Four-wave mixing and phase conjugation in semiconductor laser media

Govind P. Agrawal

AT&T Bell Laboratories, Murray Hill, New Jersey 07974

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A theory of four-wave mixing in semiconductor laser media is developed by considering the contributions of both the gain and index gratings created by the carrier-density modulation occurring at the beat frequency of the pump and the probe waves. The general formalism can be applied to semiconductor lasers operating below or above threshold. As an illustration, we consider the case in which the semiconductor laser is operated as a traveling-wave amplifier. The results show that the dominant contribution to the four-wave mixing process comes from the index grating. Further, the index grating makes the probe transmission asymmetric with respect to the pump–probe detuning.

Four-wave mixing and the related phenomenon of optical phase conjugation[1] have been studied extensively. Recently it was found[2] that nearly degenerate four-wave mixing (NDFWM) inside a semiconductor laser can generate phase-conjugate signals with high efficiency while requiring pump powers of only a few milliwatts. In a theoretical treatment of this process,[3] the semiconductor-laser medium was modeled as an inverted two-level system. Such an approach, although capable of explaining the qualitative behavior, has several limitations: (1) The parameters of the two-level system cannot be directly related to the known device parameters, (2) spatial effects related to the laser waveguide are not incorporated, and (3) the effects of carrier-induced index changes are not included.

My objective in this Letter is to present a theory of NDFWM after including the contributions of both the gain and the index gratings created by the carrier-density modulation occurring at the beat frequency of the pump and the probe waves. The general formalism can be applied to semiconductor lasers operating below or above threshold. As an illustrative application, I consider the case wherein the semiconductor laser operates as a traveling-wave amplifier. The results show that the dominant contribution to the NDFWM process comes from the index grating. Further, the index grating is responsible for making the probe transmission asymmetric with respect to the pump–probe detuning.

In the standard NDFWM geometry, two counterpropagating pump waves at the frequency ω_0 interact with a probe wave at the frequency ω_1 and generate a conjugate wave at the frequency $\omega_2 = 2\omega_0 - \omega_1$. The nonlinear interaction in the scalar approximation is governed by the wave equation

$$\nabla^2 E - \frac{n^2}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 P}{\partial t^2}, \tag{1}$$

where n is the refractive index, ϵ_0 is the vacuum permittivity, and c is the velocity of light in vacuum. The total electric field E is given by

$$E(x, y, z, t) = U(x, y)\sum_j E_j(z)\exp(-i\omega_j t), \tag{2}$$

where $j = 0, 1, 2$ for pump, probe, and conjugate waves, respectively. The semiconductor-laser structure is assumed to support only the fundamental waveguide mode with the distribution $U(x, y)$. This is generally the case for strongly index-guided laser.[4] Similar to Eq. (2), the induced polarization P can also be written as

$$P(x, y, z, t) = U(x, y)\sum_j P_j(z)\exp(-i\omega_j t), \tag{3}$$

In a semiconductor laser the induced polarization is calculated using

$$P = \epsilon_0 \chi(N) E, \tag{4}$$

where the susceptibility[4]

$$\chi(N) = -\frac{nc}{\omega}(\beta + i)g(N), \tag{5}$$

and the gain is assumed to vary linearly with the carrier density N, i.e.,

$$g(N) = a(N - N_0). \tag{6}$$

Here a is the gain coefficient and N_0 is the carrier density required to achieve transparency. The parameter β in Eq. (5) accounts for the carrier-induced index change that occurs invariably in semiconductor lasers whenever the gain changes. It is often referred to as the linewidth-enhancement factor or the antiguiding parameter and has typical values in the range 3–6 depending on the operating wavelength of the semiconductor laser (see Ref. 4 for a detailed discussion).

