Temporal Analog of Reflection and Refraction at a Temporal Boundary

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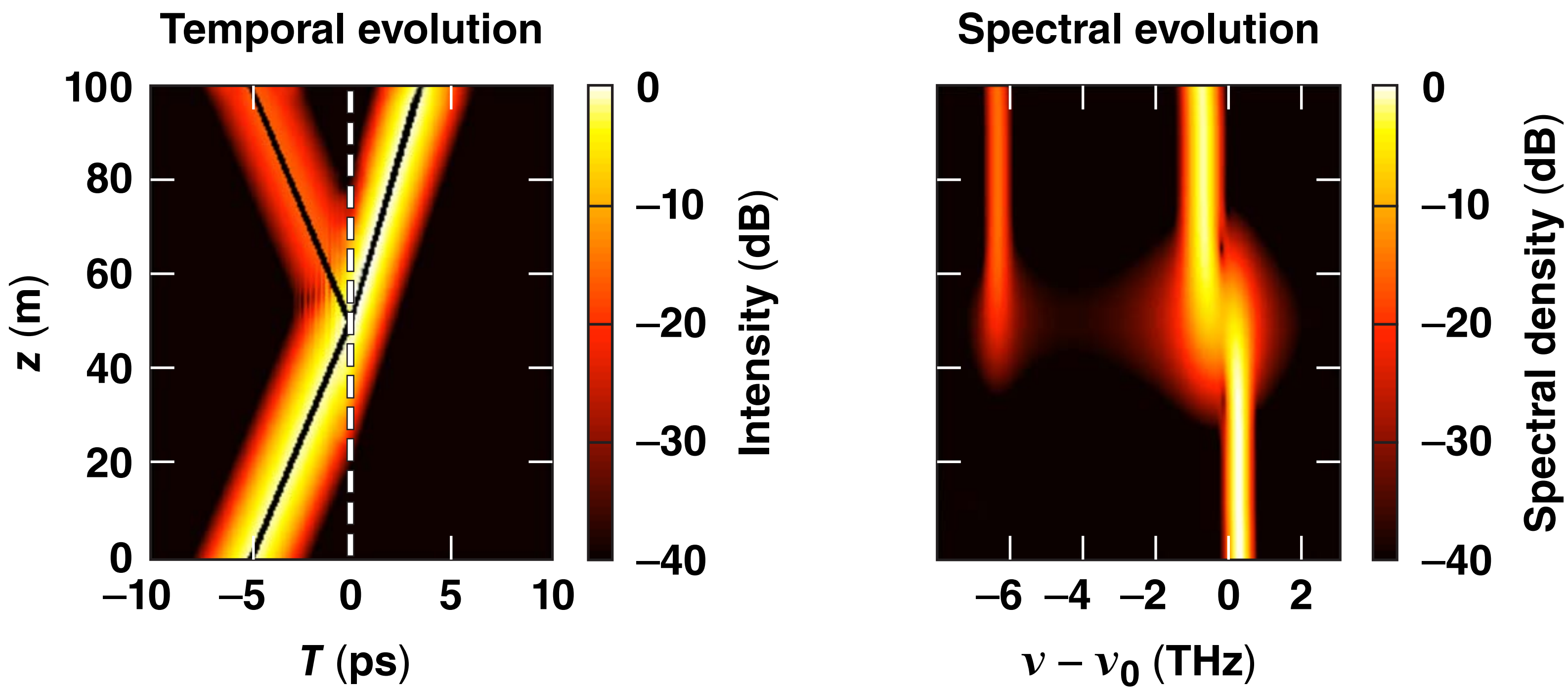

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Summary

A temporal analog of reflection and refraction occurs when a pulse encounters a moving temporal boundary

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- A moving temporal boundary breaks translational symmetry in time and space, allowing both photon momentum and energy to change
- We performed numerical simulations to demonstrate that such a moving boundary produces temporal reflection and refraction
- We derive temporal analogs for Snell's laws of reflection and refraction based on the conservation of photon momentum in the moving frame
- Using these equations, a temporal analog of total internal reflection was found and demonstrated using numerical simulations
- We are developing an experiment to verify these results using a traveling-wave phase modulator to produce the moving refractive index boundary

