phenomenological approach is valid.

## IV. Comparison to Previous Work

The asymmetry in the longitudinal-mode interaction was first studied by Bogatov *et al.* [16]. Their treatment is applicable when the beat frequency $\Omega_{ij}$ is comparable to the spontaneous recombination rate ($\sim 10^9$ s$^{-1}$) of electrons. For a semiconductor laser of typical length 250 $\mu$m, the beat frequency between the two neighboring modes exceeds $10^{11}$ Hz. The carrier density in the active region cannot respond at such high frequencies. Noting that the intraband relaxation time of electrons is typically $< 1$ ps, Ishikawa *et al.* [17] suggested that results of [16] can be used if the spontaneous carrier lifetime is replaced by the intraband relaxation time. Based on this suggestion, Manning *et al.* [10] used the following expression for the nonlinear gain (in our notation):

$$\bar{g}_{\mathrm{NL}}(\omega_j) = -B \sum_k \frac{1 + \alpha\,\Omega_{jk}\tau}{1 + \Omega_{jk}^2\tau^2} |E(\omega_k)|^2 \tag{30}$$

where $B$ is the strength parameter. The parameters $\alpha$ and $\tau$ do not have a clear physical interpretation as they have been introduced phenomenologically. In the analysis of [16], $\alpha$ is the linewidth enhancement factor defined as the ratio of the real-to-imaginary parts of the carrier-induced change in the susceptibility. This interpretation for $\alpha$ in (30) is incorrect since the total carrier density remains unaffected in the intraband relaxation processes. Similarly, it is not clear whether the intraband relaxation time $\tau$ in (30) should be identified with $\tau_v$, $\tau_c$, or $\tau_{in}$ since all occur in the general expression (27) for the nonlinear gain.

Let us consider the conditions under which the general result (27) takes the form of (30). One possibility consists of neglecting the second term in (27) and then identifying $\alpha$ by $\alpha_k$ and $\tau$ by $\tau_{in}/2$. The second term is indeed expected to small for Fabry–Perot lasers because of a near cancellation of $\chi_2$ and $\chi_{\mathrm{FWM}}$ contributions to the nonlinear gain. Using the results of [5], we estimate that the contribution of the second term is $\sim 4$ percent or less. Thus, to a good degree of approximation, the nonlinear gain can be obtained by setting $b_j = 0$ in (27). For Fabry–Perot lasers, the nonlinear gain then becomes

$$g_{\mathrm{NL}}^{\mathrm{FP}}(\omega_j) = -\sum_k \frac{C_{jk} g_L(\omega_k)}{C_k} \frac{1 + \alpha_k \Omega_{jk}\tau_{in}/2}{1 + (\Omega_{jk}\tau_{in}/2)^2} \frac{|E(\omega_k)|^2}{I_s}. \tag{31}$$

A comparison of (30) and (31) explains why the phenomenological expression (30) has been so successful in explaining the experimental data [10], [14]. In general, both $\alpha$ and $B$ in (30) should be made mode dependent for a more accurate representation of the nonlinear gain.

The situation is different for DFB lasers where the dominant mode is usually displaced from the gain peak. The four-wave mixing contribution can then become vanishingly small because of the phase mismatch. An approximate expression for $g_{\mathrm{NL}}$ is obtained from (27) by setting $b_j = 1$. To simplify it further, we consider the relative magnitudes of $\tau_c$, $\tau_v$, and $\tau_{in}$. Numerical values of the intraband relaxation times for the InGaAsP material are not known accurately, and their estimates vary widely. Asada and Suematsu [6] have used $\tau_c = 0.2$ ps, $\tau_v = 0.07$ ps, and $\tau_{in} = 0.1$ ps in their calculation of the nonlinear gain. On the other hand, Kazarinov *et al.* [5] have used $\tau_c = 1$ ps and $\tau_{in} = 0.1$ ps in fitting their data on the gain spectra of GaAs lasers. In general, $\tau_c$ is expected to be large compared to both $\tau_v$ and $\tau_{in}$. If we keep the dominant contribution in (27), the nonlinear gain for DFB lasers can be approximated by