The carrier density N is obtained by solving the rate equation

$$\frac{dN}{dt} = \frac{I}{qV} - \frac{N}{\tau_s} - \frac{g(N)}{\hbar\omega_0}\langle |E|^2 \rangle, \tag{7}$$

where the diffusion term is ignored by assuming that the transverse waveguide dimensions are smaller than

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the diffusion length. The angle brackets denote averaging over the waveguide cross section in the x and y directions and over a distance comparable with the diffusion length ($\sim$2–3 μm) in the z direction. The reason for neglecting the standing-wave effects in Eq. (7) is that carrier diffusion washes out spatial holes burned by counterpropagating pump waves. In Eq. (7) I is the injected current, τ_s is the spontaneous carrier lifetime, and V is the active volume.

The physical process behind NDFWM can be understood from Eqs. (5)–(7). Beating of the pump and the probe waves modulates the carrier density N at the beat frequency

$$\Omega = \omega_1 - \omega_0 = \omega_0 - \omega_2. \tag{8}$$

Such a modulation creates a dynamic grating for the gain as well as for the index. Diffraction of the other pump wave from these gratings generates a conjugate wave. The effectiveness of the grating is related to the spontaneous lifetime τ_s ($\simeq$2–3 nsec) and is governed by the condition $\Omega\tau_s \lesssim 1$.

To account for the carrier-density modulation, an approximate solution of Eq. (7) is of the form

$$N(t) = \overline{N} + [\Delta N \exp(-i\Omega t) + \text{c.c.}], \tag{9}$$

where $\overline{N}$ is the static carrier density. Using Eqs. (2) and (9) in Eq. (7) and assuming that the probe and the conjugate waves are much weaker compared with the pump waves, we obtain

$$\Delta N = \frac{\Gamma(\overline{N} - N_0)(E_0{}^*E_1 + E_0E_2{}^*)}{(1 + \overline{|E_0|^2}/P_s + i\Omega\tau_s)P_s}, \tag{10}$$

where

$$P_s = \hbar\omega_0/(\Gamma a\tau_s) \tag{11}$$

is the saturation intensity ($\sim$3 mW/μm^2). The confinement factor $\Gamma = \langle|U(x, y)|^2\rangle$ results from spatial averaging in the transverse directions. The bar over $|E_0|^2$ in Eq. (10) denotes averaging in the z direction.

The induced polarization is calculated by using Eqs. (3)–(6) together with Eqs. (9) and (10). This leads to the following expression for the polarization components P_j ($j = 1, 2$):

$$P_j = -\frac{\epsilon_0 nc}{\omega_j}(\beta + i)\Gamma g(\overline{N}) \times \left[E_j + \frac{\Gamma(|E_0|^2E_j + E_0{}^2E_{3-j}{}^*)}{(1 + \overline{|E_0|^2}/P_s + i\Omega\tau_s)P_s}\right]. \tag{12}$$

The mode gain $\Gamma g(\overline{N})$ is obtained using the steady-state solution of Eq. (7) and is given by

$$\Gamma g(\overline{N}) = \frac{g_0}{1 + \overline{|E_0|^2}/P_s}, \tag{13}$$

where the small-signal gain

$$g_0 = \Gamma a[(I\tau_s/qV) - N_0] \tag{14}$$

is a function of the injected current I.

This completes the general formalism. For its application, one should distinguish whether the laser is operating below or above threshold. In the latter case, the saturated gain is clamped at its threshold value g_{th}. Thus, in the above-threshold operation of the semiconductor laser, $\Gamma g(\overline{N}) = g_{\text{th}}$. Using Eqs. (13) and (14), we obtain

$$\frac{\overline{|E_0|^2}}{P_s} = \frac{g_0}{g_{\text{th}}} - 1 = \frac{I - I_{\text{th}}}{I_{\text{th}} - I_0}, \tag{15}$$

where $I_0 = qVN_0/\tau_s$ is the current at transparency and $I_{\text{th}} = I_0 + qVg_{\text{th}}/(a\Gamma\tau_s)$ is the threshold current. Equation (15) relates the intracavity pump intensity to the device current.