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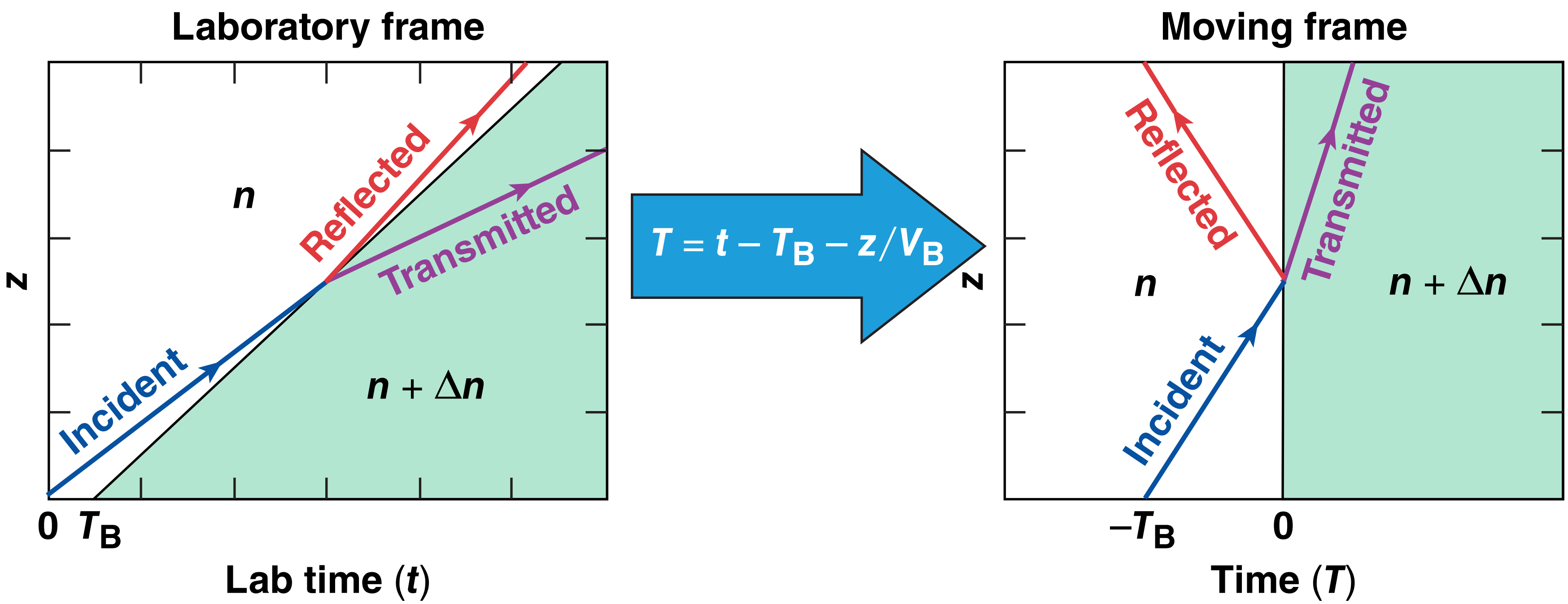
From a physics perspective, a refractive index boundary breaks translational symmetry

- A spatial boundary breaks translational symmetry in space, which requires the photon energy (ω) to be conserved while its momentum (θ) changes
- A temporal boundary breaks translational symmetry in time, which requires the photon momentum (β) to be conserved while its energy (ω) changes
- From this comparison, we see that the frequency of the optical pulse is the temporal analog of the angle in space
- If we allow for a moving temporal boundary, we expect both energy and momentum to change

A moving temporal boundary is a refractive index change that propagates through a dispersive medium



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A change in refractive index in time shifts the dispersion relation of the material

- An index change of Δn will shift the dispersion relation by $\beta_B = k_0 \Delta n$
- We can Taylor expand the dispersion relation in the moving frame to give

$$\beta(\omega) = \beta_0 + \Delta\beta_1 (\omega - \omega_0) + \frac{\beta_2}{2} (\omega - \omega_0)^2 + \beta_B H(T)$$

- The final term vanishes for $T < 0$ and has value β_B for $T > 0$
- Using this dispersion relation together with Maxwell's equations leads to the time-domain equation for the pulse envelope $A(z, T)$

$$\frac{\partial A}{\partial z} + \Delta\beta_1 \frac{\partial A}{\partial T} + i\frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} = i\beta_B H(T) A$$