$$g_{\mathrm{NL}}^{\mathrm{DFB}}(\omega_j) = -\sum_k \frac{C_{jk} g_L(\omega_k)}{C_k(1+\delta_{jk})} \cdot \left(1 + \frac{1+\alpha_k \Omega_{jk}\tau_c}{1+\Omega_{jk}^2 \tau_c^2}\right) \frac{|E(\omega_k)|^2}{I_s} \quad (32)$$

where the two terms in (27) have been combined by using the $\delta$-function notation ($\delta_{jk} = 1$ if $j = k$ and zero otherwise). This result should be compared to the phenomenological expression (30). It shows that the relevant intraband relaxation time is the energy relaxation time $\tau_c$ of conduction-band electrons.

The asymmetric nature of the nonlinear gain is governed by the parameter $\alpha_k$ [defined by (20)] which is related to the slope of the gain profile. To estimate $\alpha_k$, we approximate the gain profile by

$$g(\omega) \simeq g(\omega_p)\left[1 - \frac{1}{2}\left(\frac{\omega - \omega_p}{\Delta\omega}\right)^2\right] \quad (33)$$

where $\omega_p$ is the frequency at which the gain peaks and $2\Delta\omega$ is the full width at half maximum (FWHM). Using (29) and (33), we obtain

$$\alpha_k = \frac{1}{\tau_{in}\Delta\omega}\left(\frac{\omega_p - \omega_k}{\Delta\omega}\right)\frac{g(\omega_p)}{g(\omega_k)}. \quad (34)$$

Typical values of the gain bandwidth (FWHM) for InGaAsP lasers are 40–50 nm. Using $\tau_{in} \simeq 0.1$ ps, we estimate that $\tau_{in}\,\Delta\omega \sim 1$. Thus, an order-of-magnitude estimate of $\alpha_k$ is given by $\Delta\lambda_k/20$ where $\Delta\lambda_k$ is the wavelength separation of the mode (in nanometers) from the gain peak. Such values of $\alpha_k$ are too small to explain the observed shift of the mode spectrum toward longer wavelengths with an increase in the drive current. It appears that some other effect is required to explain the data of [10] and [17].

## V. Self and Cross Saturation

The nonlinear gain given by (27) is responsible for the reduction in optical gain under lasing conditions. The physical mechanism behind gain reduction is spectral hole burning that leads to gain saturation. Both self-saturation [$j = k$ in (31) and (32)] and cross-saturation terms contribute in determining the net gain reduction occurring for a particular longitudinal mode. For its use in multimode rate equations, it is useful to write the nonlinear gain in terms of photon density defined by [4]–[6]

$$S_k = \frac{2\epsilon_0 n n_g}{\hbar\omega_0}|E(\omega_k)|^2 \quad (35)$$

where $n_g$ is the group index and $\hbar\omega_0$ is the photon energy. The nonlinear gain reduction then becomes

$$g_{\mathrm{NL}}(\omega_j) = -g_L(\omega_0)\sum_k \beta_{jk} S_k \quad (36)$$

where $g_L(\omega_0)$ is the gain of the dominant mode. From (32) and (36), the saturation coefficient is given by

$$\beta_{jk} = \frac{\mu^2\omega_0\tau_{in}(\tau_c + \tau_v)}{2\epsilon_0\hbar n n_g}\frac{g_L(\omega_k)}{g_L(\omega_0)}\frac{C_{jk}}{(1+\delta_{jk})C_k} \cdot \left(1 + \frac{1+\alpha_k\Omega_{jk}\tau_c}{1+\Omega_{jk}^2\tau_c^2}\right) \quad (37)$$

where we have used (20) for the saturation intensity $I_s$. A slightly different expression is obtained for Fabry–Perot lasers using (31) and (36). The saturation coefficient depends on the spatial distribution of the optical mode through $C_{jk}/C_j$. To evaluate it, we consider a strongly index-guided semiconductor laser whose waveguide supports a single lateral and transverse mode. The field distribution is then approximated by

$$F_j(r) = U(x, y)\sin(k_j z) \quad (38)$$

where $k_j = n_j\omega_j/c$ and $U(x, y)$ is the spatial distribution of the fundamental waveguide mode. Averaging over the active volume, we obtain [4]