Although Eqs. (12) and (15) can be used to discuss NDFWM under above-threshold operation, the analysis is complicated since the boundary conditions at the laser facets couple the forward and backward components of the pump, probe, and conjugate waves.[3] To show the main qualitative features of NDFWM as simply as possible, we consider the case of a traveling-wave amplifier whose facets have negligible reflectivities (by the use of an antireflection coating). In the collinear geometry, the pump, probe, and conjugate fields are

$$E_0 = \sqrt{P_s}[A_f \exp(ik_0z) + A_b \exp(-ik_0z)],$$
$$E_1 = \sqrt{P_s}A_1 \exp(ik_1z), \qquad E_2 = \sqrt{P_s}A_2 \exp(-ik_2z), \tag{16}$$

respectively, where $k_j = n\omega_j/c$. Using Eqs. (2) and (16) in Eq. (1), we obtain

$$\sqrt{P_s}\frac{\mathrm{d}}{\mathrm{d}z}A_j = \pm\frac{i\omega_j}{2\epsilon_0 nc}P_j \exp(\mp ik_jz). \tag{17}$$

The transverse effects are included through the confinement factor Γ in expression (12) for P_j. Substituting Eq. (12) into Eq. (17) and keeping only the nearly phase-matched terms, we obtain the coupled-wave equations

$$\mathrm{d}A_1/\mathrm{d}z = -\alpha_1A_1 + i\kappa_1A_2{}^* \exp(i\Delta kz), \tag{18a}$$

$$\mathrm{d}A_2{}^*/\mathrm{d}z = +\alpha_2{}^*A_2{}^* + i\kappa_2{}^*A_1 \exp(-i\Delta kz), \tag{18b}$$

where $\Delta k = k_2 - k_1$ and

$$\alpha_j = \frac{i}{2}\frac{(\beta + i)g_0}{1 + P_0}\left(1 + \frac{\Gamma P_0}{1 + P_0 \pm i\Omega\tau_s}\right), \tag{19}$$

$$\kappa_j = -\frac{1}{2}\frac{(\beta + i)g_0}{1 + P_0}\left(\frac{\Gamma P_0}{1 + P_0 \pm i\Omega\tau_s}\right). \tag{20}$$

In Eqs. (17), (19), and (20) the upper or lower sign is chosen for $j = 1$ or 2, respectively. If we assume that the pump beams of equal intensity P_{in} are incident at the two ends of an amplifier of length L, the average intracavity pump intensity P_0 is related to P_{in} by

$$P_0 = 2P_{\text{in}}\{[\exp(g_0L) - 1]/g_0L\}. \tag{21}$$

Owing to their linearity, Eqs. (18) are readily solved. The expression for the phase-conjugate reflectivity R and the probe transmittivity T are[5,6]

$$R = \left|\frac{A_2{}^*(0)}{A_1(0)}\right|^2 = \left|\frac{\kappa_2 \sin(pL)}{p \cos(pL) + \alpha \sin(pL)}\right|^2,$$

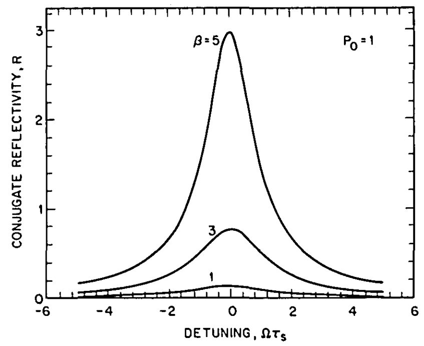

Fig. 1. Variation of the conjugate reflectivity R with the pump–probe detuning (in normalized units) for three values of the linewidth-enhancement factor β.