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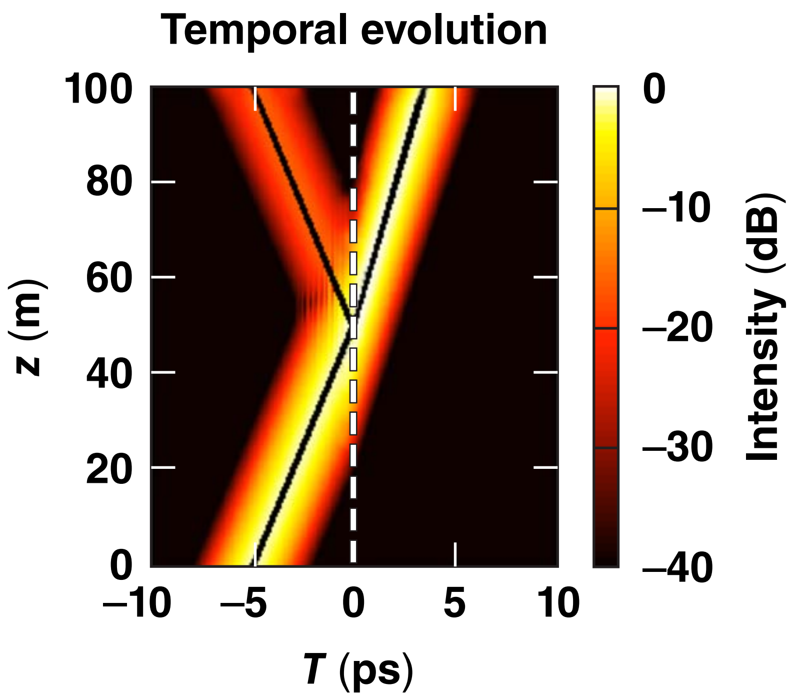
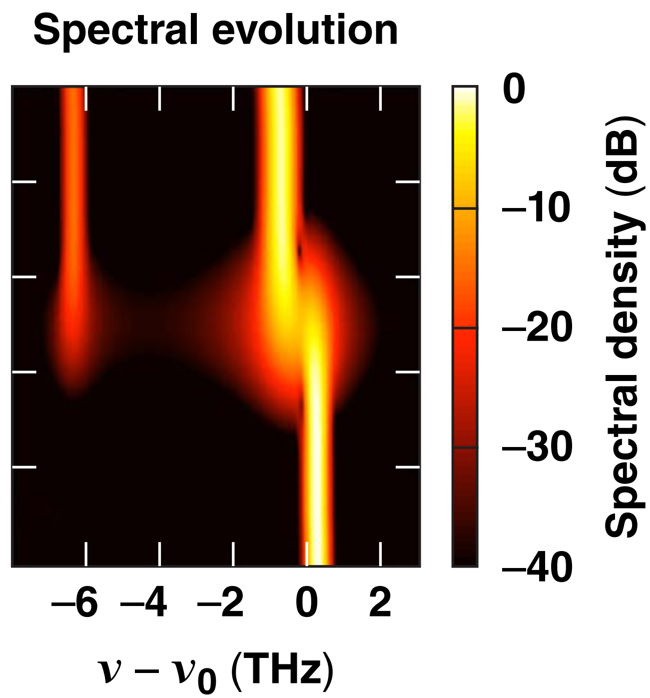
The temporal evolution is strikingly similar to an optical beam hitting a spatial boundary





$\beta_B = 0.5\ m^{-1}$ $\Delta\beta_1 = 0.1$ ps/m $\beta_2 = 0.005\ ps^2/m$ $\Delta n = 8 \times 10^{-8}$