$$\frac{C_{jk}}{C_k} = \frac{\langle |F_j(r) F_k(r)|^2\rangle}{\langle |F_k(r)|^2\rangle} = \frac{(2+\delta_{jk})\Gamma'}{4\Gamma} \quad (39)$$

where $\Gamma$ is the confinement factor and

$$\Gamma' = \int\int_\sigma |U(x,y)|^4\,dxdy \Big/ \int\int_{-\infty}^{\infty} |U(x,y)|^2\,dxdy \quad (40)$$

where the integration is over the waveguide cross section $\sigma$. Both $\Gamma'$ and $\Gamma$ are dimensionless quantities since we have used a normalization scheme where $U(x, y)$ is dimensionless.

A quantity of practical interest is the self-saturation coefficient $\beta_{00}$ for the main mode. Using (37) and (39), it is given by

$$\beta_{00} = \frac{3\mu^2\omega_0\tau_{in}(\tau_c+\tau_v)\Gamma'}{8\epsilon_0\hbar n n_g\Gamma}. \quad (41)$$

It can be easily verified that $\beta_{00}$ is the same whether we use (31) or (32) for the nonlinear gain. The self-saturation coefficient $\beta_{00}$, which leads to a nonlinear suppression of the dominant mode gain by a factor of $1 - \beta_{00}S_0$, is known to dramatically affect the dynamic response of semiconductor lasers [10]–[14].

We now use (41) to estimate $\beta_{00}$ and compare the estimated value to the known experimental values. The only geometry-dependent parameter is the ratio $\Gamma'/\Gamma$. Its evaluation requires knowledge of the waveguide mode $U(x, y)$. To simplify the calculation, we assume that $U(x, y)$ can be approximated by a Gaussian in both the lateral and transverse directions, i.e.,

$$U(x, y) = \exp(-x^2/2\sigma_T^2)\exp(-y^2/2\sigma_L^2).$$

The integration over the waveguide cross section in (40) can be readily carried out in terms of the error functions. For an active layer of width $w$ and thickness $d$, we obtain

$$\frac{\Gamma'}{\Gamma} = \frac{1}{2} \frac{\operatorname{erf}(d/\sqrt{2}\sigma_T) \operatorname{erf}(w/\sqrt{2}\sigma_L)}{\operatorname{erf}(d/2\sigma_T) \operatorname{erf}(w/2\sigma_L)}.$$

Using typical values $d/\sigma_T = 0.4$ and $w/\sigma_L = 3$, we estimate that $\Gamma'/\Gamma \simeq 0.7$. We use this value for our calculation of $\beta_{00}$. The dipole moment $\mu$ for semiconductor lasers has been estimated by Asada and Suematsu [6]. Using their result,

$$\mu = q\langle r \rangle, \langle r \rangle\,(\text{Å}) = 3.5\lambda_g(\mu\text{m})$$

where $\lambda_g$ is the bandgap wavelength. For a 0.82 $\mu$m GaAs laser and a 1.3 $\mu$m InGaAsP laser, $\mu = 4.6 \times 10^{-29}$ cm and $\mu = 7.3 \times 10^{-29}$ cm, respectively.

Consider first the case of GaAs lasers. The intraband relaxation times for GaAs have been estimated both theoretically and experimentally [21], [22]. Using the representative values $\tau_{in} = 0.1$ ps, $\tau_v = 0.07$ ps, $\tau_c = 0.2$ ps, $n = 3.4$, and $n_g = 4$ in (41), we obtain $\beta_{00} = 2.7 \times 10^{-18}$ cm$^3$. Johnson and Mooradian [23] have recently obtained a self-saturation coefficient of 1.3 W$^{-1}$ from their transient measurements of the carrier density in a GaAlAs laser. To obtain $\beta_{00}$, we need the relationship between the photon density and the output power for the specific laser used in the experiment. This is not known since several laser parameters have not been specified in [23]. If we use a representative value of $3 \times 10^{14}$ photons/cm$^3$ at 1 mW of output power, $\beta_{00} = 4.3 \times 10^{-18}$ cm$^3$. This value is in reasonable agreement with the theoretical estimate.