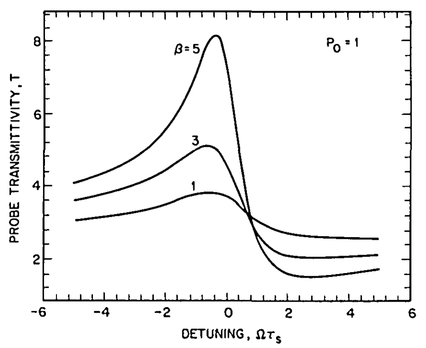

Fig. 2. Variation of the probe transmittivity T with the pump–probe detuning (in normalized units) for three values of β. Note the asymmetric enhancement of T for negative values of $\Omega\tau_s$.

$$T = \left|\frac{A_1(L)}{A_1(0)}\right|^2 = \left|\frac{p\exp(-\overline{\alpha}L)}{p\cos(pL) + \alpha\sin(pL)}\right|^2, \qquad (23)$$

where

$$p = (\kappa_1\kappa_2{}^* - \alpha^2)^{1/2}, \qquad \alpha = \tfrac{1}{2}(\alpha_1 + \alpha_2{}^* - i\Delta k), \quad (24)$$

$$\overline{\alpha} = \tfrac{1}{2}(\alpha_1 - \alpha_2{}^* + i\Delta k). \qquad (25)$$

Figures 1 and 2 show the variation of R and T with the detuning $\Omega\tau_s$ for three values of β after using $g_0L = 2$, $\Gamma = 0.5$, and $P_0 = 1$. The phase mismatch Δk is neglected since $\Delta kL = (2nL/c)\Omega \sim 10^{-2}$ under typical experimental conditions.

The most interesting features of Figs. 1 and 2 are dramatically different line shapes for R and T and considerable enhancement of R and T with an increase in β. Both of these features are related to the carrier-induced index change governed by the parameter β. The enhancement factor for R is $1 + \beta^2$, as seen from Eqs. (20) and (22). Physically, the modulation of carrier density at the beat frequency Ω creates a gain grating and an index grating. However, the index-grating contribution to the NDFWM process is β^2 times larger than that of the gain grating. Since β is generally larger ($\beta \simeq 5$) for InGaAsP lasers compared with GaAs lasers, InGaAsP lasers may be more efficient for the generation of the phase-conjugate signal. The line shape of R is approximately Lorentzian, with a power-broadened linewidth (FWHM) given by

$$\Delta\nu = (1 + P_0)/(\pi\tau_s). \qquad (26)$$

Thus NDFWM may be useful for estimating the carrier lifetime in semiconductor lasers.

The asymmetric line shape for the probe transmittivity T can be understood by considering the probe gain in the absence of nonlinear interaction ($\kappa_j = 0$). From Eq. (19), the probe gain is

$$G = -2\,\mathrm{Re}(\alpha_1) = \frac{g_0}{1 + P_0}\left[1 + \frac{\Gamma P_0(1 + P_0 - \beta\Omega\tau_s)}{(1 + P_0)^2 + (\Omega\tau_s)^2}\right]. \qquad (27)$$

Since $G(-\Omega) > G(\Omega)$, the probe gain is larger when the probe is detuned toward the longer-wavelength side of the pump. The asymmetric nature of nonlinear interaction in semiconductor lasers was first studied by Bogatov *et al.*[7] and is a consequence of the simultaneous presence of the gain and the index gratings during the four-wave mixing process. The asymmetry in probe transmission can be exploited to infer the value of β for semiconductor lasers.

In the case of finite facet reflectivities, new qualitative features may arise from the intracavity nature of NDFWM.[8–11] When the semiconductor laser operates below threshold, the intracavity pump intensity in general exhibits bistability.[9] As a result, both R and T can become bistable under appropriate conditions. Similar to the case of intracavity degenerate four-wave mixing,[9] the bistable behavior is sensitive to the cavity detuning and other related parameters. Such a bistable behavior in semiconductor laser amplifiers has been observed experimentally.[10] When the semiconductor laser operates above threshold, the bistability disappears since the intracavity pump intensity is a single-valued function of the device current I [see Eq. (15)]. However, the intracavity nature of NDFWM modifies the spectra of both R and T. These spectra then exhibit an additional peak in the wings[2,3] that is related to the relaxation–oscillation characteristics of the semiconductor laser. The formalism presented here can be used for a quantitative study of these features with the inclusion of the appropriate boundary conditions at the laser facets.

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