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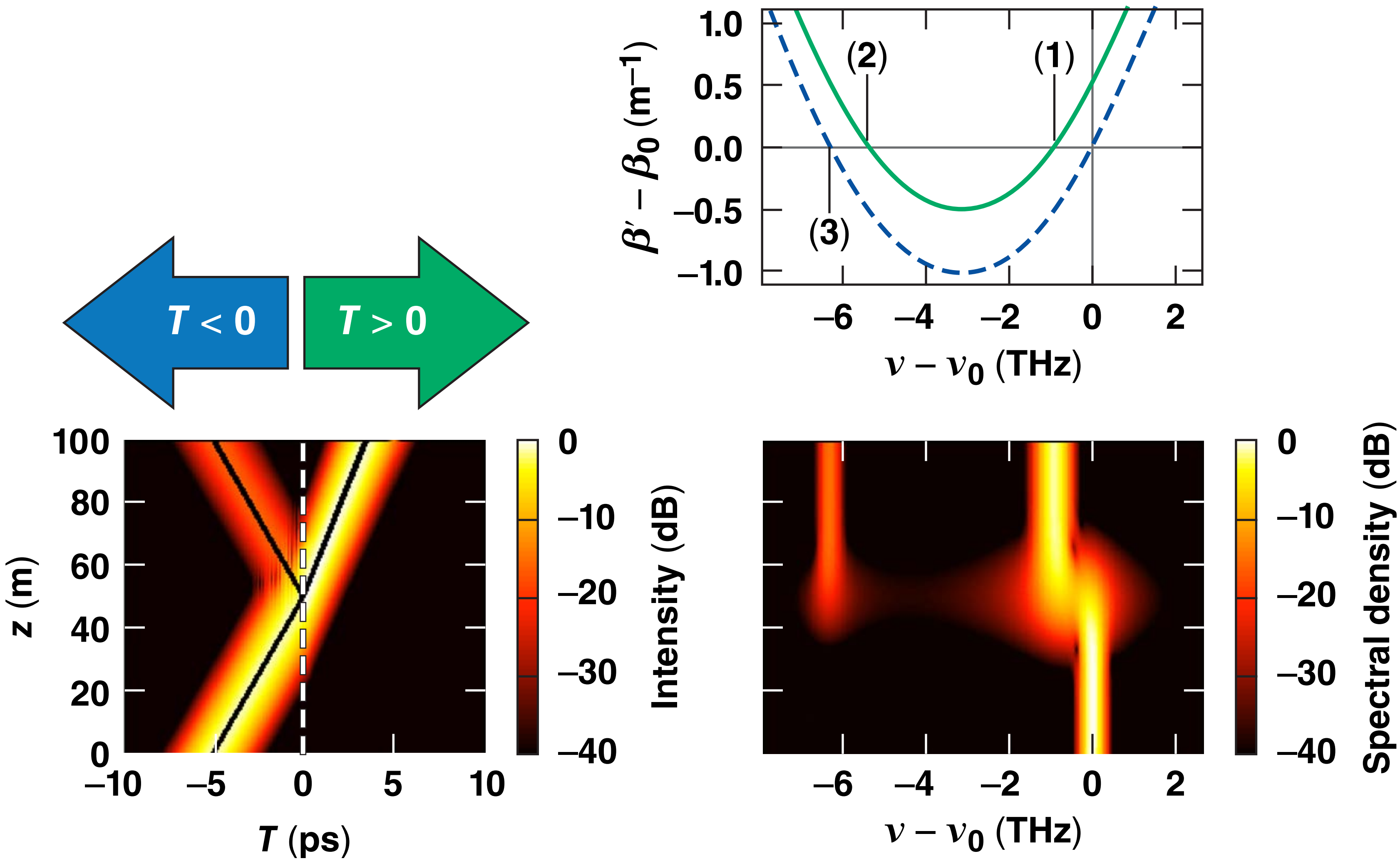
The dispersion relation in the moving frame can be used to explain the observed frequency shifts



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Solving for point (1) and point (3), we can find equations analogous to Snell's laws

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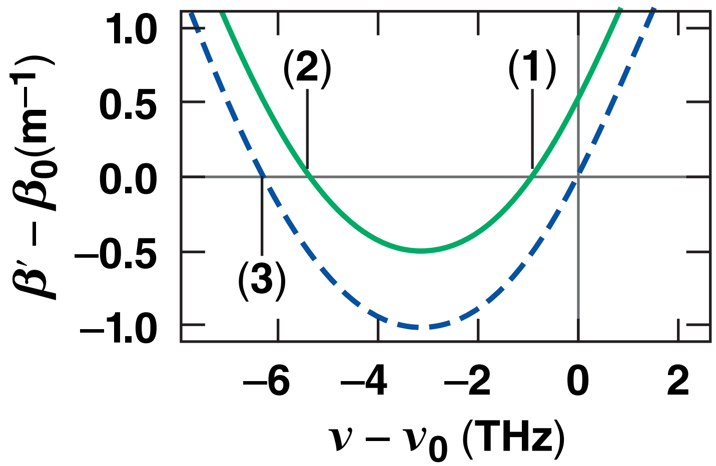
- Setting $\beta = \beta_0$ we find the reflected and transmitted frequencies

$$\omega_r = \omega_0 - 2\frac{\Delta\beta_1}{\beta_2},$$

$$\omega_t = \omega_0 + \frac{\Delta\beta_1}{\beta_2}\left[-1 + \sqrt{1 - \frac{2\beta_B\,\beta_2}{(\Delta\beta_1)^2}}\right]$$