We now consider the case of InGaAsP lasers. The intraband relaxation times for InGaAsP are not known accurately. Their values are expected to be slightly larger than those for GaAs lasers; for our estimate, we have chosen $\tau_{in} = 0.12$ ps, $\tau_v = 0.07$ ps, and $\tau_c = 0.3$ ps. For 1.3 $\mu$m InGaAsP lasers, we obtain $\beta_{00} = 6.6 \times 10^{-18}$ cm$^3$ if we use $n = 3.3$, $n_g = 4$, and $\mu = 7.3 \times 10^{-29}$ cm. Since the dipole moment $\mu$ increases linearly with the wavelength, $\beta_{00}$ scales linearly with the wavelength. Thus, $\beta_{00}$ increases by nearly 20 percent for 1.55 $\mu$m InGaAsP lasers if we assume that all other parameters in (41) remain the same.

Experimental estimates of $\beta_{00}$ for InGaAsP lasers vary widely. Tucker [11] has used a value as high as $6.7 \times 10^{-17}$ cm$^3$, while others have estimated it to be lower as much as by one order of magnitude. One can estimate $\beta_{00}$ using the measurements of small-signal modulation response [11], [13], [24]. One method consists of relating $\beta_{00}$ to the height $M_p$ of the resonance peak [11]:

$$M_p \simeq \frac{A}{2\pi \nu_R \beta_{00}} \tag{42}$$

where $A$ is the gain coefficient and $\nu_R$ is the relaxation-oscillation frequency. We use the measurements of Bowers *et al.* [13, Fig. 11] and adopt their value $A = 1.8 \times 10^{-6}$ cm$^3$/s. For a specific 1.3 $\mu$m InGaAsP laser, their data show that $M_p \simeq 11$ at $\nu_R = 3.5$ GHz, while $M_p \simeq 5$ at $\nu_R = 8$ GHz. From (42), we obtain $\beta_{00} = 7.2 \times 10^{-18}$ cm$^3$, which is in excellent agreement with the theoretical value. A similar value is inferred from the measurements shown in [24, Fig. 12].

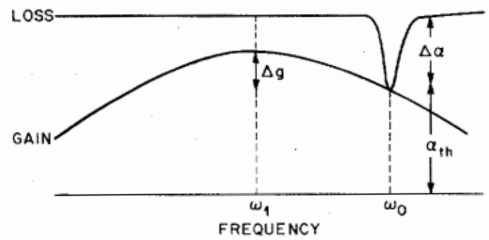


Fig. 1. Schematic illustrations of the gain and loss profiles for a DFB semiconductor laser. $\Delta\alpha - \Delta g$ is the net gain margin for the side mode (occurring near the gain peak at $\omega_1$) and $\alpha_{th}$ is the threshold gain of the main mode near the Bragg wavelength $\omega_0$.

The experimental and theoretical values of $\beta_{00}$ agree with each other within a factor of 2 for both GaAs and InGaAsP lasers. One cannot expect a better agreement in view of the uncertainities associated with the material parameters as well as experimental measurements. The agreement between the theory and the experiment lends support to the hypothesis that spectral hole burning is the dominant mechanism for the nonlinear gain in semiconductor lasers. Expression (41) relates the self-saturation coefficient to the material and laser parameters and predicts the $\beta_{00} = 0.6$–$1 \times 10^{-17}$ cm$^3$ for InGaAsP lasers, depending on the laser wavelength and other design parameters.

## VI. Application to DFB Lasers

As a specific application of the nonlinear-gain theory, we apply it to study the range of single-longitudinal-mode operation of DFB lasers. Such lasers can be modeled by considering only two modes, referred to as the main and side modes. For a DFB laser with uncoated facets, the Fabry–Perot mode closest to the gain peak is often the dominant side mode. The wavelengths of the main and side modes can differ significantly (as much as by 10 nm or more), depending on the deviation of the gain peak from the Bragg wavelength. The coupled-mode equations for the main and side modes are

$$\dot{S}_0 = v_g[g_0(1 - \beta_{00}S_0 - \beta_{01}S_1) - \alpha_{th}]S_0 + R_{sp} \tag{43}$$

$$\dot{S}_1 = v_g[g_0(1 - \beta_{10}S_0 - \beta_{11}S_1) - (\alpha_{th} + \Delta\alpha - \Delta g)]\,S_1 + R_{sp} \tag{44}$$

where $v_g = c/n_g$ is the group velocity, $R_{sp}$ is the rate of spontaneous emission, $\alpha_{th}$ is the threshold gain of the main mode (in the absence of mode coupling), $\Delta\alpha$ is the gain margin (excess gain required by the side mode to reach threshold), and $\Delta g$ is the reduction in $\Delta\alpha$ due to gain roll-off (see Fig. 1).

The gain $g_0$ varies with the carrier density $N$ and is obtained by solving

$$\dot{N} = \frac{I}{qV} - \frac{N}{\tau_s} - \frac{v_g}{\Gamma}[g_0S_0 + (g_0 + \Delta g)S_1] \tag{45}$$

where we have neglected the nonlinear-gain terms as they are small compared to $g_0$. They are, however, retained in (43) and (44) since their magnitude is comparable to $g_0 - \alpha_{th}$. For simplicity, we also neglect $\Delta g$ in (45) since $\Delta g << g_0$ in most practical cases of interest. In (45), $I$ is the injected current, $V$ is the active volume, and $\tau_s$ is the spontaneous carrier lifetime. Assuming a linear variation of the gain with the carrier density, the steady-state solution of (45) yields

$$g_0(N) = \Gamma a(N - N_0) = \frac{\alpha_{th}(I/I_{th})}{1 + (S_0 + S_1)/\bar{S}} \quad (46)$$

where $a$ is the gain coefficient, $N_0$ is the carrier density at transparency, and

$$I_{th} = \frac{qV}{\tau_s}\left(N_0 + \frac{\alpha_{th}}{\Gamma a}\right) \quad (47)$$

is the threshold current. The parameter $\bar{S} = (v_g a \tau_s)^{-1}$ governs gain saturation occurring due to interband transitions. Using typical parameter values appropriate for InGaAsP lasers, we estimate that $\bar{S} \sim 10^{15}$ cm$^{-3}$.

To obtain the stable steady-state solutions of the coupled-mode equations, we substitute (46) in (43) and (44). We neglect the spontaneous-emission term $R_{sp}$ for simplicity since it does not affect the steady-state behavior significantly. Defining a dimensionless measure of the photon density (or, equivalently, the mode power) by

$$A_i = S_i/\bar{S} \qquad (i = 0, 1) \quad (48)$$

we obtain the coupled-mode equations

$$\tau_p \dot{A}_0 = \left(\frac{I/I_{th}}{1 + A_0 + A_1} - 1 - \beta_{00}\bar{S}A_0 - \beta_{01}\bar{S}A_1\right)A_0 \quad (49)$$

$$\tau_p \dot{A}_1 = \left(\frac{I/I_{th}}{1 + A_0 + A_1} - 1 - r - \beta_{10}\bar{S}A_0 - \beta_{11}\bar{S}A_1\right)A_1 \quad (50)$$

where $\tau_p = (v_g \alpha_{th})^{-1}$ is the photon lifetime and

$$r = (\Delta\alpha - \Delta g)/\alpha_{th} \quad (51)$$

is the relative gain margin. In the absence of gain nonlinearities ($\beta_{ij} = 0$), the only stable solution of (45) and (46) when $r > 0$ is

$$A_0 = I/I_{th} - 1, \qquad A_1 = 0, \quad (52)$$

showing that the side mode remains suppressed at all pumping levels as long as $r > 0$.