- By shifting the reference frequency to $\omega_c = \omega_0 - (\Delta\beta_1/\beta_2)$ and using the notation $\Delta\omega = (\omega - \omega_c)$ we find the temporal analogs to Snell's laws

$$\Delta\omega_r = -\Delta\omega_0 \qquad \Delta\omega_t = \Delta\omega_0\sqrt{1 - \frac{2\beta_B\beta_2}{(\Delta\beta_1)^2}}$$

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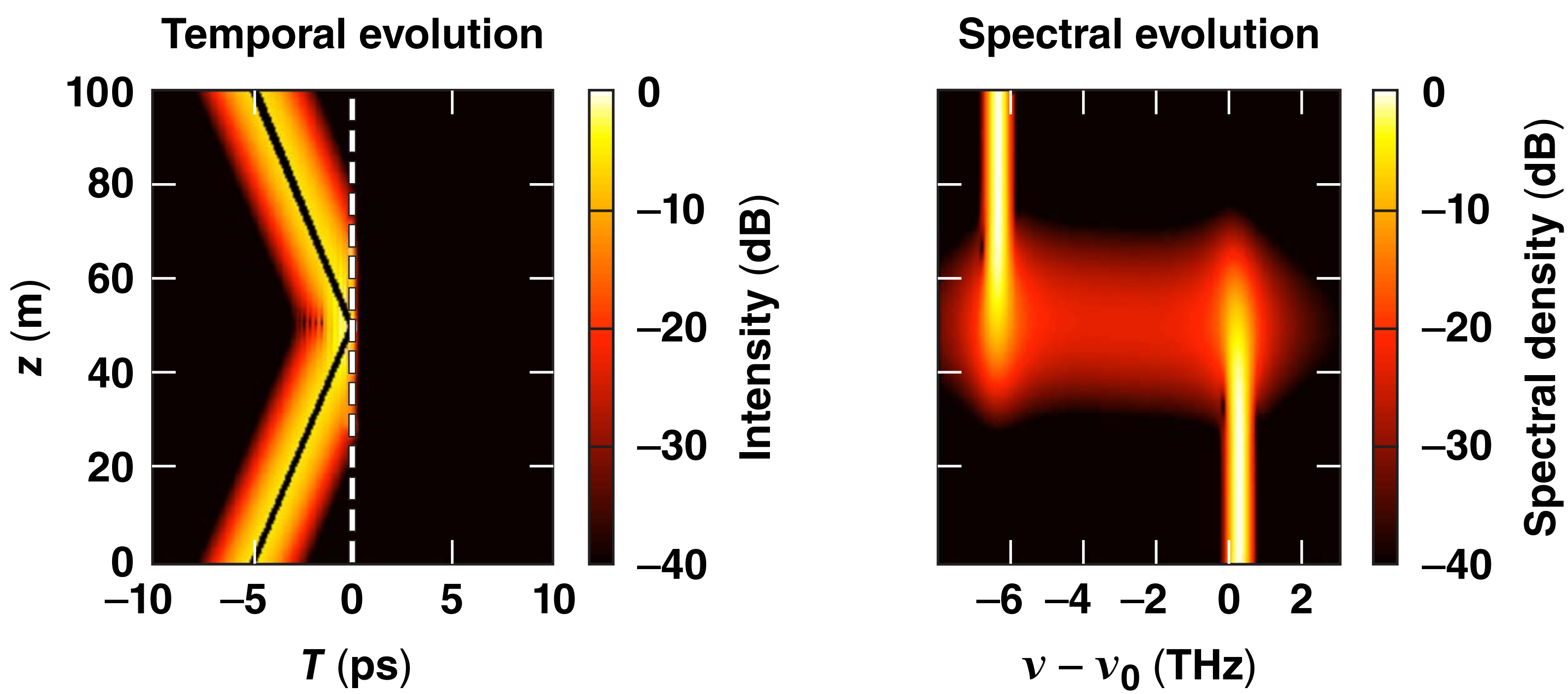
When β_B is large, the transmitted frequency can no longer propagate

$$\Delta\omega_t = \Delta\omega_0 \sqrt{1 - \frac{2\beta_B \beta_2}{(\Delta\beta_1)^2}}$$ becomes complex