We now show that in the presence of gain nonlinearities, the solution (52) becomes unstable above a critical current $I_{cr}$. This can be seen most readily within the framework of third-order perturbation theory. Linearizing (49) and (50) in $A_0$ and $A_1$, we obtain

$$\tau_p \dot{A}_0 = (\alpha_0 - \beta_0 A_0 - \theta_{01}A_1)A_0 \quad (53)$$

$$\tau_p \dot{A}_1 = (\alpha_1 - \beta_1 A_1 - \theta_{10}A_0)A_1 \quad (54)$$

where

$$\alpha_0 = \frac{I}{I_{th}} - 1, \qquad \alpha_1 = \frac{I}{I_{th}} - 1 - r \quad (55)$$

$$\beta_0 = (1 + \beta_{00}\bar{S}), \qquad \beta_1 = (1 + \beta_{11}\bar{S}) \quad (56)$$

$$\theta_{01} = (1 + \beta_{01}\bar{S}), \qquad \theta_{10} = (1 + \beta_{10}\bar{S}). \quad (57)$$

The stable steady-state solutions of (53) and (54) are well known [2] and depend on the coupling coefficient

$$C = \frac{\theta_{01}\theta_{10}}{\beta_1\beta_2} = \frac{(1 + \beta_{01}\bar{S})(1 + \beta_{10}\bar{S})}{(1 + \beta_{00}\bar{S})(1 + \beta_{11}\bar{S})}. \quad (58)$$

Since $\beta_{01}$ can be larger or smaller than $\beta_{00}$ depending on the mode separation [see (37)], the numerical value of $C - 1$ changes from positive to negative as the mode separation $|\Omega_{01}|$ increases. Since (53) and (54) permit the two-mode solution for $C < 1$, both modes can oscillate simultaneously when their separation exceeds a critical value. This result shows how the single-frequency range of DFB lasers can depend on the mismatch of the Bragg wavelength from the gain peak.

To obtain the critical value of mode separation above which $C < 1$, we neglect the asymmetry parameter $\alpha_k$ in (37) for simplicity and obtain

$$\frac{\beta_{10}}{\beta_{11}} = \frac{\beta_{01}}{\beta_{00}} = \frac{2(2 + \delta^2)}{3(1 + \delta^2)} \quad (59)$$

where

$$\delta = |\omega_0 - \omega_1|\tau_c \quad (60)$$

is a dimensionless measure of mode separation. Since $\beta_{01} = \beta_{00}$ when $\delta = 1$, $C \leq 1$ in (58) when $\delta \geq 1$. Thus, the two-mode solution is possible when the mode separation $\Delta\nu \geq 1/(2\pi\tau_c)$. Using $\tau_c = 0.3$ ps, the condition becomes $\Delta\nu > 0.5$ THz. This corresponds to a wavelength separation $\Delta\lambda > 4$ nm at 1.55 $\mu$m.

The critical pumping level $I_{cr}$ above which the side mode starts to oscillate can be obtained by the requirement [2] that the effective mode gain

$$\alpha_1' = \alpha_1 - (\theta_{10}/\beta_1)\alpha_0 > 0. \quad (61)$$

Using (55)–(57), we obtain

$$\frac{I_{cr}}{I_{th}} = 1 + \frac{(1 + \beta_{11}\bar{S})r}{(\beta_{11} - \beta_{10})\bar{S}}. \quad (62)$$

Since $\beta_{11}\bar{S} << 1$, it can be neglected in the numerator. Using (51) and (59) in (62), the critical pumping level above which the side mode starts oscillating is given by

$$\frac{I_{cr}}{I_{th}} = 1 + \frac{3(\delta^2 + 1)}{(\delta^2 - 1)}\left(\frac{\Delta\alpha - \Delta g}{\alpha_{th}}\right)\left(\frac{1}{\beta_{11}\bar{S}}\right). \quad (63)$$

Note that the gain rolloff $\Delta g$ is also $\delta$ dependent. For a parabolic gain profile (33), it varies as

$$\Delta g = \frac{1}{2}g(\omega_p)\left(\frac{\delta}{\Delta\omega\tau_c}\right)^2. \quad (64)$$

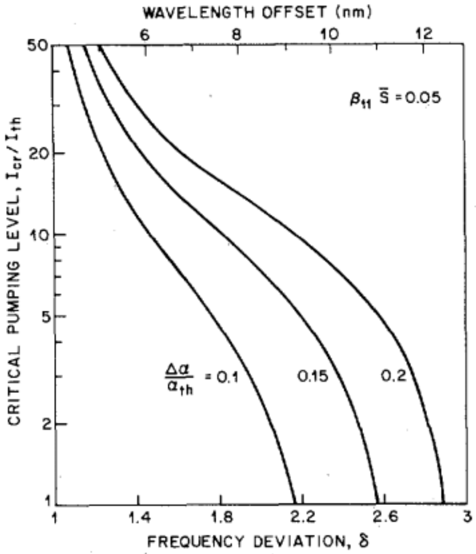


Fig. 2. Variation of the critical pumping level $I_{cr}/I_{th}$ with the normalized mode separation $\delta$ for three values of the relative gain margin $\Delta\alpha/\alpha_{th}$. Upper scale shows the mode separation in nanometers using $\lambda = 1.55$ $\mu$m and $\tau_c = 0.3$ ps.

Fig. 2 shows the variation of $I_{cr}/I_{th}$ as a function of $\delta$ for $\beta_{11}\bar{S} = 0.05$, $\tau_c = 0.3$ ps, and $\Delta\omega\tau_c = 5$. Three curves are shown corresponding to three values of the relative gain margin $\Delta\alpha/\alpha_{th}$ ranging from 0.1 to 0.2. The side mode can reach the threshold at relatively low values of the pumping level when the DFB-wavelength offset from the gain peak exceeds a certain value. These results are different from those given in [15] because a different expression (30) for the nonlinear gain was used therein. Nonetheless, the main conclusion remains the same: one should attempt to reduce the mismatch of the Bragg wavelength from the gain peak in order to increase the single-frequency range of DFB lasers. The tolerable mismatch depends on parameters such as the relative gain margin and the intraband relaxation time and can be as small as 2 nm for $\Delta\alpha/\alpha_{th} \lesssim 0.1$ and $\tau_c = 1$ ps.

Let us briefly consider the case $\delta < 1$. Since the cross saturation is more effective than self saturation [see (59)], the coupling coefficient $C > 1$. Even though the two modes cannot oscillate simultaneously in the strong-coupling regime, a jump from main mode to side mode can occur in a bistable manner when the effective mode gain for the main mode becomes negative, i.e., when [2]

$$\alpha_0' = \alpha_0 - (\theta_{01}/\beta_0)\alpha_1 < 0. \tag{65}$$

Using (55)–(57), this occurs when

$$\frac{I}{I_{th}} > 1 + \frac{3r}{\beta_{00}\bar{S}} \frac{1 + \delta^2}{1 - \delta^2}. \tag{66}$$

If we use typical values $r = 0.1$ for the relative gain margin and $\beta_{00}\bar{S} = 0.05$ for the nonlinear-gain suppression, we find that the side mode does not reach threshold until $I > 7I_{th}$. Thus, a large range of single-longitudinal mode operation can be ensured in the strong coupling regime by making $\delta < 1$. Note that the third-order perturbation theory is not expected to hold for such large values of $I/I_{th}$. However, the qualitative conclusions remain valid even when (49) and (50) are used to obtain the steady-state solutions.

## VII. Conclusions

We have obtained an analytic expression for the nonlinear gain in multimode semiconductor lasers using the density-matrix formalism together with several simplifying assumptions. In general, the nonlinear gain is found to consist of the symmetric and asymmetric components. However, the asymmetry does not have its origin in the carrier-induced index change, but is related to the slope of the gain profile. We discuss in detail the similarities and the differences between our result and a phenomenological expression of the nonlinear gain used in previous work [10], [14], [17]. The general expression for the nonlinear gain is used to obtain the range of single-longitudinal-mode operation of DFB lasers using a two-mode model. It is shown that gain nonlinearities can significantly reduce the range of single-longitudinal-mode operation when the Bragg wavelength has a large offset from the gain peak. We have also used the theory to obtain an analytic expression for the self-saturation coefficient that leads to a reduction in the gain with an increase in the laser power. The predicted values of this nonlinear-gain parameter are in agreement with the experimentally deduced values for both GaAs and InGaAsP lasers.

## Acknowledgment

Helpful discussions with D. M. Fye, C. H. Henry, and R. F. Kazarinov are gratefully acknowledged. The author is thankful to P. J. Anthony and J. E. Geusic for their support and encouragement.

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