$\beta_B = 1.5\ \text{m}^{-1}$ $\Delta\beta_1 = 0.1$ ps/m $\beta_2 = 0.005\ \text{ps}^2/\text{m}$ $\Delta n = 2.4 \times 10^{-7}$

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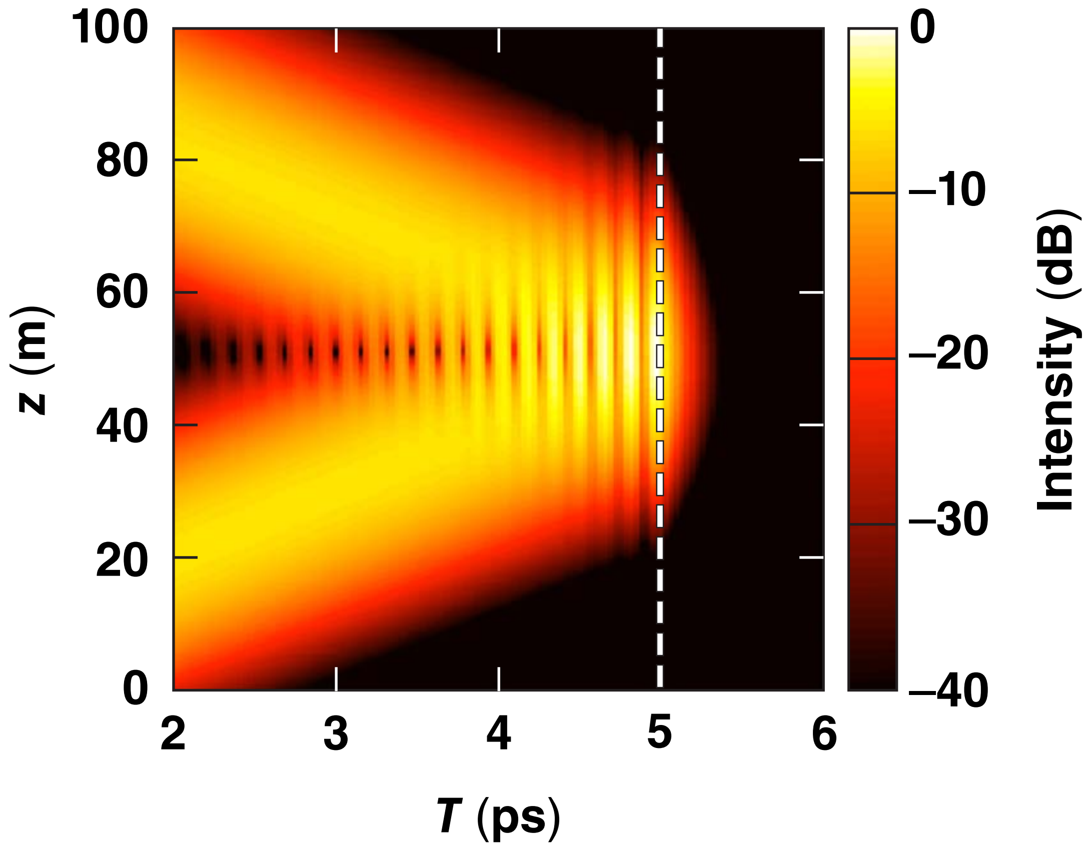
Looking closer at the reflecting pulse, a temporal analog of the evanescent wave can be seen



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We have imposed momentum conservation, even though a moving boundary can change both photon momentum and energy

- Momentum was only conserved in the moving frame where the dispersion relation is given by

$$\beta(\omega) = \beta_0 + \Delta\beta_1 (\omega - \omega_0) + \frac{\beta_2}{2} (\omega - \omega_0)^2$$

where $\Delta\beta_1 = \beta_1 - 1/V_B$ is a measure of the difference in speed

- In the lab frame, the dispersion relation is given by

$$\beta'(\omega) = \beta_0 + \beta_1 (\omega - \omega_0) + \frac{\beta_2}{2} (\omega - \omega_0)^2$$

- Comparing these equations we find that $\beta' = \beta + \frac{\omega - \omega_0}{V_B}$

- Even though β is constant, β' changes because the frequency changes

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